Corrections and clarifications to

*Nonlinear Potential Theory on Metric Spaces*,

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p. 6, l. – 7. Replace “ε-net” by “5ε-net”.

p. 21, l. 13–14. Replace “Then there exist upper gradients $g_j$ so that” by “Then there is $\tilde{\rho} \in L^p(X)$ such that $g_j := g + \tilde{\rho}/j$, $j = 1, 2, \ldots$, are upper gradients. If moreover $g < \infty$ a.e., then”.

p. 21, l. 20–21. Replace the last two sentences by “Let finally $\tilde{\rho} = \rho + \infty \chi_{g' \neq g}$ and $g_j = g + \tilde{\rho}/j$. Then $g_j$ is an upper gradient of $f$. If moreover $g < \infty$ a.e., then (1.6) holds.”

p. 36. Add the sentence “Proposition 1.14 is from Cheeger [91].”

p. 39, l. 9, 10. Insert “Assume that $\text{supp} \, \mu$ is locally compact.” before “If”.

p. 46, l. 20. Delete “minimal”.

p. 46, l. 21. Add the sentence “Moreover, $|\varphi'| \circ u|g_u$ is a minimal $p$-weak upper gradient of $\varphi \circ u$, provided that $\varphi$ is Lipschitz, $\varphi \circ u \in D^p(X)$ or $|\varphi'| \circ u|g_u \in L^p(X)$.”

p. 47, l. 1. After 2.14 add “where the last inequality is only required to hold if $\varphi \circ u \in D^p(X)$”.

p. 47, l. 8–9. Replace these lines by “Lemma 2.14 and (2.5) imply that $|\varphi'| \circ u|g_u$ is a $p$-weak upper gradient of $\varphi \circ u$. Hence $|\varphi'| \circ u|g_u \geq g_{\varphi \circ u}$ a.e. if $\varphi \circ u \in D^p(X)$, which in particular holds if $|\varphi'| \circ u|g_u \in L^p(X)$, which in turn is true if $\varphi$ is Lipschitz. To show the minimality in this case, observe that (2.5) and (2.4) yield”

p. 47, l. 17–18. Replace sentence by “Then $g_u/u$ is a $p$-weak upper gradient of $v = \log u$, which is minimal if $v \in D^p(X)$ or $g_u/u \in L^p(X)$.”

p. 61, l. 10. Insert “and $p > 1$” after “space”.

p. 62, l. 13, 15. To avoid confusion between $g_1$, $g_2$ and $g_j$, replace $g_j$ by $\tilde{g}_j$.

p. 64, l. 14–15. Replace sentence by “Lemma 2.37 for open $E$ appeared in [45].” (When [49] was finalized the proof of Lemma 2.37 was omitted, and so the book is the original reference for Lemma 2.37 in the general form.)

p. 88 l. – 3. Replace by

$$
\frac{1}{p} = \left( \int_B |u - u_B|^q \, d\mu \right)^{1/q} = \lim_{j \to \infty} \left( \int_B \min\{j, |u - u_B|^q\} \, d\mu \right)^{1/q} = \lim_{j \to \infty, k \to \infty} \left( \int_B \min\{j, |u_k - (u_k)_B|^q\} \, d\mu \right)^{1/q} \leq C \text{diam}(B) \left( \int_B g^p \, d\mu \right)^{1/p},
$$

Alternatively Fatou’s lemma can be used.

p. 107, l. – 2. Replace “complete” by “proper”.

p. 131, l. 5. Replace $u \geq \frac{1}{2}$ by $u(x) \geq \frac{1}{2}$.

p. 145, l. 15. Replace $\|g_u\|_{L^p(X)}$ by $\|g_u\|_{L^p(X)}$.

p. 145, l. – 8. Replace $C(r^p + 1)$ by $C(r^p + 1) \mu(2B)$.


p. 159, l. – 8. Insert “Assume that $\text{supp} \, \mu$ is locally compact.” before “If”.

p. 161, l. 1. Replace 6.7 (xi) by 6.19 (x).

p. 182, l. 7. Replace “open” by “bounded open”.

p. 246, l. – 12. Replace [46] by [45].

p. 259, l. 5. Insert “Let $h_j = \max\{f_j, \psi_j\}$. As $h_j - f_j = (\psi_j - f_j)_+$, $\in N^1_{p} (\Omega)$, by Proposition 7.4, we have $h_j \in \mathcal{K}_{\psi_j, f_j}$.” before “Let”.
p. 259, l. 9–10. Replace by
\[ \|w_j\|_{N^{1,p}(X)} \leq C\|g_{w,j}\|_{L^p(\Omega)} \leq C(\|g_{w,j}\|_{L^p(\Omega)} + \|g_{f,j}\|_{L^p(\Omega)}) \leq C(\|g_{h,j}\|_{L^p(\Omega)} + \|\psi_j\|_{N^{1,p}(\Omega)}) \leq C. \]

p. 259, l. 12. Replace by
\[ \|u_j\|_{N^{1,p}(\Omega)} \leq \|w_j\|_{N^{1,p}(\Omega)} + \|f_j\|_{N^{1,p}(\Omega)} \leq C(\|f_j\|_{N^{1,p}(\Omega)} + \|\psi_j\|_{N^{1,p}(\Omega)}) \leq C. \]

p. 278, l. 18 Replace “sup_{\partial B}” by sup_{\partial B} u”.

p. 345, l. 6 Replace “1 < p \leq n < q” by “1 < q \leq n < p”.

p. 345, l. 18–19 Replace R^n by R^2 twice.

p. 348, l. 9 Insert “l_\gamma \leq Ad(x,y)” and after “such that”.


p. 372, [56] Add “in metric spaces” after “functions”.