TOPSIS method with a pool of distance measures

David Koloseni

University of Dar es salaam, Tanzania

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Introduction

- Technique for order preference by similarity to an ideal solution (TOPSIS) belongs to the classical multi-criteria decision making (MCDM) methods for solving the decision making problems.
- TOPSIS method relied on the idea that the chosen alternative should have the farthest distance from the negative ideal point and the shortest distance from the positive ideal point concurrently.
- In this paper a TOPSIS method with a pool of distance measures is created and applied in calculating the distances between the ideals solutions, instead of a single distance measure.

- In applying TOPSIS method, different distance measure gives different ranking of the alternatives for each criterion.
- The proposed TOPSIS method allows decision makers to obtain the best ranking by proper aggregation of the closeness coefficients of each the alternatives from each distance measure.
- The proposed modified TOPSIS algorithm saves computations cost, time, effort, facilitate the preference elicitation and solves the problem of trial and error in choosing the distance measure which can be the best in the decision making process.

Pool of Distance measures

$$d_1(x,y) = \left(\sum_{i=1}^{n} |x_i - y_i|^2\right)^{\frac{1}{2}} \quad (1)$$

$$d_2(x,y) = \sum_{i=1}^{n} |x_i - y_i| \quad (2)$$

$$d_3(x,y) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{\max\{|x_i|, |y_i|\}} \quad (3)$$

$$d_4(x,y) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{/1 + \min\{|x_i|, |y_i|\}} \quad (4)$$

$$d_5(x,y) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{1 + \max\{|x_i|, |y_i|\}} \tag{5}$$

$$d_6(x,y) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{\{1 + |x_i| + |y_i|\}} \tag{6}$$

$$d_7(x,y) = \sum_{i=1}^{n} \left| \frac{x_i}{1 + |x_i|} - \frac{y_i}{1 + |y_i|} \right| \tag{7}$$

$$d_8(x, y) = \sum_{i=1}^{n} (x_i - y_i) \times log(\frac{x_i}{y_i}) \tag{8}$$

$$d_9(x, y) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{(1 + |x_i|) \times (1 + |y_i|)} \tag{9}$$

Some Definitions

Definition

A fuzzy measure on the set X of criteria is a set function $\mu : P(X) \to [0, 1]$ satisfying the following axioms:

i. $\mu(\emptyset) = 0$

ii. $A \subset B$ implies $\mu(A) \leq \mu(B)$ where $\mu(A)$ represents the weight of importance of the set of criteria A.

A fuzzy measure is said to be additive if $\mu(A \cup B) = \mu(A) + \mu(B)$ whenever $(A \cap B) = \emptyset$. Also, supper additive if $\mu(A \cup B) \geq \mu(A) + \mu(B)$ and subadditve if $\mu(A \cup B) \leq \mu(A) + \mu(B)$.

Definition

Let μ be a fuzzy measure on X. The sugeno integral of a function $f : X \to [0, 1]$ with respect to μ is defined by:

$$S_\mu(f(x_{(1)}), f(x_{(2)}), \cdots, f(x_{(n)})) = max(min(f(x_{(i)}), \mu A_{(i)})) \quad (10)$$

where $\cdot_{(i)}$ indicates that the indices have been permuted so that $0 \leq f(x_1) \leq f(x_2) \leq \cdots \leq f(x_n) \leq 1$ and $A_{(i)} := \{x_{(i)}, \cdots, x_{(n)}\}$

Proposed Method

The procedures of TOPSIS can be expressed in the following steps.

1. Obtain a decision matrix for $A = (a_{nk})$ ranking with n alternatives over k criteria.
2. Raw measurements are usually normalized, converting raw measures a_{nk} into normalized measures s_{nk}.

$$s_{nk} = \frac{a_{nk}}{\sqrt{\sum_{n=1}^{k} a_{nk}^2}} \tag{11}$$

3. Develop a set of importance weights w_k, for each of the criteria where

$$\sum_{k=1}^{n} w_k = 1. \tag{12}$$

4. Calculate the weighted normalized decision matrix. The weighted normalized value

$$v_{nk} = s_{nk} \times w_k \tag{13}$$

5. Determine positive and negative ideal alternatives respectively

$$\begin{aligned} A^+ &= \{\{\max_n v_{nk} | k \in K\}, \{\min_n v_{nk} | k \in K'\}\} | n = 1, 2, ..., m\} \\ &= \{v_1^+, v_2^+, ..., v_m^+\} \end{aligned} \tag{14}$$

$$\begin{aligned} A^- &= \{\{\min_n v_{nk} | k \in K\}, \{\max_n v_{nk} | k \in K'\}\} | n = 1, 2, ..., m\} \\ &= \{v_1^-, v_2^-, ..., v_m^-\} \end{aligned} \tag{15}$$

6. Calculate the separation measures of each alternative from the ideal solution using the distance measures in equation 1 to 9. whereby

$$d^{+}(v_i^{+}, v) \tag{16}$$

calculates the shortest distance from the positive ideal point and

$$d^{-}(v_i^{-}, v) \tag{17}$$

calculates the farthest distance from the negative ideal point.

7. Calculate the relative closeness to the ideal solution.

$$CC = \frac{d^+}{d^+ + d^-} \tag{18}$$

8. Rank the preference order for each distance measure. The best alternative is the one with the greatest relative closeness to the ideal solution.
9. The best rank is obtained by aggregating the closeness coefficient using sugeno fuzzy integral.

More on Fuzzy measure and Fuzzy Integrals

- Fuzzy measure represents the background knowledge on the information sources.
- Typically, fuzzy measure represents the importance or relevance of the sources when computing the aggregation.
- Fuzzy integrals combine the data supplied by several information sources according to a fuzzy measure.
- Sugeno integral is a natural operator to combine the conclusions of several rules in a fuzzy rule based system when such rules are not independent.

Numerical Example

Suppose the criteria for evaluating car for a certain company use are represented by speed, buying cost, fuel consumption, engine capacity, number of passengers, and availability of spare parts. These criteria are named as criteria1, criteria2, criteria3, criteria4, criteria5 and criteria6 respectively. Let the twelve proposed car's alternatives in short form 'Alt' and their corresponding evaluation ratings be described as shown in Table 1.

Table: Decision matrix

	Criteria1	Criteria2	Criteria3	Criteria4	Criteria5	Criteria6
Alt1	9	6	7	7	6	9
Alt2	6	8	2	8	6	1
Alt3	4	2	10	8	9	4
Alt4	7	2	5	3	9	5
Alt4	8	4	4	9	9	4
Alt6	2	9	5	10	7	6
Alt7	6	9	9	5	9	7
Alt8	7	7	9	6	7	8
Alt9	6	6	8	2	2	9
Alt10	9	6	2	1	4	10
Alt11	8	5	1	5	5	10
Alt12	1	4	4	7	6	6

The normalized decision matrix of the preferred ratings in the interval $[0, 1]$ using the vector normalization.

Table: Normalized decision matrix

	Criteria1	Criteria2	Criteria3	Criteria4	Criteria5	Criteria6
Alt1	0.4143	0.2771	0.3300	0.2909	0.2634	0.3659
Alt2	0.2762	0.3694	0.0943	0.3325	0.2634	0.0407
Alt3	0.1841	0.0924	0.4714	0.3325	0.3951	0.1626
Alt4	0.0921	0.2309	0.1414	0.3740	0.2195	0.2033
Alt5	0.3682	0.1847	0.1886	0.3740	0.3951	0.1626
Alt6	0.0921	0.4156	0.2357	0.4156	0.3073	0.2439
Alt7	0.2762	0.4156	0.4243	0.2078	0.3951	0.2846
Alt8	0.3222	0.3232	0.4243	0.2494	0.3073	0.3252
Alt9	0.2762	0.2771	0.3771	0.0831	0.0878	0.3659
Alt10	0.4143	0.2771	0.0943	0.0416	0.1756	0.4066
Alt11	0.3682	0.2309	0.9471	0.2078	0.2195	0.4066
Alt12	0.0460	0.1847	0.1886	0.2909	0.2634	0.2439

Table: CC and Ranking

	dist1		dist2		dist3		dist4		dist5		dist6		dist7		dist8		dist9		
	CC	R	CC	R	CC	R	CC	R	CC	R	CC	R	CC	R	CC	R	CC	R	Overall Ranking
Alt1	0.2410	12	02513	12	0.2373	12	0.2502	12	0.2593	12	0.2523	12	0.2516	12	0.0672	12	0.2791	12	12
Alt2	0.5148	5	0.5364	4	0.4425	3	0.5294	4	0.5301	4	0.5245	4	0.5236	4	0.4996	3	0.5578	4	4
Alt3	0.5310	4	0.4741	5	0.3641	6	0.4672	5	0.4631	5	0.4576	5	0.4565	5	0.3673	5	0.5028	5	5
Alt4	0.7712	1	0.7383	1	0.4567	1	0.7343	1	0.7230	1	0.7194	1	0.7188	1	0.7252	1	0.7574	1	1
Alt5	0.3000	10	0.3272	8	0.3176	9	0.3241	8	0.3306	8	0.3258	8	0.3252	8	0.1831	8	0.3480	8	8
Alt6	0.6578	2	0.5735	2	0.3232	8	0.5677	2	0.5542	2	0.5491	2	0.5483	2	0.5188	2	0.6038	2	2
Alt7	0.3722	7	0.2762	11	0.2142	12	0.2651	11	0.2667	11	0.2576	11	0.2561	11	0.1080	10	0.2960	11	11
Alt8	0.3395	9	0.3147	9	0.2490	10	0.3027	10	0.3077	10	0.2976	10	0.2962	10	0.0914	11	0.3320	10	10
Alt9	0.5561	3	0.5508	3	0.4206	4	0.5433	3	0.5408	3	0.5345	3	0.5336	3	0.4394	4	0.5747	3	3
Alt10	0.3650	8	0.4163	7	0.4457	2	0.4129	7	0.4220	7	0.4190	7	0.4187	7	0.3420	6	0.4429	7	7
Alt11	0.3919	6	0.4364	6	0.4054	5	0.4301	6	0.4370	6	0.4317	6	0.4311	6	0.3477	6	0.4599	6	6
Alt12	0.2665	11	0.3139	10	0.3318	7	0.3120	9	0.3240	9	0.3219	9	0.3220	9	0.1640	9	0.3368	9	9

Next move is how to construct fuzzy measures

- Construct the correlation matrix of the closeness coefficient from each distance measure
- Find eigenvalues and eigenvectors from the correlation matrix. and select the eigenvector with the highest eigenvalue.
- The selected eigenvector will contain nine fuzzy measures for each closeness coefficient.
- Calculate the degree of interaction by using the formula:

$$(\lambda + 1) = \prod_{i=n}^{n} (1 + \lambda * g_i) \qquad (19)$$

THANKS!