Power-type Quasiminimizers

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Power-type Quasiminimizers on weighted $\Re^n$

Definition

Let $\Omega \subset \Re^n$ be a non-empty open set and $1 < p < \infty$. A function $u \in W^{1,p}_{loc}(\Omega)$ is a Q-quasiminimizer, $Q \geq 1$, in Ω if

$$\int_{\varphi \neq 0} |\nabla u|^p dx \leq Q \int_{\varphi \neq 0} |\nabla (u + \varphi)|^p dx \tag{1}$$

for all $\varphi \in W^{1,p}_0(\Omega)$.

This definition could be extended to Q-quasiminimizers on weighted $\Re^n$.

The interest in this, is to find the best Q- quasiminimizer constant for radial symmetric functions of the power type (say $|x|^{\alpha}$) with Ω a unit ball in $\Re^{n}$.

Some properties of the quasiminimizers

- If $Q = 1$ then the 1-quasiminimizer is a minimizer and is a weak solution to the p-Dirichlet problem on the LHS of (1).
- Minimizers and quasiminimizers differ as being a minimizer is a local property while being a quasiminimizer is not a local property.
- Quasiminimizers of interest are the Power-type i.e of the form $|x|^{\alpha}$. These functions are upto a constant multiple fundamental solutions to the p-Laplace operator, $\triangle_p$.

p-admissible weights

Definition

The weight w or μ is p-admissible if the following four conditions are satisfied.

(a) $0 < w < \infty$ almost everywhere in $\Re^n$ and the measure μ is doubling.

(b) If D is an open set and $\phi_i \in C^\infty(D)$ is a sequence of functions such that $\int_D |\phi_i|^p d\mu \to 0$ and $\int_D |\nabla \phi_i - v|^p d\mu \to 0$ as $i \to \infty$ then $v = 0$.

(c) There are constants $\gamma > 1$ and $C_1 > 0$ such that

$$\left(\frac{1}{\mu(B)} \int_B |\phi|^{\gamma p} d\mu \right)^{\frac{1}{\gamma p}} \leq C_1 r \left(\frac{1}{\mu(B)} \int_B |\phi|^p d\mu \right)^{\frac{1}{p}}.$$

whenever $B = B(x_0, r)$ is a ball in $\Re^n$.

p-admissible weights

(d) There is a constant $C_2 > 0$ such that

$$\int_B |\phi - \phi_B|^p d\mu \leq C_2 r^p \int_B |\nabla \phi|^p d\mu$$

whenever $B = B(x_0, r)$ is a ball in $\Re^n$ and $\phi \in C^\infty(B)$ is bounded with $\phi_B = \frac{1}{\mu(B)} \int_B \phi d\mu$.

Some literature

Anders and jana in 2011 proved that;

Theorem

If $1 < p \neq n,\ \alpha \leq \beta = \frac{p-n}{p-1}$ and $u(x) = |x|^{\alpha}$. Then u is a Q-quasiminizer in $\mathbf{B} \backslash \{0\}$ and a Q-quasisuperharmonic function in $\mathbf{B}$, where

$$Q_{\alpha,n,p} = \left(\frac{\alpha}{\beta}\right)^{p} \frac{\beta p - p + n}{\alpha p - p + n}$$

is the best quasiminimizer constant.

Also results for $p = n$ were obtained.

Impact and Applications of My Research

Quasiminimizers act as tools in studying regularity of minimizers of variation integrals.

Thank you!