Cancellation of Nonlinearities Using Indirect Input Measurements

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Summary

Dynamic network concepts can be applied to complex systems that are not networks in the traditional sense, for instance, by considering that each degree of freedom of a mechanical system is a module in a network. However, unlike the common assumption of linearity in dynamic network literature, many mechanical systems exhibit some kind of nonlinear behavior. Here, some results for cancellation of these nonlinearities using indirect input measurements are presented. The resulting model is linear in the inputs and has a larger operational domain than a model based on linearization techniques.

Dynamics of a Furuta Pendulum

The dynamics of a Furuta pendulum can be described by

$$\boldsymbol{M}(\theta_2)\begin{bmatrix}\ddot\theta_1\\ \ddot\theta_2\end{bmatrix} = \begin{bmatrix} -b_1\dot\theta_1 + m_2 l_2 L_1 \sin(\theta_2)\dot\theta_2^2 - I_2\sin(2\theta_2)\dot\theta_1\dot\theta_2 + \tau_1 + \delta_1 \\ -b_2\dot\theta_2 - m_2 l_2 g\sin(\theta_2) + \frac{I_2}{2}\sin(2\theta_2)\dot\theta_1^2 + \tau_2\end{bmatrix} \quad (1)$$

where θ_i and τ_i are the angle and the disturbance acting around the i^{th} joint, respectively, δ_1 is the user-controllable torque around the first joint and the parameters are defined in the figure below. The system inertia matrix has full rank for $\theta_2 \in \mathbb{R}$ and is given by

$$\boldsymbol{M}(\theta_2) = \begin{bmatrix} I_0 + I_2\sin^2(\theta_2) & m_2 l_2 L_1\cos(\theta_2) \\ m_2 l_2 L_1 \cos(\theta_2) & I_2 \end{bmatrix},$$

where $I_0 = J_1 + m_1 l_1^2 + m_2 L_1^2$ and $I_2 = J_2 + m_2 l_2^2$.

Inertial Measurement Unit (IMU)

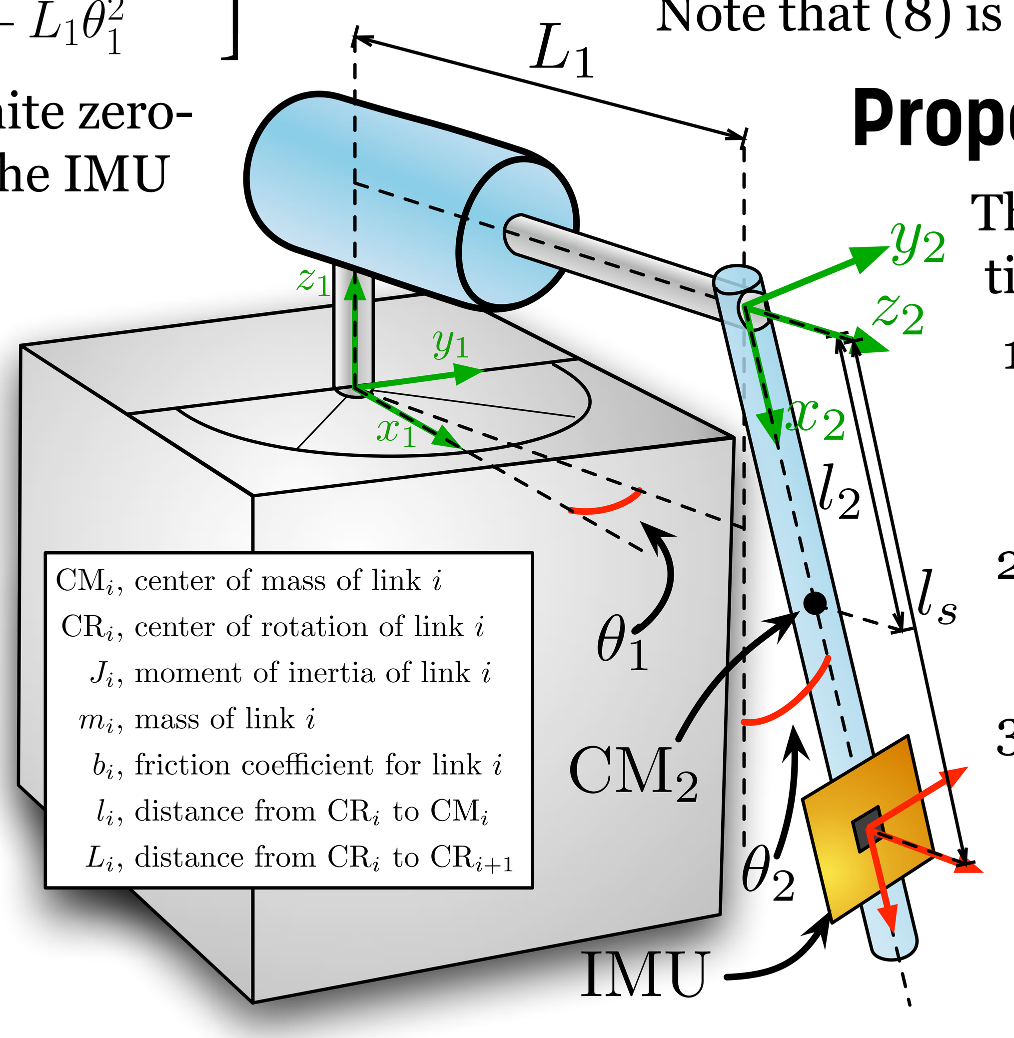
An IMU rigidly mounted to the second link in the x_2–y_2–plane will measure the acceleration

$$\boldsymbol{a} = \begin{bmatrix} -l_s\sin^2(\theta_2)\dot\theta_1^2 - l_s\dot\theta_2^2 + L_1\sin(\theta_2)\ddot\theta_1 - g\cos(\theta_2) \\ l_s\ddot\theta_2 + L_1\cos(\theta_2)\ddot\theta_1 - \frac{l_s}{2}\sin(2\theta_2)\dot\theta_1^2 + g\sin(\theta_2) \\ -l_s\sin(\theta_2)\ddot\theta_1 - 2l_s\cos(\theta_2)\dot\theta_1\dot\theta_2 - L_1\dot\theta_1^2 \end{bmatrix} + \boldsymbol{e}_a \quad (2)$$

where $\boldsymbol{e}_a$ is a vector of independent white zero-mean Gaussian measurement noise. The IMU also measures the angular velocities

$$\boldsymbol{\omega} = \begin{bmatrix} -\cos(\theta_2)\dot\theta_1 \\ \sin(\theta_2)\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \boldsymbol{e}_\omega \quad (3)$$

where $\boldsymbol{e}_\omega$ is a vector of independent white zero-mean Gaussian measurement noise.



Submodel of the Second Link

Assume that it is desired to track changes in the second link's dynamics using data. Two natural options are a nonlinear model

$$\dot\theta_2 = \mathcal{M}(\delta_1, \tau_1, \tau_2) \qquad \text{– highly nonlinear}$$

or a linearized model

$$\Delta\dot\theta_2 = G(\mathrm{p})\delta_1 + H(\mathrm{p})\begin{bmatrix}\tau_1 & \tau_2\end{bmatrix}^T \qquad \text{– limited operation range}$$

An alternative is to look at the system as a **dynamic network** and model a part of the system. A natural choice would be to use the joint angles as variables in the network representation. However, it turns out that it is beneficial to use alternative variables.

Consider the second row of (1) given by

$$I_2\ddot\theta_2 = -b_2\dot\theta_2 - m_2 l_2 g\sin(\theta_2) - m_2 l_2 L_1 \cos(\theta_2)\ddot\theta_1 + \frac{I_2}{2}\sin(2\theta_2)\dot\theta_1^2 + \tau_2 \quad (4)$$

which would be a nonlinear second-order system in θ_2 if the orange marked terms would be known. Let us introduce the signals

$$u_1 = \cos(\theta_2)\ddot\theta_1, \quad u_2 = \sin(\theta_2)\dot\theta_1 \quad \text{and} \quad u_3 = \cos(\theta_2)\dot\theta_1 \quad (5)$$

Then noting that $\sin(2x) = 2\sin(x)\cos(x)$, (4) can be written

$$I_2\ddot\theta_2 = -b_2\dot\theta_2 - m_2 l_2 g\sin(\theta_2) - m_2 l_2 L_1 u_1 + I_2 u_2 u_3 + \tau_2 \quad (6)$$

The signals $u_i, i = 1, 2, 3,$ are unknown but note that u_2 and u_3 are **directly measured** by the IMU. Furthermore, u_1 is **indirectly measured** since the second row of (2) can be written as

$$a_2 = l_s\ddot\theta_2 + g\sin(\theta_2) + L_1 u_1 - l_s u_2 u_3 \quad (7)$$

Solving (7) for u_1 and inserting into (6) gives the **indirect model**

$$(I_2 - m_2 l_2 l_s)\ddot\theta_2 = -b_2\dot\theta_2 - m_2 l_2 a_2 - (I_2 - m_2 l_2 l_s)\omega_1\omega_2 + \tilde\tau_2 \quad (8)$$

where u_2 and u_3 have been replaced with their measurements. Note that (8) is a linear first order model from a_2 and $\omega_1\omega_2$ to $\dot\theta_2$.

Properties of the Indirect Model

The indirect model (8) has three interesting properties worth pointing out.

1. The disturbances are non-additive since

$$\tilde\tau_2 = \tau_2 + m_2 l_2 e_{a_2} + (I_2 - m_2 l_2 l_s)(\omega_1 e_{\omega_2} + e_{\omega_1}\omega_2 + e_{\omega_1} e_{\omega_2})$$

2. The external signals τ_1 and δ_1 provide upstream excitation to the signals of the model.
3. The model is correct for all $\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} \in \mathbb{R}^2$.

Estimation of the Indirect Model

The indirect model (8) can be discretized using the bilinear transform which results in a discrete-time linear predictor model. The instrumental variable (IV) method is one method that can be used to estimate the parameters. The estimate is the solution to

$$\hat\vartheta = \underset{\vartheta}{\operatorname{argmin}} \sum_{k=1}^{K} \frac{1}{N_k} \left\| \begin{bmatrix}\zeta_1^k & \dots & \zeta_{N_k}^k\end{bmatrix} \left(\begin{bmatrix} (\varphi_1^k)^T \\ \vdots \\ (\varphi_{N_k}^k)^T \end{bmatrix} g_k(\vartheta) - \begin{bmatrix} \omega_{3,1}^k \\ \vdots \\ \omega_{3,N_k}^k \end{bmatrix} \right) \right\|^2 \quad (9)$$

where ϑ are the physical parameters, φ_t^k is the regression vector, $g_k(\vartheta)$ is a nonlinear transformation and ζ_t^k is the instrument vector. Furthermore, K datasets are used to gain identifiability.

Assume that the data is collected while using the control law

$$\delta_1 = -L_\theta\begin{bmatrix}\theta_1 & \theta_2 & \dot\theta_1 & \dot\theta_2\end{bmatrix}^T + L_r r$$

where L_θ and L_r are of suitable dimensions and r is the user-controllable reference angle for θ_1. A natural choice of instruments in closed-loop IV for linear systems is to use

$$\zeta_t^k = \begin{bmatrix} r_t^k & \dots & r_{t-n_r}^k \end{bmatrix}^T \quad (10)$$

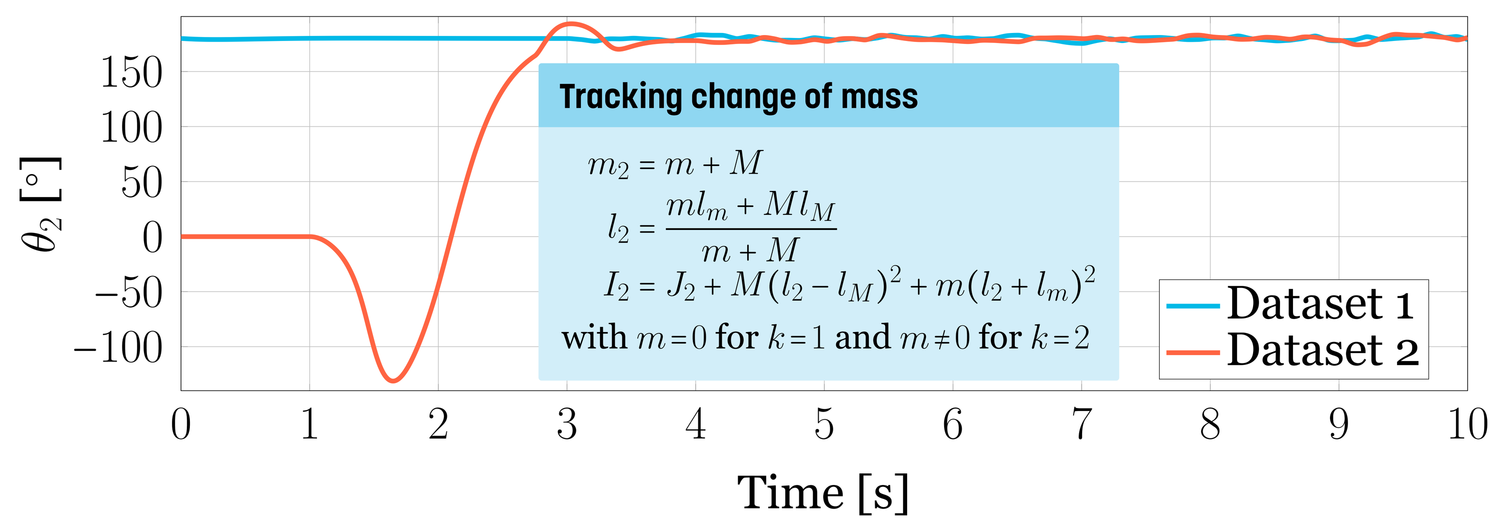
This also seems to be a reasonable choice for this system since

$$E r\tilde\tau_2 = E r\big(E\tau_2 + c_1\underbrace{E e_{a_2}}_{0} + c_2\underbrace{E e_{\omega_1} E e_{\omega_2}}_{0}\big) + c_2 E r\omega_1 \underbrace{E e_{\omega_2}}_{0} + c_2\underbrace{E e_{\omega_1}}_{0} E r\omega_2$$

and thus that the instruments are uncorrelated with $\tilde\tau_2$ if $E r E\tau_2 = 0$. Moreover, due to strong coupling, r correlates with the states of (1). Finally, the transformations from the states to u_1 and $u_2 u_3$ are non-even functions and hence, r should correlate with the regressors.

An example for tracking of the change in mass of the second link is seen below. Two datasets were used, one collected during swing-up and the other around the unstable equilibrium.

Parameter	J_2	b_2	m	l_m
True	0.00385	0.00031	0.03077	0.08734
Est.	0.00393	0.00030	0.03000	0.09000
Rel. Err.	1.81%	1.84%	2.57%	2.95%



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