

ICTMA22



Conference Program and Booklet

Updated August 12, 2025



The 22nd
International
Conference on
Teaching of
Mathematical
Modelling and
Applications
(ICTMA22)



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Welcome to ICTMA22 in Linköping!

In 1989 the fourth International Conference on Teaching of Mathematical Modelling and Application (ICTMA4) was arranged in Roskilde in Denmark. For ICTMA22 in 2025 we extend an invitation to all researchers, teachers and mathematicians interested in the teaching and learning of mathematical modelling and applications to return to Scandinavia again – but this time to Sweden and the city of Linköping!





About ICTMA

The International Community of Teachers of Mathematical Modelling and Applications (the 'Community') is a membership organisation that exists to promote Applications and Modelling in all areas of mathematics education - primary and secondary schools, colleges and universities. The ICTMA conferences began in the first half of the 1980s and have been held more or less every other year since then:

ICTMA-1	1983	University of Exeter, England
ICTMA-2	1985	University of Exeter, England
ICTMA-3	1987	University of Kassel, Germany
ICTMA-4	1989	University of Roskilde, Denmark
ICTMA-5	1991	Freudenthal Institute, Netherlands
ICTMA-6	1993	University of Delaware, USA
ICTMA-7	1995	University of Ulster, Northern Ireland
ICTMA-8	1997	University of Queensland, Australia
ICTMA-9	1999	University of Lisbon, Portugal
ICTMA-10	2001	Tsinghua University, Beijing, China
ICTMA-11	2003	Marquette University, Milwaukee, USA
ICTMA-12	2005	City University, London, UK
ICTMA-13	2007	Indiana University, USA
		Kathmandu University, Nepal (Satellite Meeting)
ICTMA-14	2009	University of Hamburg, Germany
ICTMA-15	2011	Australian Catholic University, Australia
ICTMA-16	2013	Universidade Regional De Blumenau, SC, Brazil
ICTMA-17	2015	University of Nottingham, U.K.
ICTMA-18	2017	University of Stellenbosch, Cape Town, South Africa
ICTMA-19	2019	University of Hong Kong, Hong Kong
ICTMA-20	2022	University of Würzburg, Germany
ICTMA-21	2023	Yokohama National University, Japan

Learn more about the history of ICTMA here: [ICTMA, the first century](#).

Read more about ICTMA here: <https://www.ictma.net>.



Conference Theme for ICTMA22 – *Challenges and opportunities for mathematical modelling education*

In general, we welcome contributions to ICTMA22 that include, but are not limited to, theoretical, empirical, philosophical, and methodological topics related to the promotion, teaching, learning, and evaluation of mathematical modelling and application, such as the following:

- Pre-primary, primary and secondary school education
- Teacher and university education
- Technology, simulations and modelling
- Design of task, activities and courses focusing on modelling and applications
- Modelling competences
- Modelling assessment
- Problem-posing in modelling
- Creativity in modelling
- Modelling in STEM education and in other interdisciplinary settings
- Modelling in the 21st century
- Modelling in and for everyday life, work, and planet earth
- Modelling teaching and learning in the age of artificial intelligence

In an educational setting, challenges for mathematical modelling education offer us as researchers and teachers of mathematical modelling education, opportunities to reflect, to use creativity and to find new pathways for the future, not only for our students, teachers and for conducting research, but also for society at large. Challenges are opportunities, and as Winston Churchill famously put it: “*Never let a good crisis go to waste.*”. That is, challenges present opportunities to reflect upon, to be used creatively, and to find new perspectives and pathways to advance, learn and develop from.

For ICTMA22 we especially encourage participants and speakers to shed their light on the theme *challenges and opportunities in mathematical modelling education*, such as

- Challenges and Opportunities for implementing mathematical modelling in schools and curricula
- Challenges and Opportunities for mathematical modelling competitions
- Challenges and Opportunities for mathematical modelling education for our environment and planet earth
- Challenges and Opportunities for assessment of mathematical modelling
- Challenges and Opportunities for ethics in mathematical modelling
- Challenges and Opportunities for mathematical modelling education and digital tools
- Challenges and Opportunities for mathematical modelling in primary education
- Challenges and Opportunities for theorizing the teaching and learning of mathematical modelling
- Challenges and Opportunities for teaching and learning of mathematical modelling in the age of artificial intelligence

Conference organization

Local Organization Committee (LOC)

- Jonas Bergman Ärleback (Chair)
- Peter Frejd (co-chair)
- Madeleine Engström
- Malin Aurell
- Karin Johansson
- Ulrika Johansson
- Sara Julsgård
- Sara Låthén
- Cornelia Larsson
- Naira Nadeem
- Tim Nordström
- Marting Purfürst



Scientific Program

The ICMTA22 conference includes many types of scientific activities: keynote and plenary lectures, a panel discussion, long- and short oral communications in parallel sessions, and a poster session. In addition, it includes a new format called *Reflection fika*.

Keynote, plenaries and panel discussion

The scientific program includes one (1) 60-minute long keynote lecture followed by a 15-minute discussion, and four (4) 45-minute long plenary lectures followed by a 15-minute discussion:

Keynote, Monday August 11, 10:30-12:00, entitled *Perspectives on mathematical modelling education – quo vadis?* by **Professor Gabriele Kaiser** (University of Hamburg, Germany; East China Normal University in Shanghai, China).

Plenary, Tuesday August 12, 9:00-10:00, entitled *From an Imprecise Imprecision: Reflections on Culture and Modelling* by **Professor Daniel Orey** (Federal University of Ouro Preto, Brazil).

Plenary, Tuesday August 12, 13:30-14:30, entitled *Unpacking the Modelling Mindset in Teacher Education: A Behind-the-Scenes Journey to Empower Future Mathematics Teachers* by **Associate Professor Serife Sevinc** (Middle East Technical University, Turkey; TU. Dresden, Germany).

Dual plenary, Wednesday August 12, 9:00-10:00, entitled by *AI and mathematical modelling education – challenges and opportunities* by **Senior Lecture Kerri Spooner** (Auckland University of Technology, New Zealand) and **Associate Professor Takashi Kawakami** (Utsunomiya University, Japan).

Plenary, Thursday August 12, 9:00-10:00, entitled *Modeling Moral Change: A Case Study in Developing Explanatory and Predictive Mathematical Models* by **Professor Kimmo Ericsson** (Mälardalen University, Sweden).

Panel discussion, Friday August 15, 9:00-10:30, moderated by **Professor Pauline Vos** (Western Norway University of Applied Sciences, Norway) on *Mathematical Modelling and Responsible and Critical Citizenship*. The participating panellists are **Professor Marcelo C. Borba** (State University of São Paulo, Brazil), **Professor Vince Geiger** (Australian Catholic University, Australian), **Associate Professor Britta Eyrych Jessen** (University of Copenhagen, Denmark), **Professor Hans-Stefan Siller** (Universität Würzburg, Germany).

Long or Short Oral Communications and Presentations

Long oral communications (30 minutes + 15 minutes discussion) or short oral communications (20 minutes + 10 minutes discussion) are organised in seven parallel sessions at ICTMA22. Oral communications can be empirical, theoretical, philosophical or methodological. They may be based on ongoing or completed research.

Chairing of the Long and Short oral Communications and Presentations

Each session have designated chairpersons, as listed in the programme. The role of the chair is essential to the smooth running of the conference parallel presentations. We kindly ask all chairs to manage their sessions with both sensitivity and firmness, ensuring that presentations and discussions stay on schedule.

For timing:

- **Long presentations:** Allow **30 minutes** for the talk, followed by **up to 15 minutes** for questions and discussion.
- **Short presentations:** Allow **20 minutes** for the talk, followed by **up to 10 minutes** for questions and discussion.

Timing cards will be available in each room to help presenters stay within their allotted time. These cards indicate when **5**, **2**, and **1 minute** remain. Please hold them up clearly from the audience so the presenter can adjust accordingly and finish on time.

Reflection fika

Fika (https://en.wikipedia.org/wiki/Coffee_culture#Sweden) is important in Sweden and to Swedes. Therefore, the ICTMA22 programme will include so-called *Reflection Fika* to take advantage of this uniquely Swedish version of a coffee and cake break on two afternoons of the conference. A comfortable seating area will provide a relaxed and open setting in which everyone can reflect on the day in conversation. Under the informal guidance of the moderator **Associate Professor Susana Carreira** (University of Algarve, Portugal), the two senior researchers **Professor Mogens Niss** (Roskilde University, Denmark) and **Professor Peter Galbraith** (University of Queensland, Australia), will share their impressions and reflections on the day's sessions and put these reflections into different perspectives. The floor will then be open to the audience, who can ask questions, expand on their reflections, dream, criticise or just watch. A reflection fika is meant to feed our minds and stomachs. A similar interactive academic format was implemented at PME Umeå (2018) and was very well received.

Poster session

Poster presentations are scheduled during the **coffee breaks on Tuesday and Wednesday**, from **10:00 to 10:30**. Posters will be displayed in the same area where coffee and refreshments are served, allowing participants to engage with presenters in an informal and interactive setting.

We encourage all attendees to take this opportunity to explore current research, exchange ideas, and connect with colleagues during these dedicated poster sessions.

Early career researcher day

On **Sunday August 10, 12.00-17.30**, a special activity for early career researchers will be offered:

Leader - Prof Dr Pauline Vos



Guests - Prof Dr Vince Geiger

Prof Dr Peter Galbraith

Ass. Prof Lisa Steffensen

12:00 – 12:40	- Welcome and lunch - introducing ourselves and our research (PV) - getting to know each other (game)
12:45 – 13:30	Workshop – ICTMA president's advice to Early Career Researchers (VG)
Break	
13:45– 14:30	Workshop – Observation research of modelling classrooms (LS)
Break	
14:45 – 15:30	Workshop - Academic writing and getting published (PV)
Break	
15:45 – 16:30	Workshop - The role of theory in research and practice as influenced by the history and culture of ICTMA (PG)
16:30-17:00	Evaluation and Goodbye
Break	
18:00	Joining the ICTMA22 Welcome Reception

Schematic overview of the conference schedule

Time	Sunday Aug 10	Monday Aug 11	Tuesday Aug 12	Wednesday Aug 13	Thursday Aug 14	Friday Aug 15
08:30		Registration 08:00-09:00				
09:00		Opening ceremony 09:00-10:00	Plenary 1 (Orey) 09:00-10:00	Plenary 3 (Kawakami & Spooner) 09:00-10:00	Plenary 4 (Eriksson) 09:00-10:00	Plenary Panel 09:00-10:30
09:30		Coffee break 10:00-10:30	Coffee break / Poster session 10:00-10:30	Coffee break / Poster session 10:00-10:30	Coffee break 10:00-10:30	
10:00						
10:30		Keynote Speech (Kaiser) 10:30-12:00	Parallel session 3 10:30-12:00	Parallel session 5 10:30-12:00	Parallel session 6 10:30-12:00	Coffee break 11:00-12:30
11:00						Henry Pollak award and closing ceremony 11:00-12:30
11:30						
12:00	Early Career Lunch					
12:30						
13:00						
13:30						
14:00	Early Career Researcher Day					
14:30		Parallel session 1 13:30-15:00	Plenary 2 (Sevinc) 13:30-14:30		Parallel session 7 13:30-15:00	
15:00		Coffee break 15:30-17:00	Reflection of the day (FIKA) 14.30-16.00		Reflection of the day (FIKA) 15:00-16:30	
15:30		Parallel session 2 15:30-17:00	Parallel session 4 16.00-17.30		Business meeting 16.30-17.30	
16:00						
16:30						
17:00						
17:30						
18:00						
18:30	Welcome reception 18:00-20:00 Vallfarten, Campus Valla					
19:00						
19:30					Conference dinner 19:00-	
20:00						

Presentation of keynote- and plenary speakers and panellists

Keynote speaker Gabriele Kaiser holds a master's degree as a teacher for mathematics and humanities for lower and upper secondary level. She completed her doctorate in mathematics education in 1986 with a study on applications and modelling supervised by Werner Blum. The thesis was covering theoretical aspects such as an analysis of the historical and the recent discussion on applications and modelling as well as empirical aspects, namely the possibilities of this kind of examples in mathematics teaching. These research works were extended in new projects on modelling in mathematics teaching together, amongst others with colleagues from the mathematics department.

International comparison in mathematics teaching is one of the main fields of research of Gabriele Kaiser and therefore became the subject of her post-doctoral study, which comprised a comparative study on mathematics teaching in England and Germany.



Since 1998, she is full professor for mathematics education at the Faculty of Education of the University of Hamburg. Her areas of research are empirical studies on teacher education and teachers' professionalism, modelling and applications in school, international comparative studies, gender and cultural aspects in mathematics education. She was convenor of the 13th International Congress on Mathematical Education, which took place in Hamburg in 2016 with more than 3500 participants. Forty books have come out in the frame of this congress edited by her.

Gabriele Kaiser's most recent projects deal with teacher education, partly under an international perspective starting from the IEA Teacher Education Study in Mathematics (TEDS-M) on the efficiency of teacher education in various countries at primary and secondary level. Several follow-up studies – with Sigrid Blömeke and Johannes König – have been carried out in the frame of longitudinal studies and studies integrating knowledge and teacher noticing. A most recent study focuses on inclusive mathematics education and teacher professionalism

Gabriele Kaiser has been directing the large-scale project ProfaLe "Professional Teacher Action for Promoting Subject-based Learning under Changing Societal Conditions" within the framework of the „Quality Campaign for Teacher Education“, which has focused amongst other the integration of subject teaching and teaching of mathematics education, the integration of the perspective of inclusion and language within teacher education. The project lasted from 2015 to 2023.

From 2005 to 2024 she has been the editor-in-chief of the high-ranking journal ZDM –Mathematics Education, which is since 2020 in SSCI. From 2025 onwards, she serves as associate editor. In addition, she is editing several book series at national and international level amongst others International Perspectives on Mathematical Modelling. From 2007 to 2011 and from 2019 to 2023 she has been president of the ICMI affiliate group "The International Community of Teachers of Mathematical Modelling and Applications" (ICTMA). From 2017 to 2021 she was a professorial fellow at the Institute for Learning Sciences and Teacher Education at the Australian Catholic University in Brisbane (Australia), where she holds currently an honorary professorship. In addition, she holds an honorary professorship at the Educational University Hong Kong. Furthermore, since July 2025 she holds a part-time professorship at East China Normal University in Shanghai, China.



Plenary speaker Kimmo Eriksson is a professor of mathematics at Mälardalen University, Sweden, and a researcher at the Institute for Futures Studies. He is also the chairperson of the Global Social Norms research network (globalsocialnorms.org). Kimmo received his PhD degree in mathematics from the Royal Institute of Technology in 1993, starting his academic career as an algebraic combinatorialist. He later branched out into game theory, mathematical statistics, cultural evolution, social psychology, sociology, and mathematics education, and in 2018 he obtained a second PhD degree in social psychology from University of Kent. In his mathematics education research he has mainly analyzed data from TIMSS and PISA, often focusing on how within-country differences (e.g., by gender or socioeconomic status) vary between countries. His main current research interest is to understand how and why moral opinions and social norms vary across space and time, using cross-national surveys and mathematical models of cultural evolution.

Plenary speaker Daniel Clark Orey, Ph.D. is Professor Emeritus of Mathematics and Multicultural Education at California State University, Sacramento.

He has taught and lived in Oregon, Brazil, Guatemala, Mexico, Nepal and the United States.

He is a Fulbright Senior specialist with experiences at the Pontifical Catholic University of Campinas in Brazil (1998) and at Kathmandu University in Nepal (2007).

He is currently professor of mathematics education in the Department of Mathematics Education and serves in the Post-Doctoral Academic Masters and Doctoral Program in Mathematics Education at the Universidade Federal de Ouro Preto, Brazil.





Plenary speaker Serife Sevinç (Şerife Sevinç; in her native language) is a mathematics education researcher and teacher educator who has worked at universities in Germany, Turkey, and the United States. She is currently an Associate Professor of Mathematics Education at Middle East Technical University, Türkiye, and a Postdoctoral Research Associate in the Department of Primary Education/Mathematics Education at Technische Universität Dresden (TU Dresden), Germany.

Dr. Sevinç earned her Ph.D. in Curriculum and Instruction with a specialization in Mathematics Education, as well as a minor Ph.D. in Inquiry Methodology, from Indiana University Bloomington, U.S.A. Her doctoral research, supervised by Prof. Richard Lesh, focused on the nature of pre-service teachers' knowledge in designing modeling problems and pre-service teachers' conceptions of problem

characteristics that facilitate the modeling process. Through this work, she has made significant contributions to the field of mathematics education, publishing theoretical and empirical studies in prestigious international journals. Additionally, she has served as a reviewer for several leading mathematics education journals.

Her research interests center on students' and pre-service teachers' mathematical thinking in problem-solving and problem-posing processes, with a particular focus on mathematical modeling. She continues to investigate how the modeling process enhances students' and teachers' conceptual understanding of mathematics, as well as the knowledge required by pre-service teachers to design modeling problems. Her involvement in several international projects has enabled her to collaborate with esteemed colleagues in the U.S.A. and Europe. These projects range from exploring young students' experiences with mathematical modeling to examining cognitive and intercultural aspects of the modeling process among prospective mathematics teachers, contributing to cutting-edge advancements in the field.

In recent years, Dr. Sevinç has been an active member of the ICTMA (International Community of Teachers of Mathematical Modelling and Applications) community, regularly participating in its conferences and contributing to the ICTMA series books by publishing chapters, each highlighting significant aspects of mathematical modeling. She is also an active participant in the CERME (Congress of European Research in Mathematics Education) community, attending conferences, publishing in the CERME Proceedings, and serving as a co-leader of TWG6: Applications and Modelling at CERME14.

Beyond her research and academic contributions, Dr. Sevinç is deeply committed to advancing the field of mathematics education, particularly the modeling community. She achieves this through publishing research reports, practitioner articles, book chapters, and conference proceedings; organizing teacher training workshops; designing mathematical modeling courses for prospective teachers; and supervising master's theses and doctoral dissertations.

Dual plenary speaker Takashi Kawakami is an Associate Professor of Mathematics Education at the Cooperative Faculty of Education, Utsunomiya University, Japan.

Born in 1983, the same year as ICTMA-1, he began his teaching career as a private primary school teacher after earning a master's degree in Education from Chiba University (Japan) in 2008. In 2013, he transitioned to academic research as a full-time lecturer at Nishikyushu University (Japan), where he contributed to the training of kindergarten and primary school teachers and conducted research on mathematics and statistics education. Since 2017, he has imparted mathematics and statistics education to primary and secondary mathematics teachers at the Utsunomiya University. In 2023, he earned a Ph.D. in Education from Hyogo University of Teacher Education (Japan).



Takashi Kawakami's research interests include mathematical modelling, statistics and data science education, STEM education, and mathematics teachers' professional learning. His research began with mathematical modelling education in primary and secondary schools, which was the subject of his master's thesis. His current primary focus is exploring the intersection of mathematical modelling and statistics/data science education using conceptual and empirical approaches.

In terms of the conceptual approach, he has been systematically reviewing the research literature on data-based modelling in statistics and mathematics education with Jonas Bergman Ärlebäck. In terms of the empirical approach, Takashi Kawakami's doctoral thesis explored primary and secondary school students' flexible use of mathematical/deterministic and statistical/stochastic reasoning to make informed predictions and decisions when modelling data-rich situations. More recently, in collaboration with Akihiko Saeki and other colleagues, Takashi Kawakami extended this work to initiatives that promote data-informed interdisciplinary modelling as a part of STEM education and data-informed sociocritical modelling as a component of citizen education.

Takashi Kawakami's research was supported by grants from the Japan Society for the Promotion of Science and the CASIO Science Promotion Foundation. He has received several awards, both in Japan and overseas, including the Highly Commended Award for an Early Career Researcher at the 10th International Conference on Teaching Statistics (ICOTS-10) in 2018, the Excellent Paper Award from the Japan Society for Science Education, the Excellent Paper Award from the Mathematics Education Society of Japan, and the Utsunomiya University President's Special Award.

Takashi Kawakami has contributed to international and national academic conferences. He has been actively involved in ICTMA conferences since 2013. He served on the Local Organising Committee for ICTMA-21 in Japan. He participated in the Survey Team "Statistics and Data Science Education as a Vehicle for Empowering Citizens" at ICME-15 in 2024. He reviewed the common discourse between research on mathematical modelling education and statistics/data science education. Since 2014, Takashi Kawakami has served as an organiser or co-organiser of thematic sessions on mathematical modelling education at the annual conference of the Japan Society for Science Education (JSSE). He is currently a board member of the JSSE and the guest editor-in-chief of a special issue of a JSSE journal focused on data science education.



Dual plenary speaker Dr. Kerri Spooner is a senior lecturer in the Department of Mathematical Sciences at Auckland University of Technology (AUT), New Zealand. Her academic journey is marked by her dedication to mathematics education. She holds a Doctoral degree in Tertiary Mathematical Modelling Education from AUT and a Master's degree in Secondary School Mathematical Modelling Education from the University of Auckland.

Kerri's journey into mathematical modelling began in 2002 when she attended her first Mathematics in Industry Study Group (MISG) while working as a secondary school mathematics teacher. This experience sparked her interest in mathematical modelling and led

her to participate in several MISGs and Mathematics in Industry New Zealand (MINZ) workshops. Kerri further developed her modelling skills by completing two tertiary modelling courses. These courses deepened her understanding of the mathematical modelling cycle and differential equations, equipping her with the skills to create mathematical models for various situations.

In 2011, Kerri worked as part of a professional mathematical modelling team. This experience, along with her participation in MISGs and MINZs, and her study in modelling, provided her with first-hand knowledge of professional modelling behaviours and their complexities. This awareness influences her teaching approach, aiming to create authentic and engaging modelling experiences for her students.

Kerri's experiences with mathematical modelling, as well as her experiences as a secondary school teacher and tertiary lecturer, made her aware of the challenges students can face when modelling. She emphasizes the importance of being open to new contexts and ideas, learning new mathematics, and working collaboratively. She uses insights from her own experiences of modelling to inform her teaching and research, always striving to be open to new modelling behaviours presented by her students.

In 2012, encouraged by research showing that students as young as 13 could engage in modelling, Kerri developed and taught a holistic mathematical modelling unit for her secondary class. She aimed to provide students with an authentic experience of mathematical modelling, allowing them to engage fully with the process. She believes in the importance of students having an authentic experience of modelling that reflects professional practices. She emphasizes students' active engagement in the modelling process, the benefits of group work, mimicking professional modelling teams, and using prompts to facilitate a successful holistic modelling experience. Kerri's ideal way of teaching modelling involves taking genuine real-world problems and having students attempt to model them. However, she has also found students can also have valuable learning experiences modelling fictional situations.

As part of her dedication to providing students with experiences of modelling, since 2016 Kerri has overseen the New Zealand section of the International Mathematical Modelling Competition (IMMC). In 2019 and 2022 New Zealand teams have been outstanding award winners.

Throughout her career, Kerri has been recognized for her excellence in teaching, winning five teaching awards and being nominated for a national teaching excellence award. She is known for promoting active learning and student collaboration to engage students with course content and learning material. Her keen interest in successful ways to engage students in their learning drives her to attempt to create learning experiences that are authentic, engaging, and accessible.

Panel chair Pauline Vos, PhD (2002), is a Professor of Mathematics Education at the Western Norway University of Applied Sciences (HVL), in the city of Bergen.

She leads theoretical research into the why, when and how of using/learning what type of mathematics by which students/users, and practical research into the design of learning trajectories that deviate from deductive, teacher-centered approaches.

She builds on earlier experiences as mathematics teacher and teacher educator in The Netherlands, Zimbabwe and Moçambique. She has contributed to many past ICTMA conferences from ICTMA12 in London (2005) onwards.

To complementing her interest in mathematical modeling, she also has an interest in mathematical literacy and vocational mathematics.

Within the Congress of the European Society for Research in Mathematics Education, she leads the Thematic Working Group "Mathematics Education for Work, Society and Personal Development".



Panelist speaker Marcelo C. Borba is a Professor of the Graduate Program in Mathematics Education and of the Mathematics Department at UNESP (State University of São Paulo) in Brazil, where he chairs the research group GPIMEM (Research Group of Informatics, other Media and Mathematics Education).

Marcelo researches the use of digital technology in mathematics education, mathematical video produced by teachers and students, online distance education, modeling as a pedagogical approach, videos and ethnomatematics and qualitative research methodology.

He was an Associate Editor of ZDM, The International Journal of Mathematics Education between 2011 and 2023 and he was, from 2018 to 2022, the chair of the Teaching Committee of CAPES, a funding agency of the

National Ministry of Education in Brazil. He is currently leading a project of CAPES-PrInt that promotes internationalization of research in technology in mathematics education in Brazil.

He was a Keynote speaker at PME-44 (Psychology of Mathematics Education Study Group), Thailand-Virtual (2020); at Plenary Panel of ICME 14 (International Congress of Mathematics Education), held in a hybrid format in China (2021); and at ICM-22 (International Congress of Mathematicians), which was originally scheduled to take place in Russia but was held virtually (2022).

He published around 20 books in English, Portuguese and Spanish, and more than a hundred papers in these three languages. Maybe more importantly, he is actively involved in planting trees within reforestation projects!

Panelist speaker Professor Vince Geiger is the Research Director of the STEM in Education Program in the Institute of Learning Science and Teacher Education at Australian Catholic University. His work is situated in an interdisciplinary research space focused on the enabling and transformative role of mathematics within the STEM disciplines and other areas across the curriculum. This work, including over 150 peer reviewed publications, is driven by an understanding that the capacity to know and use mathematics confidently when addressing real-world problems is key to the empowerment of critically informed citizenship – the foundation for personal well-being and cohesive socially just societies. His contributions to research in STEM education are motivated by an awareness that capability in this space is vital for an individual's career prospects and the prosperity of nations.



Vince's research interests include mathematical modelling education, critical mathematical thinking, numeracy, and the role of digital resources in supporting mathematical teaching and learning. His research has attracted over \$7 million (AUD) in research funding both nationally and internationally and includes the projects: Enabling students' critical mathematical thinking (Australian Research Council); Using mathematics to solve real world problems – The role of enablers (Australian Research Council), Principals as STEM Leaders (Department of Education and Training), Strengthening teachers' instructional capabilities with Big Data (Australian Universities-German DAAD Joint Research Cooperation Scheme).

Most recently he is contributing to the DASME (Data Science in Mathematics Education) project, funded by the Novo Nordisk Foundation and led by Associate Professor Britta Jessen at the University of Copenhagen.

Vince's research awards include the role of Giovanni Prodi Professor at the University of Wurzburg and a Mathematics Education Research of Australasia (MERGA) Research Award. He is the current President of the International Community of Teachers of Mathematics and Applications, a Past President of the Australian Association of Mathematics Teachers, Editor-in-Chief of Mathematics Education Research Journal (Scopus med% 8); Editorial Board Member for the International Journal of Science and Mathematics Education, member of the Numeracy Expert Group for the 2nd Cycle of the Programme for International Assessment of Adult Competencies, and a member of the International Organising Committee for the International Commission for Mathematics Instruction Study-27 on mathematics education and the socio-ecological.



Panelist speaker Britta Eyrich Jessen is an associate professor at University of Copenhagen. Her research concerns mathematical modelling, digital technologies in modelling education, interdisciplinarity and curriculum analyses linked to modelling application. Lately her research interests have moved towards statistical modelling and data science, and she plans to explore potential mergers between those domains. For this purpose, she has secured funding for the research project DASME – Data Science in Mathematics Education running July 2025 - December 2028.

Most often she works with tools and notions from the Anthropological Theory of the Didactics such as Study and Research Paths, as well as Herbartian schema. This allows her to focus on practices and reasoning linked to students' engagement with modelling activities.

Her work mainly focuses on upper secondary education, and professional development of teachers at this level through lesson study and similar activities. She teaches and supervises pre-service and in-service teachers and is responsible for a course on “the mathematically modelling science” at the masters' program on STEM education. She is a co-leader of the TWG06 on modelling and application at CERME.

Before her research carrier, she was an upper secondary teacher in mathematics and physics.

Panelist speaker Hans-Stefan Siller is a full professor at the Chair of Mathematics V – Didactics of Mathematics at the Julius Maximilian University of Würzburg. As a professor of didactics of mathematics, his remit encompasses the didactics of the subject of mathematics in all teaching positions, (vocational) grammar schools, secondary modern schools, middle schools, primary schools and schools with special educational needs. His research and work is focused at the interface of current research in mathematics didactics, conception and mathematical topics; in recent years increasingly at the interface of mathematics, MINT, education for sustainable development, digitalization and in research on giftedness.



Following his studies in mathematics and physics at the University of Graz (Austria), he acquired practical teaching experience at Austrian grammar schools in the city and province of Salzburg. He then successfully completed his doctoral studies at the Paris Lodron University of Salzburg (Austria). From 2011, he assumed responsibility for the Austrian school system at both the research and organizational level as project manager for the central written matriculation examination in Austria and the introduction of the central written matriculation examination for mathematics at Austrian grammar schools. In 2012, he habilitated at the University of Vienna and in April of the same year accepted his first professorship at the University of Koblenz-Landau, Koblenz campus. Since October 2017, he has been conducting research as a full professor of mathematics education at the Chair of Mathematics V - Mathematics Education. In the summer semester of 2021, he was a visiting professor at the University of Semarang (Indonesia).

Hans-Stefan Siller is conducting research on various questions in the domain of mathematics education, with a particular focus on interdisciplinary research settings. His research encompasses both the traditional subjects in the teaching of mathematics, in which teaching and learning processes are examined from cognitive and affective perspectives, and the digitalization of the teaching of mathematics, including the utilization of digital tools and media, the didactic preparation of content – taking into account fundamental concepts – interdisciplinary work, and the subject of mathematical modelling. In addition to these areas, he has demonstrated a commitment to addressing significant (and emerging) challenges faced by educators, including the integration of education for sustainable development (ESD) within mathematics lessons and the utilization of artificial intelligence (AI), with a particular focus on generative AI. Since 2021, for instance, he has led the mathematics working group on the integration of education for sustainable development on behalf of the German Standing Conference of the Ministers of Education and Cultural Affairs (KMK).

In all his research fields, Hans-Stefan Siller has demonstrated a commitment to both theoretical and evidence-based research findings, which have resulted in numerous high-ranking publications. Notably, his work is strongly oriented towards third-party funding, and seeks to integrate different perspectives, as is being implemented in one of his current projects, DIM²ENSION - Digital Supported Mathematical Modelling for Sustainable Development Goals in European Education, together with colleagues.

Since 2024, he has been contributing his expertise as an editor to the Mathematics Education Research Journal (MERJ).

Programme and schedule of presentations

Early career researcher day, Sunday, August 10, 2025

Sunday, August 10, 2025	
Time/Location	Activity
U1, C-huset	Early career researcher day
12.00 – 12.40	Welcome, lunch, introducing ourselves and our research (Pauline Vos)
12.45 – 13.30	Workshop – ICTMA president’s advice to Early Career Researchers (Vince Geiger)
13.30 – 13.45	Break
13.45 – 14.30	Workshop – Observation research of modelling classrooms (Lisa Steffensen)
14.30 – 14.45	Break
14.45 – 15.30	Workshop - Academic writing and getting published (Pauline Vos)
15.30 – 15.45	Break
15.45 – 16.30	Workshop - The role of theory in research and practice as influenced by the history and culture of ICTMA (Peter Galbraith)
16.30 – 17.00	Evaluation and Goodbye

Sunday, August 10, 2025

Sunday, August 10, 2025	
Location/Time	Activity
C4, C-huset	Registration
11.30 – 17.00	Conference registration
Determinanten, B-huset	Executive board meeting
15.00 – 17.00	A meeting for executive board members
Vallfarten	Welcome Reception
18.00 – 20.00	The ICTMA22- Welcome Reception

Monday, August 11, 2025

Time/Location	Activity
C4, C-huset	Registration
08.00 – 09.00	Conference registration
C4, C-huset	Opening ceremony
09.00 – 10.00	ICTMA22 will open
10.00 – 10.30	Coffee break
C4, C-huset	Keynote Speech
10.30 – 12.00 Chair: Rita Borromeo Ferri	[1.] Gabriele Kaiser <i>Perspectives on mathematical modelling education – quo vadis?</i>
12.00 – 13.30	Lunch
U1, C-huset	Parallel session 1 “Long presentations”
13.30– 15.00 Chair: Amy Brass	[2.] Peter Galbraith <i>Enabling excellence in modelling: Investigating an IMMC performance</i>
Chair: Peter Galbraith	[3.] Amy Brass, Rose Mary Zbiek, and Gary Kaufman <i>Decomposition of modelling sub-competencies into elements to assess mathematical modelling</i>
U2, C-huset	Parallel session 1 “Long presentations”
13.30– 15.00 Chair: Peter Collignon	[4.] Toshikazu Ikeda, Kosuke Mineno, Kensuke Koizumi, and Yuki Wada <i>From interpreting designs to creating new concepts: Integrating empathy into the mathematical modelling process</i>
Chair: Toshikazu Ikeda	[5.] Peter Collignon and Regina Möller <i>Algorithms as parts of modelling processes: Reflections for mathematics education in the age of AI</i>
U3, C-huset	Parallel session 1 “Long presentations”
13.30– 15.00 Chair: Elena Naftaliev	[6.] Milton Rosa and Daniel Orey <i>Ethnomathematical knowledge of dry rice producers in northeast Brazil through the development of ethnomodels</i>
Chair: Milton Rosa	[7.] Elena Naftaliev <i>Freedom of choice: Ethnomathematical modeling with interactive tools</i>

U4, C-huset	Parallel session 1 “Long presentations”
13.30– 15.00 Chair: Annika Rosendahl	[8.] Katrin Vorhölter and Hans-Stefan Sille <i>Modelling for implementing sustainability in mathematics education: How tasks affect</i>
Chair: Katrin Vorhölter	[9.] Annika Rosendahl, Catharina Beckschulte, Jascha Quarder, and Stanislaw Schukajlow <i>Assessing portfolios in the context of mathematical modelling – Perceived expectancy and values of pre-service teachers</i>
U6, C-huset	Parallel session 1 “Long presentations”
13.30– 15.00 Chair: Jennifer Czoher	[10.] Ortal Nitzan-Tamar, Zehavit Kohen and Hans Stefan Siller <i>Adapting mathematical modelling tasks to a variety of levels for teaching in a heterogeneous classroom</i>
Chair: Ortal Nitzan-Tamar	[11.] Jennifer Czoher, Sindura Kularajan and Elizabeth Roan <i>Can we use modeling cycles to predict which instructor scaffolding moves elicit specific modeling competencies?</i>
U7, C-huset	Parallel session 1 “Short presentations”
13.30– 15.00 Chair: Simon Barlovits	[12.] Ayşe Oztürk <i>Second graders’ moves in emergent modeling through a puzzle block game</i>
Chair: Shigekazu Komeda	[13.] Simon Barlovits, Simone Jablonski, Matthias Ludwig, Jonas Bergman Ärlebäck, Susanna Carreira, Georgios Fesakis, Alissa Fock, Hélia Jacinto, Sara Julsgård, Maria Anastasia Moustaka, and Hans-Stefan Siller <i>Teachers' perspectives on mathematical modelling: A European view on characteristics, benefits and challenges</i>
Chair: Ayşe Oztürk	[14.] Shigekazu Komeda <i>Development and evaluation of a beginner-level workshop to improve modelling lessons for in-service secondary mathematics teachers with no prior modelling experience</i>
U10, C-huset	Parallel session 1 “Short presentations”
13.30– 15.00 Chair:	[15.]
Chair:	[16.]
Chair:	[17.]

U11, C-huset	Parallel session 1 “Short presentations”
13.30– 15.00 Chair: Louise Meier Carlsen	[18.] Ruth Rodriguez Gallegos and Rafael Ernesto Bourguet Díaz <i>Integration of artificial intelligence in mathematical modeling: A qualitative approach for engineering students in multivariable calculus</i>
Chair: Ruth Rodriguez Gallegos	[19.] Louise Meier Carlsen and Britta Eyrich Jessen <i>Studying the epistemic value of generative AI in modelling activities</i>
Chair:	[20.]
U14, C-huset	Parallel session 1 “Short presentations”
13.30– 15.00 Chair: Kristy Litster	[21.] Leonie Reuter, Jascha Quarder, Stanislaw Schukajlow, Gilbert Greefrath and Katharina Wiehe <i>Finance-specific modelling tasks in mathematics instruction – Opportunities and challenges from the perspective of students and teachers</i>
Chair: Dayat Hidayat	[22.] Kristy Litster <i>Supporting prospective elementary teachers’ use of mathematical modeling through real-world societal issues</i>
Chair: Leonie Reuter	[23.] Dayat Hidayat and Signe Kastberg <i>Mathematical modeling through a Socio-ecological perspective: Promoting sustainable development in mathematics education</i>
U15, C-huset	Parallel session 1 “Short presentations”
13.30– 15.00 Chair: Kaneko Masafumi	[24.] Uwe Schürmann and Georg Bruckmaier <i>Modelling tasks from written comparison tests in Germany</i>
Chair: Carlos Segura	[25.] Kaneko Masafumi, Tsuchida Satoshi, Nishikori Hisashi and Kawano Yuichiro <i>The importance of experimental experiences that activate data-driven mathematization in high school chemistry</i>
Chair: Uwe Schürmann	[26.] Carlos Segura, Clara García-Cerdá and Irene Ferrando <i>Effect of reading literacy on performance and confidence solving modelling problems with missing or excess data</i>
15.00 – 15.30	Coffee break

U1, C-huset		Parallel session 2 “Long presentations”	
15.30– 17.00		[27.] Hissan Yunus and Tomas Højgaard	<i>Different mathematical competencies afforded by tasks: An analytical framework for determining modeling competency</i>
Chair: Irina Lyublinskaya			
		[28.] Irina Lyublinskaya	<i>Developing modeling competencies of mathematics preservice teachers: Problem-based approach</i>
Chair: Hissan Yunus			
U2, C-huset		Parallel session 2 “Long presentations”	
15.30– 17.00		[29.] Neslihan Bulut and Rita Borromeo Ferri	<i>AI-powered learning: Exploring secondary school students’ use of ChatGPT for solving mathematical modelling problems</i>
Chair: Mustafa Cevikbas			
		[30.] Mustafa Cevikbas and Gabriele Kaiser	<i>Solving mathematical modelling problems benefitting from generative AI: Affordances, challenges, and reliability</i>
Chair: Neslihan Bulut			
U3, C-huset		Parallel session 2 “Long presentations”	
15.30– 17.00		[31.] Pauline Vos and Peter Frejd	<i>The role of mathematical models in the lives of ‘non-mathematical’ citizens</i>
Chair: Liv-Marit Braatø			
		[32.] Liv-Marit Braatø, Suela Kacerja, Lisa Steffensen and Inger Elin Lilland	<i>The use of toolkits for socio-critical modelling in Norway and South Africa</i>
Chair: Pauline Vos			
U4, C-huset		Parallel session 2 “Long presentations”	
15.30– 17.00		[33.] Clara Nehrkorn and Katja Eilerts	<i>Understanding the role of beliefs in pre-service primary teachers’ learning to teach mathematical modelling</i>
Chair: Eva Thanheiser			
		[34.] Eva Thanheiser	<i>Making the invisible visible: Using mathematics modeling to understand systemic inequity</i>
Chair: Clara Nehrkorn			

U6, C-huset	Parallel session 2 “Long presentations”
15.30– 17.00 Chair: Erica Dorethea Spangenberg	[35.] Luis Miguel Paz-Corrales, Avenilde Romo-Vázquez and Luis Moreno-Armella <i>An instructional proposal to address the intra-mathematisation of mathematical modelling in teacher education</i>
Chair: Luis Miguel Paz-Corrales	[36.] Erica Dorethea Spangenberg <i>Developing preservice teachers’ creativity through a mathematical modelling week</i>

U15, C-huset	Parallel session 2 “Short presentations”
15.30– 17.00 Chair: Tian Wang	[49.] Hadas Handelman and Zehavit Kohen <i>The critical role of extra-mathematical knowledge in STEM-based hands-on mathematical modelling activities</i>
Chair: Hadas Handelman	[50.] Tian Wang, Danni Lin, Michiel Veldhuis and Jian Liu <i>Online assessment of mathematical modelling competency for grade 6 students in China</i>
Chair:	[51.]

Tuesday, August 12, 2025

Time/Location	Activity
C4, C-huset	Plenary 1
09.00 – 10.00 Chair: Suela Kacerja	[52.] Daniel Orey <i>From an Imprecise Imprecision: Reflections on Culture and Modelling</i>
10.00 – 10.30	Coffee break/ Poster presentations
	[P1.] Simone Jablonski <i>Fermi Problems for reasoning? – The research project Fermat</i>
	[P2.] Iwona Krzyżanowska <i>Students' perceptions of learning mathematics in mathematical modeling and data analysis</i>
	[P3.] Aleksandra Nowel <i>Mathematical modeling studies at the University of Gdańsk: Program structure, courses, and student activities</i>
	[P4.]
	[P5.]
U1, C-huset	Parallel session 3 “Long presentations”
10.30– 12.00 Chair: Rina Durandt	[53.] Sonia Yvain-Prébiski <i>Training primary school teachers to create modelling problems and the impact on their professional knowledge of modelling: A case study in France within an action research project</i>
Chair: Sonia Yvain-Prébiski	[54.] Rina Durandt <i>How do primary mathematics student teachers respond to a modelling unit? Some answers from empirical research</i>
U2, C-huset	Parallel session 3 “Long presentations”
10.30– 12.00 Chair: Burçak Boz Yaman	[55.] Lukas Bayer, Martin Bracke and Nadine Dittert <i>Integrating computational thinking into mathematical modelling: A conceptual framework with example projects</i>
Chair: Lukas Bayer	[56.] Burçak Boz Yaman and Mustafa Hicret Yaman <i>Dialoguing with AI: A case study of peer-like collaboration between ChatGPT and a pre-service teacher in a mathematical modelling task</i>

U3, C-huset	Parallel session 3 “Long presentations”
10.30– 12.00 Chair: Mathilde Kjær Pedersen	[57.] Lisa Steffensen and Faaiz Gierdien <i>Pre-service teachers’ discussions of “best” economic model for water consumption and sustainability – in Norway and South Africa</i>
Chair: Lisa Steffensen	[58.] Mathilde Kjær Pedersen, Maria Kirstine Østergaard, Morten Misfeldt, and Uffe Thomas Jankvist <i>The concept of model in mathematics education and science education</i>
U4, C-huset	Parallel session 3 “Long presentations”
10.30– 12.00 Chair: Sara Julsgård	[59.] Vince Geiger, Hans-Stefan Siller, Nina Unshelm and Sarah Digan <i>Conceptualising and generating a typology for evidentiary practices for students’ work on application problems related to sustainability</i>
Chair: Vince Geiger	[60.] Sara Julsgård <i>Logarithms in biology education: A content analysis of YouTube videos on survivorship curves</i>
U6, C-huset	Parallel session 3 “Long presentations”
10.30– 12.00 Chair: Michele Fiorentino	[61.] Orit Cohen-Nissan and Zehavit Kohen <i>Structured-to-directed iterative modelling: Fostering mathematical modelling competencies through task-based scaffolding</i>
Chair: Orit Cohen-Nissan	[62.] Michele Fiorentino and Antonella Montone <i>Modelling the concept of proportion through the Brousseau’s puzzle: a synergic approach in an integrated STEM environment</i>
U7, C-huset	Parallel session 3 “Short presentations”
10.30– 12.00 Chair: Beate Lode	[63.] Mia Viermann, Dominik Schlüter, Rebecca Henkel and Maike Hagen <i>Attitudes of student teachers towards teaching and learning mathematical modelling</i>
Chair: Lily Roth	[64.] Beate Lode <i>Designing an instrument for measuring mathematics teacher educators’ rationale for mathematical modelling in teacher education (RMMT) – a methodological exploration</i>
Chair: Mia Viermann	[65.] Lily Roth, Sindura Kularajan and Julisa Gonzales <i>Exploring the roles between a teacher and researcher in designing and implementing a mathematical modeling unit</i>

U10, C-huset	Parallel session 3 “Short presentations”
10.30– 12.00 Chair: Duncan Mhakure	[66.] Irene Ferrando and Olga Shemberko-Melnyk <i>Can Fermi graphs boost problem-solving performance? Comparing student approaches across educational levels</i>
Chair: Shengtian Zhou	[67.] Duncan Mhakure and Arne Jakobsen <i>Fostering and supporting high school students’ productive struggles during mathematical modelling and its applications to South African national curriculum</i>
Chair: Irene Ferrando	[68.] Shengtian Zhou, Nils Henry Rasmussen and Lisa Stefensen <i>Aspects of relevance to the students in a modelling project</i>
U11, C-huset	Parallel session 3 “Short presentations”
10.30– 12.00 Chair: Ragnhild Hansen	[69.] Andreas Back, Gilbert Greefrath and Stanislaw Schukajlow <i>Teaching modelling with GeoGebra and affective characteristics towards digital tools</i>
Chair: Andreas Back	[70.] Ragnhild Hansen <i>Perspectives on the implementation of modelling with digital tools</i>
Chair:	[71.]
U14, C-huset	Parallel session 3 “Short presentations”
10.30– 12.00 Chair: David Zaldivar-Rojas	[72.] Anna Lundberg and Björn Textorius <i>Enhancing sustainable critical thinking through mathematical modelling: Utilizing textbook tasks in grade 6 education</i>
Chair: Dorte Moeskær Larsen	[73.] David Zaldivar-Rojas and Samantha Quiroz-Rivera <i>“The Cat in the Hat”: children’s literature as a context for mathematical modelling</i>
Chair: Anna Lundberg	[74.] Dorte Moeskær Larsen and Maiken Westen Holm Svendsen <i>How to integrate mathematic modelling into interdisciplinary STEM teaching</i>
U15, C-huset	Parallel session 3 “Short presentations”
10.30– 12.00 Chair: Sindura Kularajan	[75.] Hiroki Otani <i>Statistical and mathematical models of bivariate data from Japanese junior high school science textbooks: A textbook analysis</i>
Chair: Juan Felipe Medina-Mendieta	[76.] Sindura Kularajan, Hillary Swanson, Afaf Bahajj, and Lily Roth <i>Constructive resources in mathematical modeling</i>
Chair: Hiroki Otani	[77.] Juan Felipe Medina-Mendieta and Carolina Guerrero-Ortiz <i>Modelling using numerical methods and analysis of error sources</i>
12.00 – 13.30	Lunch

C4, C-huset	Plenary 2
13.30 – 14.30 Chair: Irene Ferrando	[78.] Serife Sevinc <i>Unpacking the Modelling Mindset in Teacher Education: A Behind-the-Scenes Journey to Empower Future Mathematics Teachers</i>
C4, C-huset	Reflection fika (including coffee break)
14.30 – 16.00 Moderator: Susana Carreira	[79.] Mogens Niss and Peter Galbraith
U1, C-huset	Parallel session 4 “Long presentations”
16.00– 17.30 Chair: Akihiko Saeki	[80.] Shophika Vaithyanathasarma <i>Leveraging the concept of relationship to knowledges to understand mathematical modelling’s presence in teaching practices</i>
Chair: Shophika Vaithyanathasarma	[81.] Akihiko Saeki, Takashi Kawakami and Toshikazu Ikeda <i>The relationship between mathematics curriculum and mathematical modelling research in Japan</i>
U2, C-huset	Parallel session 4 “Long presentations”
16.00– 17.30 Chair: Emine Nur Ünveren Bilgiç	[82.] Kikuchi Ruka and Matsuzaki Akio <i>How do the computer results output by generative AI affect mathematical models of modeller?</i>
Chair: Kikuchi Ruka	[83.] Emine Nur Ünveren Bilgiç, Esra Selcen Yakıcı Topbaş and Hatice Aydan Kaplan <i>Metanoticing in mathematical modeling instruction: A new lens on teacher noticing</i>
U3, C-huset	Parallel session 4 “Long presentations”
16.00– 17.30 Chair:	[84.]
Chair:	[85.]
U4, C-huset	Parallel session 4 “Long presentations”
16.00– 17.30 Chair: Sofie Ye	[86.] Rita Borromeo Ferri and Neslihan Bulut <i>Education for sustainable development through mathematical modelling activities in primary school: Implemented tasks, suitable teaching methods and learners’ solutions</i>
Chair: Rita Borromeo Ferri	[87.] Sofie Ye, Maja Elmgren, Magnus Jacobsson and Felix Ho <i>Problem solving at the interface of maths and chemistry: Modelling processes, cognitive resources and epistemic framing</i>

U6, C-huset	Parallel session 4 “Long presentations”
16.00– 17.30 Chair: Pauline Vos	[88.] Dominik Schlüter, Michael Besser, Gilbert Greefrath and Pauline Vos <i>Empirical analysis of authentic aspects in reality-based mathematical test items</i>
 Chair: Dominik Schlüter	[89.] Pauline Vos and Désiré Baartman <i>“How big is the panther?” - mathematical modelling in kindergarten</i>
U7, C-huset	Parallel session 4 “Short presentations”
16.00– 17.30 Chair: Andreas Rui	[90.] Oguz Koklu and Muhammet Şahal <i>Pre-service mathematics teachers’ investigations of a genuine campus problem through statistical data modelling</i>
 Chair: Won Jung	[91.] Andreas Rui and Suela Kacerja <i>Programming as a support for mathematical modelling – a teacher perspective</i>
 Chair: Oguz Koklu	[92.] Won Jung, Kyungwon Lee and Oh Nam Kwon <i>Integrating mathematical modelling into mathematics classrooms: An analysis of Korean secondary teachers’ documentation work</i>
U10, C-huset	Parallel session 4 “Short presentations”
16.00– 17.30 Chair: Rina Durandt	[93.] Sevinç Göksen-Zayim, Derk Pik and Carla van Boxtel <i>Effects of collaborative modelling lesson series and debriefing on students’ ability to engage in mathematical modelling, assumption making and on students’ self-efficacy</i>
 Chair: Belinda Cornelissen	[94.] Rina Durandt, Alfred Lindl and Werner Blum <i>Exploring the link between tertiary students’ prior mathematical knowledge and their performance in a modelling unit</i>
 Chair: Sevinç Göksen-Zayim	[95.] Belinda Cornelissen <i>Exploring pre-service mathematical literacy teachers’ mathematical modelling competency through the solving of simultaneous equations</i>

U11, C-huset	Parallel session 4 “Short presentations”
16.00– 17.30 Chair: Morten Finset Ingvaldsen	[96.] Adi Nur Cahyono <i>Empowering mathematical modelling through AI and coding: An exploratory study in Indonesian secondary schools</i>
Chair: Bárbara Nivalda Palharini Alvim Sousa	[97.] Morten Finset Ingvaldsen, Andreas Brandsæter and Ingeborg Berget <i>Student conceptions of the role of data in AI models</i>
Chair: Adi Nur Cahyono	[98.] Bárbara Nivalda Palharini Alvim Sousa, Lourdes Maria Werle de Almeida and Jhony Alexander Villa-Ochoa <i>Meaning in mathematical modelling activities mediated by technology: From philosophical to didactic implications</i>

U14, C-huset	Parallel session 4 “Short presentations”
16.00– 17.30 Chair: Duncan Mhakure	[99.] Minoru Ohtani <i>Student voice while modelling socially pressing issues with digital technology in a Japanese high school mathematics lesson</i>
Chair: Minoru Ohtani	[100.] Arne Jakobsen, Wisani Hlangwani, Miranda Moodley, Jacob Jaftha and Duncan Mhakure <i>Evaluating a South African model-eliciting activity using rubric-based assessment: A case study on measurement concepts</i>
Chair:	[101.]

U15, C-huset	Parallel session 4 “Short presentations”
16.00– 17.30 Chair: Yuan Yan	[102.] Daiki Fujiwara <i>Model construction and selection in predictive modelling by 8th-grade students: Focusing on the determinism of models</i>
Chair: Mathieu Thibault	[103.] Yuan Yan and Xiaoli Lu <i>Integrating practical problem-solving activities into the mathematics curriculum: A multi-perspective analysis of three seventh-grade textbook series in China</i>
Chair: Daiki Fujiwara	[104.] Mathieu Thibault, Stéphanie Rioux and Sandrine Michot <i>Minecraft education: Didactic benefits and challenges from the perspective of mathematical modelling</i>

Wednesday, August 13, 2025

Time/Location	Activity
C4, C-huset	Plenary 3
09.00 – 10.00 Chair: Jill Brown	[105.] Takashi Kawakami & Kerri Spooner <i>AI and mathematical modelling education – challenges and opportunities</i>
10.00 – 10.30	Coffee break/ Poster presentations
	[P1.] Simone Jablonski <i>Fermi Problems for reasoning? – The research project Fermat</i>
	[P2.] Iwona Krzyżanowska <i>Students' perceptions of learning mathematics in mathematical modeling and data analysis</i>
	[P3.] Aleksandra Nowel <i>Mathematical modeling studies at the University of Gdańsk: Program structure, courses, and student activities</i>
	[P4.]
	[P5.]
U1, C-huset	Parallel session 5 “Long presentations”
10.30– 12.00 Chair: Ingeborg Lid Berget	[106.] Sebastian Kuntze, Martin Bracke, Michael Fischer and Stefanie Hartmann <i>Analysing non-optimal implementation of modelling tasks – What do mathematics teachers notice in classroom cartoon vignettes?</i>
Chair: Sebastian Kuntze	[107.] Ingeborg Lid Berget and Shaista Kanwal <i>Analysing teachers’ examples of modelling activities</i>
U2, C-huset	Parallel session 5 “Long presentations”
10.30– 12.00 Chair: Yukiko Asami-Johansson	[108.] Jennifer Czoher, Brendan Kelly, Callie Angel, Stephanie Tarigan and Garima Gogia <i>Scaling and disseminating a modelling-first curriculum for life sciences majors: Drivers of success</i>
Chair: Jennifer Czoher	[109.] Yukiko Asami-Johansson, Berta Barquero and Ignasi Florensa <i>Conditions for modelling in mathematics curriculum reforms in engineering programmes in Sweden and Spain</i>

U3, C-huset		Parallel session 5 “Long presentations”	
10.30– 12.00	Chair: Michele Fiorentino	[110.] Nina Unshelm, Hans-Stefan Siller and Vince Geiger	<i>Mathematical Modelling as a method for civic understanding, interpretation and evaluation of media claim</i>
		[111.] Michele Fiorentino, Antonella Montone and Pauline Vos	<i>Boundary artefacts for mathematical modelling in vocational education</i>
	Chair: Nina Unshelm		
U4, C-huset		Parallel session 5 “Long presentations”	
10.30– 12.00	Chair: Felix M Ho	[112.] Suzuki Yumi and Matsuzaki Akio	<i>What kinds of models did modelers make while solving the problem in the real world? The case of an arranged ice cream problem</i>
		[113.] Felix M Ho	<i>In search of synergy in students’ conceptual understanding and the teaching of calculus and chemistry</i>
	Chair: Suzuki Yumi		
U6, C-huset		Parallel session 5 “Long presentations”	
10.30– 12.00	Chair: Sindura Kularajan	[114.] Deborah Lehrmann and Hans-Stefan Siller	<i>Influence of creativity on mathematical modelling activities</i>
		[115.] Sindura Kularajan, Hillary Swanson, Afaf Bahajj, and Lily Roth	<i>A mathematical modeling professional development course for secondary teachers</i>
	Chair: Deborah Lehrmann		
U7, C-huset		Parallel session 5 “Short presentations”	
10.30– 12.00	Chair: Barbara Bustos-Osorio	[116.] Marlena Meyer and Gilbert Greefrath	<i>How students’ knowledge of benchmarks influences their process of solving Fermi problems</i>
		[117.] Barbara Bustos-Osorio and Ramos-Rodríguez	<i>Effectiveness of a teacher professional development program on mathematical modeling and cognitive demand for elementary school teachers</i>
	Chair: Songchai Ugsonkid		
	Chair: Marlena Meyer	[118.] Songchai Ugsonkid, Sakon Tangkawsakul and Weerawat Thaikam	<i>Developing pre-service teachers' real-world problem-posing abilities: A three-year study in Thailand</i>

U10, C-huset	Parallel session 5 “Short presentations”
10.30– 12.00 Chair: Derk Pik	[119.] Lourdes Maria Werle de Almeida and Bárbara Nivalda Palharini Alvim Sousa <i>Students' metacognitive behaviour identified on the construction of mathematical modelling cycles</i>
Chair: Tadashi Misono	[120.] Derk Pik, Sevinç Göksen-Zayim and Carla van Boxel <i>When should students start making assumptions: a thinking aloud study</i>
Chair: Lourdes Maria Werle de	[121.] Tadashi Misono <i>A misconception about number and quantity</i>
U11, C-huset	Parallel session 5 “Short presentations”
10.30– 12.00 Chair: Simon Barlovits	[122.] Ángeles Domínguez, Cesar Lozano and Corey Brady <i>The importance of mathematics in artificial intelligence</i>
Chair: Britta Eyrich Jessen	[123.] Simon Barlovits, Matthias Ludwig, Jonas Bergman Ärlebäck, Susanna Carreira, Georgios Fesakis, Alissa Fock, Hélia Jacinto, Sara Julsgård, Tim Läufer, Claudia Lázaro, Maria Anastasia Moustaka, D.-X. Ken Oehler, Juan Antonio Reyes and Hans-Stefan Siller <i>Entering the next DIM²ENSION: How mathematical modelling in the context of sustainability can be supported by a digital device</i>
Chair: Ángeles Domínguez	[124.] Britta Eyrich Jessen, Metine Rahbek Tarp and Nicolaj Pape Frantzen <i>Why model through machine learning in upper secondary mathematics? A Danish case study</i>
U14, C-huset	Parallel session 5 “Short presentations”
10.30– 12.00 Chair: Kazuki Koike	[125.] Cesar Gallart, Clara Segura, Irene Ferrando and Luis Albarracín <i>Relationship between pre-service teachers' flexibility and performance in solving Fermi problems: does the setting make a difference?</i>
Chair: Cesar Gallart	[126.] Kazuki Koike, Yu Nakajima, Shuto Yano and Masashi Ikemura <i>What cue lead students to begin mathematizing in a human-centered way? —An exploration through the example of gifting oranges to relatives</i>
Chair:	[127.]

U15, C-huset	Parallel session 5 “Short presentations”
10.30– 12.00 Chair: Trude Sundtjønn	[128.] Hiroki Hatsuda and Kensuke Koizumi <i>A case study emphasizing the importance of idealization in mathematization of real-world phenomena for pre-service teachers: Focusing on application of Fermi problems</i>
Chair: Burçak Yaman	[129.] Trude Sundtjønn <i>Authenticity in vocational mathematics exam tasks in Norway</i>
Chair: Hiroki Hatsuda	[130.] Burçak Yaman and Emine Nur Ünveren Bilgiç <i>Challenges in implementing a technology-integrated mathematical modeling (MM) course: A mentorship-based collaborative study</i>
12.00 – 13.30	Lunch to go
Various pickups places, TBA	Excursion
13.30-	

Thursday, August 14, 2025

Time/Location	Activity
C4, C-huset	Plenary 4
09.00 – 10.00 Chair: Yuriy Rogovchenko	[131.] Kimmo Eriksson <i>Modeling Moral Change: A Case Study in Developing Explanatory and Predictive Mathematical Models</i>
10.00 – 10.30	Coffee break
U1, C-huset	Parallel session 6 “Long presentations”
10.30– 12.00 Chair: Catharina Beckschulte	[132.] Kenneth Rosenhagen Madsen Megerlig and Morten Blomhøj <i>Students conceptual understanding in exam tasks with models in Danish high schools</i>
Chair: Kenneth Rosenhagen Madsen Megerlig	[133.] Catharina Beckschulte, Jascha Quarder, Annika Rosendahl and Stanislaw Schukajlow <i>Assessment of mathematical modelling in digital portfolios – What competencies can be identified?</i>
U2, C-huset	Parallel session 6 “Long presentations”
10.30– 12.00 Chair: Rebecca Ellis	[134.] Liliana Luca and Giuseppe Pappalardo <i>Inside the modeling mind: A dynamic framework for cognitive processes in mathematical problem solving</i>
Chair: Liliana Luca	[135.] Rebecca Ellis, Adewale Adeolu, Benjamin Galluzzo and Jie Chao <i>Using mediated writing to facilitate communication in mathematical modelling class</i>
U3, C-huset	Parallel session 6 “Long presentations”
10.30– 12.00 Chair: Jill Brown	[136.] Yuriy Rogovchenko and Svitlana Rogovchenko <i>Epistemological aspects of teaching mathematical modeling to biology students: Do today’s approaches meet tomorrow’s needs?</i>
Chair: Yuriy Rogovchenko	[137.] Jill Brown and Gloria Stillman <i>A practitioner’s giving meaning to differentiation in a mathematical modelling class through teaching tactics</i>

U4, C-huset	Parallel session 6 “Long presentations”
10.30– 12.00 Chair: Cyril Julie	[138.] Vince Geiger, Kim Beswick, Merrilyn Goos, Jill Fielding, Thorsten Scheiner and Mirjam Schmid <i>Teachers connecting compassion and empathy to dilemmas related to social and environmental justice decision-making</i>
Chair: Vince Geiger	[139.] Cyril Julie <i>Using an index to trace the long-term trajectory of a continuous development project to enhance achievement in a high-stakes school mathematics examination</i>
U6, C-huset	Parallel session 6
10.30– 12.00 Chair:	[140.]
Chair:	[141.]
U7, C-huset	Parallel session 6 “Short presentations”
10.30– 12.00 Chair: Pauline Riemer	[142.] Jeannette Galleguillos <i>Prospective teachers developing modeling projects: Difficulties, learning and interest</i>
Chair: Josefa Castillo-Funes	[143.] Pauline Riemer and Gilbert Greefrath <i>Investigating the structure of digital mathematical modelling competence – Introducing the ModiWe project</i>
Chair: Jeannette Galleguillos	[144.] Josefa Castillo-Funes and Carolina Guerrero-Ortiz <i>Relevance of extra-mathematical knowledge in the modeling processes of future elementary school teachers</i>
U10, C-huset	Parallel session 6 “Short presentations”
10.30– 12.00 Chair: Peter Frejd	[145.] Andreas Brandsæter, Camilla Skog, Morten Ingvaldsen and Ingeborg Berget <i>How students validate mathematical models: A case-study of Norwegian 9th grade students</i>
Chair: Tsukasa Katsuragi	[146.] Peter Frejd <i>Potentials for fostering the spirit of mathematical modelling in forest education</i>
Chair: Andreas Brandsæter	[147.] Tsukasa Katsuragi, Yasufumi Kuroda and Kosuke Iba <i>Learning possibilities created by the fusion of steam education and origamics</i>

U11, C-huset	Parallel session 6 “Short presentations”
10.30– 12.00	[148.]
Chair:	
Chair:	[149.]
Chair:	[150.]
U14, C-huset	Parallel session 6 “Short presentations”
10.30– 12.00	[151.] El Moubarik Moulay Mbarek, Rachid El Jid and Mohamed Hanini
Chair: Susana Carreira	<i>The "data processing and organization" a comfortable habitat for teaching mathematical modeling in primary Moroccan school</i>
	[152.] Susana Carreira, Alice Santos, Susana Carreira and Floriano Viseu
Chair: El Moubarik Moulay Mbarek	<i>Modelling for equity: Mathematical modelling and social justice from a sociocritical perspective</i>
	[153.]
Chair:	
U15, C-huset	Parallel session 6 “Short presentations”
10.30– 12.00	[154.] Marieke Ende and Nils Buchholtz
Chair: Frank Förster	<i>Error-diagnostic competence in the context of Math Trails – an intervention study</i>
	[155.] Frank Förster and Nadine Geffers
Chair: Ingi Heinesen Højsted	<i>“Stochastik digital” (Stochastics digital): Enhancing competence through a flipped classroom format with a focus on application</i>
	[156.] Ingi Heinesen Højsted
Chair: Marieke Ende	<i>Designing for change: Challenges and opportunities in implementing mathematical modelling in Faroese schools</i>
12.00 – 13.30	Lunch
U1, C-huset	Parallel session 7 “Long presentations”
13.30– 15.00	[157.] Matsuzaki Akio
Chair: Tomas Højgaard	<i>Confirming and supplementing of a mathematician’s modelling competencies</i>
	[158.] Tomas Højgaard
Chair: Matsuzaki Akio	<i>Mathematical modelling competency and textbook design</i>
U2, C-huset	Parallel session 7 “Long presentations”
13.30– 15.00	[159.]

Chair:	
Chair:	[160.]

U3, C-huset	Parallel session 7 “Long presentations”
13.30– 15.00 Chair: Daniel Orey	[161.] Marielena Menzel, Timo Kosiol, Stefanie Rach and Sebastian Geisler <i>How do students value modeling with and without experiments (differently) and why?</i>
Chair: Marielena Menzel	[162.] Daniel Orey, Osvaldo Rosa Filho and Milton Rosa <i>The art of sisal weaving in a local community in Brazil: An ethnomodelling investigation</i>
U4, C-huset	Parallel session 7 “Long presentations”
13.30– 15.00 Chair: Serife Sevinc	[163.] Lisa Steffensen <i>Peatland are ugly, or?</i>
Chair: Lisa Steffensen	[164.] Serife Sevinc, Gonul Erhan and Mesture Kayhan Altay <i>Pre-service mathematics teachers’ self-posed outdoor modelling problems: Analyzing outdoor aspects in the solutions</i>
U6, C-huset	Parallel session 7 “Long presentations”
13.30– 15.00 Chair: António Júlio Aroeira	[165.] Mariela Elizabeth Ramírez-Inca and Hugo Rogelio Mejía-Velasco <i>A model development sequence for understanding calculus concepts: The case of the definite integral</i>
Chair: Mariela Elizabeth Ramírez-Inca	[166.] António Júlio Aroeira, , Susana Carreira and João Pedro da Ponte <i>The students as a decision-making variable: How teachers select and develop modelling tasks</i>
U7, C-huset	Parallel session 7 “Short presentations”
13.30– 15.00 Chair: Carol Sepúlveda-Herrera	[167.] Sakon Tangkawsakul and Chatree Faikhamta <i>Mapping the path: Learning trajectories for enhancing mathematics teachers' competence in mathematical modelling</i>
Chair: Kento Nakamura	[168.] Carol Sepúlveda-Herrera, Jaime Huincahue, Milton Rosa and Daniel Clark <i>Chess players’ spacial ethnomodel: emic, etic and dialogic characterization</i>
Chair: Sakon Tangkawsakul	[169.] Kento Nakamura, Haruka Sakurada, Yuki Watanabe and Tadashi Misono <i>Development of story-based instructional materials: Fantasy and misdirection context</i>

U10, C-huset	Parallel session 7 “Short presentations”
13.30– 15.00 Chair: Samantha Quiroz Rivera	[170.] Shota Higuchi and Yuki Watanabe <i>Productive failure in mathematical modelling</i>
Chair: Duncan Mhakure	[171.] Samantha Quiroz Rivera, José David Zaldívar Rojas and Aleida Quiroz Rivera <i>Adapting the mathematical modeling cycle to work with students with autism spectrum disorder</i>
Chair: Shota Higuchi	[172.] Duncan Mhakure Miranda Moodley, Wisani Hlangwani, Arne Jakobsen and Jacob Jaftha <i>Validating an adapted mathematical modeling knowledge and attitude scale (MMKAS) for South African university students</i>
U11, C-huset	Parallel session 7 “Short presentations”
13.30– 15.00 Chair:	[173.]
Chair:	[174.]
Chair:	[175.]
U14, C-huset	Parallel session 7 “Short presentations”
13.30– 15.00 Chair:	[176.]
Chair:	[177.]
Chair:	[178.]
U15, C-huset	Parallel session 7 “Short presentations”
13.30– 15.00 Chair:	[179.]
Chair:	[180.]
Chair:	[181.]
C4, C-huset	Reflection fika (including coffee break)
15.00 – 16.30 Moderator: Susana Carreira	[182.] Mogens Niss and Peter Galbraith
C4, C-huset	Business meeting
15.00 – 16.30	
Flygvapenmuseum Malmslätt	Conference dinner
19.00 –	

Friday, August 15, 2025

Time/Location	Activity
C4, C-huset	Plenary Panel
09.00 – 10.30 Moderator: Pauline Vos	[183.] Marcelo C. Borba, Vince Geiger, Britta Eyrich Jessen, and Hans-Stefan Siller
10.30 – 11.00	Coffee break
C4, C-huset	Henry Pollak award and closing ceremony
11.00 – 12.30	
12.30 – 13.30	Lunch to go

Abstracts

[1.] **Perspectives on mathematical modelling education – quo vadis?**

Gabriele Kaiser

(University of Hamburg, Germany & Nord University, Norway)

Mathematical modelling education has a long tradition in many parts of the world. However, discussions about the primacy of real-world context or mathematics have shaped it since its very beginning, dating back to the 19th century.

Based on these central paradigmatic differences, Kaiser-Messmer (1986) distinguished two main perspectives in the discourse: A pragmatic perspective, which focuses on utilitarian or pragmatic goals, and a scientific-humanistic perspective, which is oriented towards humanistic ideals of education. Kaiser and Sriraman (2006) synthesized international research to further differentiate this distinction into a typology of international perspectives on modelling in mathematics education, providing an overview of the convoluted debate. Based on a classification of the different aims of mathematical modelling education and their epistemological background, five perspectives and a meta-perspective were distinguished. The original distinction between the two opposing positions was taken up and further developed as realistic or applied modelling and epistemological or theoretical modelling. Further perspectives were introduced, including educational modelling, contextual modelling, socio-critical modelling and, as a meta-perspective, cognitive modelling. Blum (2015) further developed these perspectives by conceptualizing them as a pair of aims and suitable examples, and by proposing six different perspectives. The influence and usage of these two typologies have been evaluated in a systematic literature review by Krawitz et al. (2025), continuing the work of Preciado Babb et al. (2023).

Following such extensive developments, questions arise regarding the future directions of mathematical modelling education and related research. It seems to be timely to ask whether all the perspectives distinguished in these two typologies are still vivid or have been further developed, or whether new perspectives have emerged. Evaluating the current discourse in ICTMA proceedings and important journal papers reveals that new topics have been proposed or have gained more relevance, such as the metacognitive skills of students and teachers, and creativity and curiosity in mathematical modelling education. New perspectives have emerged, such as ethnomodelling, which emphasizes the important role of culture and cultural sensitivity. Meanwhile, the importance of other approaches, such as mathematical literacy and critical thinking, has increased.

Following a description of the discourse, the talk will tentatively describe further developments.

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[2.] Enabling excellence in modelling: Investigating an IMMC performance

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According to Zulkarnaen (2021) “the notion of mathematical modelling in school is different from the way it is understood by practical mathematical modelers”, making meanings assigned to “mathematical modelling” in education vulnerable to whims, preferences and theories of individual researchers and practitioners. Alternatively maintaining the integrity of “mathematical modelling” as a process practised professionally *outside* education has implications for what *should* legitimately be included under its banner *within* education. See (Pollak 2011, p.64).

So what really matters is learning and practicing the mathematical modelling process. The particular field of application – whether it is everyday life, being a good citizen, or understanding some piece of science – is less important than experiencing this thinking process.

This appeals for alignment between the two interpretations because of the value of the thinking process behind modelling that must be learned and applied. Similar ideals lie behind the International Mathematical Modelling Challenge (see: IMMC, 2025), which is based on the belief that experiencing the power of mathematics in solving real world problems outside of mathematics itself, is an essential component of mathematical learning. The website includes examples of challenge questions from the past decade.

IMMC performance is reported annually by an international judging panel. Strong performances over time have been achieved by particular schools in three Australian states. Our research context is a Queensland school which achieved Meritorious ratings in 2019 (including the top international ranking) and 2024; and an Honourable Mention in 2023. Significantly, this spanned a time which saw a complete change in school personnel – both in mathematics leadership and in team advisors. Three research questions are the focus here – there are others.

RQ1. What characterises the performance of a successful team – both in terms of mathematical modelling activity and as a collaborative group?

RQ2. How is modelling quality enhanced through provision of non-specific (to the problem) modelling support?

RQ2. What actions are employed by school personnel to assure continuance and growth of a modelling culture within the school?

Data sources: These comprise (A) performance data provided in the team modelling report. (<https://www.immchallenge.org.au/supporting-resources/previous-immc-problems/previous-problem-2024>); and (B) structured interview data provided by a leadership staff member (HOD), Team Advisor, and members of the student team. Questions were open ended, enabling interviewees to initiate and add information they deemed relevant when responding.

Evidence indicates that modelling performance was influenced strongly by team cohesion, and by ensuring that all stages of the modelling process were planned for, enacted thoroughly,

and reported comprehensively. This required allocating time for reflection on issues, including dilemmas, that emerged during the process.

The team mentor provided support such as checking that all aspects of the problem were addressed, and acting as a critical friend in terms of report structuring (no modelling detail was included). Supporting a school-wide mathematical modelling culture, involved (among other things) drawing from learning approaches from primary and junior levels, and providing experience for students to work together in solving open-ended problems throughout the curriculum.

The presentation will illustrate data relevant to RQ1, RQ2, and RQ3 and other questions of interest. It will also reflect on outcomes in terms of modelling constructs such as the modelling cycle, and conceptions of modelling competencies.

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[3.] Decomposition of modelling sub-competencies into elements to assess mathematical modelling

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Modelling can be elaborated as sub-competencies (Maaß, 2006; Niss et al., 2007), such as simplifying, mathematising, working mathematically, interpreting and validating. Sub-competencies have been the basis of modelling assessment methods, including multiple choice tests (e.g., Hankeln et al., 2019) and rubrics (e.g., Toalongo et al., 2022). Multiple choice items that assess individual sub-competencies do not reveal students' model construction process or the full nature of their understandings (Tekin-Dede & Bukova-Güzel, 2018). Rubrics can be problematic due to subjectivity and overlaps in choice of aspects to include (Tekin-Dede & Bukova-Güzel, 2018; Toalongo et al., 2022). In addition, multiple modelling assessment modes and theoretical frames exist (Cevikbas et al., 2022), but few are derived theoretically or empirically (Frejd, 2013).

We introduce a non-weighted assessment scheme based on sub-competencies and supported with empirical and theoretical work. The scheme uses seven, five, six, four and seven elements for five modelling sub-competencies of simplifying, mathematising, working mathematically, interpreting and validating, respectively. Element identification involved review of modelling literature to identify types of modelling actions attributed to each sub-competency. Table 1 shows elements for simplifying and citations to a sample of relevant literature.

Table 1

Elements of Simplifying and Examples of Supporting Literature

Code	Elements of Simplifying
S1	Identifying and clarifying goal of context or client ^{3,5,7}
S2	Clarifying and understanding problem ^{3,5,6,7}
S3	Making assumptions about aspects of context ^{2,4,5,6,7}
S4	Differentiating between relevant and irrelevant information for a given purpose ^{1,4}
S5	Choosing or gathering real world information ^{2,7}
S6	Identifying quantities, geometric shapes, and constraints in real-world context ^{1,4,5,6,7}
S7	Acknowledging or describing relationships among quantities and shapes ^{1,3,4,5,6,7}

Note. References are ¹Blomhøj & Jensen, 2003; ²Blomhøj & Kjeldsen, 2011; ³Kaiser, 2007; ⁴Maaß, 2006; ⁵Stillman et al., 2007; ⁶Tekin-Dede & Bukova-Güzel, 2018; ⁷Toalongo et al., 2022.

We developed the scheme to evaluate the effectiveness of the M2Studio Project (Chao & Galluzzo, 2022) digital environment and instructional materials. Participants were 433 secondary school students who began as novice modellers and completed a 14.5-hour learning experience. We analysed written work of individual students and video recordings of small groups as they engaged in open-ended modelling tasks. We applied the elements through simultaneous deductive coding to identify instances of each. Counts of and patterns in instances of elements allowed us to identify growth, elaborate modeling paths and make comparisons. This scheme

captured fine grain growth in modelling. Coding elements of simplifying allowed us to make a distinction between frequency of modelling actions (e.g., 10 instances of simplifying) and different simplifying actions (e.g., S3, S5, S6 and S7). Patterns in elements from different phases illustrate the complexity of modelling routes (Borromeo Ferri, 2010). We used the elements to observe differences between a group's modelling conversation, its interactions in the environment and its written reports. School practitioners have embraced the scheme. We anticipate the scheme might benefit other colleagues.

Acknowledgement

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[4.] From Interpreting Designs to Creating New Concepts: Integrating Empathy into the Mathematical Modelling Process

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Mathematical modelling (hereafter "modelling") has traditionally focused on quantitatively representing real-world phenomena, often overlooking human decision-making and the creator's intent, which are distinct from descriptive models (Meyer, 1984; Niss, 1989, 2008, 2015). In contrast, approaches such as prospective modelling (Niss, 2015; Stillman, 2019) and design thinking (Grant & Berry, 2011; Gudipati & Sethi, 2016; Li et al., 2024; Trung et al., 2024) emphasize empathy, understanding user needs, and generating innovative ideas. By incorporating empathy into modelling, these approaches aim to foster a more user-centered, creative, and holistic form of problem-solving, which can better address the complexities of human decision-making.

This case study focuses on three key aspects: the purpose (i.e., the intended use of the furniture), the user perspective (focused on usability and aesthetic appeal) (Hekkert & Schifferstein, 2008; Norman, 2013), and the mathematical perspective (focused on geometric properties and transformations). By embedding empathy into modelling, this study explores how students interpret designs and generate innovative ideas. It takes into account both the creator's intent and the user's perspective, rather than relying solely on descriptive approaches. This exploration leads to the following research question: How does incorporating empathy into modelling affect students' cognitive processes and affective involvement in interdisciplinary learning?

The study involves 30 ninth-grade students (ages 14–15) who have studied parallelograms and similarity. The lesson spans three 50-minute sessions in late February 2025, with one of the authors serving as the instructor. The interdisciplinary activity revolves around furniture design, with pizza trays as a central example. In the first two sessions, students interpret existing designs, and in the third, they create their own. This progression encourages both analysis and creative design, fostering a deeper understanding of user needs, mathematical principles, and aesthetic considerations. After each session, students reflect on their learning and write down their thoughts. At the end of the third session, many students, who had not been able to complete their designs during class, were assigned the task of creating their designs as homework. They were then asked to write an explanation of their designs.

Students' responses and design explanations will be analyzed using content analysis, categorizing them into three perspectives: (1) the purpose, (2) the user perspective, and (3) the mathematical perspective. The analysis will focus on how these elements align with both user needs and mathematical principles to create well-designed objects. Following the initial content analysis, thematic analysis will be conducted to identify emerging themes that extend beyond the predefined categories, providing a deeper understanding of students' cognitive and affective engagement in the design process.

Through the design process, students faced the challenge of balancing purpose, usability, aesthetics, and the mathematical transformations of shapes. This experience sparked their interest in the work of designers, prompting them to reflect on their own design thinking from multiple perspectives and take their first steps toward creative thinking. By integrating modelling with design thinking, students engaged in a holistic learning process, where creativity, user-centered thinking, and mathematical reasoning played key roles. The study underscores the significant role of empathy, as students not only considered geometric and usability aspects but also how their designs would be perceived by users and designers. This empathetic approach to modelling aligns with Norman (2013), who emphasize the importance of empathy in understanding user needs and ensuring designs meet both practical and emotional expectations.

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[5.] Algorithms as Parts of Modelling Processes: Reflections for Mathematics Education in the Age of AI

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This talk addresses the role of algorithms within mathematical modelling from a mathematics education perspective, revisiting both theoretical foundations and recent developments. Based on frameworks such as the modelling cycle (Blum & Leiß, 2005), constructivist approaches (Ernest, 2016; Freudenthal, 1983), and Bruner's representational levels (Bruner, 1976), we examine how algorithmic thinking and modelling interact in educational contexts. We also take into consideration the development of AI within educational context, both on school level and university level (Opesemowo et al., 2024).

Algorithms are increasingly present across domains since a long time, e.g. in mathematics, computer science and education, raising the question of how their common integration shapes learners' understanding of modelling. While algorithmic structures can support orientation within complex modelling tasks and facilitate learning processes, there is a growing tendency, especially in teacher education, to interpret modelling schemes as literal algorithms. This formalization may conflict with the broader educational aims of mathematical modelling (Greefrath et al., 20216), such as fostering creativity, critical reflection, and an understanding of mathematics as a scientific and cultural practice.

In the light of current developments in artificial intelligence, particularly the emergence of generative models and automated problem-solving systems, these tensions become more pronounced. On the one hand, AI offers new didactic opportunities, e.g., for adaptive support and dynamic modelling environments. On the other, it risks reinforcing narrow, procedural conceptions of modelling and weakening students' engagement with the modelling cycle as a flexible, interpretative process.

By discussing theoretical insights and empirical observations from teacher education, the presentation invites a dialogue on how mathematics educators can preserve the epistemic and educational richness of modelling while thoughtfully integrating algorithmic tools, including AI, into teaching and learning. The aim is to promote a modelling culture that values independent thinking, meaningful mathematization, and critical engagement with technological developments as well as an adequate imbedding in categories of the theory of science and modelling (Möller, 2014).

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[6.] Ethnomathematical Knowledge of Dry Rice Producers in Northeast Brazil through the Development of Ethnomodels

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This paper presents situations related to geometric visualization developed and used by local farmers in the town of Amarante, in the state of Piauí, Brazil. In addition, dialogic ethnomodels were presented and discussed in the analytical and interpretative phases of this study. The participating family farmers have their own measurement units, such as *braças* and *tarefas*, which help them to satisfy their daily needs of survival and solving problems related to their own environment to transcend to other sociocultural contexts.

The mathematization process used by these members is related to calculations of area of regular and irregular plots of land of land using a technique named *cubação*. By using locally developed techniques, the farmers in this study determine these areas using diverse geometric figures, including quadrilaterals, triangular, circular, and hexagonal shapes, which enabled the identification of procedures used in the elaboration of dialogic ethnomodels in accordance with ethnomodelling.

The visuospatial reasoning served as an important tool that enabled the authors to understand geometric knowledge developed in distinct cultural contexts. Thus, Owens (2020) state that: “Visuospatial reasoning incorporates visual imagery, spatial capabilities, and spatial or embodied imagery from gestures and movement (p. 11). This study also discussed the relevance of geometric knowledge of participating family farmers to the elaboration of ethnomodels, which represents activities they conduct in their daily practices (Rosa & Orey, 2019).

This study contributed particularly to the authors’ personal, professional, and academic development, given that the first author has also been a member of this distinct cultural group of family farmers since his birth. In this context, as the son of family farmers, the first author grew up helping his parents in dryland rice agriculture. Thus, this study enabled the first author to strengthen his cultural roots and identity, as well as to value and respect members of other cultures.

A local (emic) approach is essential for an intuitive understanding of geometric techniques and procedures used by members of distinct cultures, while a global (etic) approach is important for comparing geometric practices developed in diverse sociocultural contexts. A glocal (cultural dynamism) approach pursues a broad understanding of geometric knowledge developed by the members of distinct cultures throughout history by showing its sociocultural influences on the visualization processes (Orey & Rosa, 2021).

The glocal approach used in this investigation allowed us to associate the knowledge of family farmers, with the geometric knowledge present in the local school/academy, mainly regarding area calculations of distinct and diverse formats, which often do not appear in schools nor have

the necessary emphasis because they are related to local geometric practices that students experience in their daily lives.

Ethnomodelling proposes a broad cultural perspective of mathematical modelling (glocal/dialogical) by connecting unique aspects of local culture (emic) with global geometric knowledge systems (etic) (Rosa & Orey, 2010). The use of these approaches enables the translation of phenomena present in the daily lives of family rice farmers through the elaboration of ethnomodels, which can represent how these members visualize geometric practices they develop daily.

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[7.] Freedom of Choice: Ethnomathematical Modeling with Interactive Tools

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Multicultural societies present unique challenges and opportunities in the teaching and learning of mathematics. In Israel, a nation distinguished by its rich cultural diversity, mathematics educators are called to navigate complex cultural dynamics and address varying traditions and worldviews within the classroom. Integrating these diverse perspectives is not merely a pedagogical challenge but a potential source of enrichment, particularly when approached through the ethnomathematics lens.

This study investigates how mathematics educators can promote respect, dialogue, and solidarity while leveraging cultural diversity as a pedagogical resource. Specifically, it explores how mathematical modelling with interactive tools can serve as a bridge between students' cultural values and learning mathematics.

The research was conducted at an academic teachers' college in Southern Israel, where Jewish and Arab students learn together. The study involved two groups comprising 49 first-year students: 34 Arab students from the Bedouin community and 15 Jewish students, including both immigrants and Israeli-born.

The participants engaged in a project in which they created mathematical models to represent culturally significant values. They were asked to select a value or issue important to them and represent it as an artwork, created using the Desmos interactive platform, where graphs of functions served as building blocks. One group selected "Freedom of Choice" as a collective theme, while the other group allowed each student to pursue an individual theme of personal significance.

Data was collected from three primary sources: (a) the mathematical projects produced by the students, (b) students' written reflections documenting their processes, and (c) the instructor's observation notes throughout the project.

Utilizing the interactive Desmos platform enabled them to identify, describe, and translate cultural phenomena into mathematical representations through mathematical modelling. An important outcome of the project was the mutual learning that occurred among students. They learned from each other's experiences, gaining insights not only into diverse cultural values and issues but also into different ways of interpreting these ideas through mathematical modelling. This collaborative process enabled students to develop a meaningful and deep understanding of mathematical concepts, such as functions and functions' transformations. Moreover, the project encouraged cross-cultural learning, as students expanded their appreciation of each other's backgrounds and perspectives. The students acted as the primary authorities in choosing their topics and questions, creating a strong sense of ownership and engagement. Through ethno-modelling processes, the students connected mathematics with their cultural identities, fostering a learning environment built on respect, dialogue, and meaningful mathematical modelling with interactive tools.

[8.] Modelling for implementing sustainability in mathematics education: How tasks affect

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Mathematical modelling in education aims to equip students with the tools to contribute to society as informed citizens. To argue wisely as an informed citizen from a social perspective and to pose the pertinent questions, mathematical modelling should be employed as a method for (complex) problems in real life (Siller, Vorhölter, et al., 2024). To this end, it is imperative that students develop the ability to identify key influencing factors in the problems they encounter and comprehend them in a manner that enables them to apply mathematical concepts and methods to solve problems based on formulated questions and, ideally, arrive at solutions. Additionally, students should be equipped with the ability to critically evaluate existing models of real situations and the solutions and measures derived from them. Furthermore, students are expected to learn to communicate their own and others' results and to derive options for action (Siller, Günster, et al., 2024).

Given the increasing frequency and complexity of disruptive events, the development of these competencies has become paramount. Thus, recognizing, evaluating, and acting are the three areas that students in Germany should acquire as part of education for sustainable development (ESD). These have been differentiated in a subject-specific way so that they can be integrated into the competence-oriented curriculum of the mathematics subject (Reiss et al., 2016; Siller, Vorhölter, Oldenburg, et al., 2025). To ensure the effective integration of these competencies, it is crucial to incorporate suitable tasks within subject lessons that not only demand but also promote the application of these competencies (UNESCO/MGIEP, 2017).

A study was conducted with students who worked in 34 different small groups. The study investigated the ESD competences that students demonstrated in maths lessons. Six distinct modelling problems that differ in the context of sustainable development were used as a basis for the intervention. Employing a structured qualitative content analysis and a combination of deductive and inductive categories, a total of 15 competencies were identified, categorized into the three domains of recognizing, evaluating, and acting. Additionally, the math-specific components of the competences were analysed (Siller, Vorhölter, Just, et al., 2025).

The presentation will focus on two of these problems, which have different contexts and also differ in terms of the work assignments and the material provided. The presentation will include a comparison of the similarities and differences in the students' completion of the tasks in relation to ESD. To this end, the work of two groups on the respective tasks is presented. The results indicate that both the tasks and the material provided have a relevant impact on the ESD competencies students demonstrate and the proportion of mathematical considerations involved. However, it is also evident that the characteristics, attitudes, and preferences of the students influence the extent to which they utilize the tasks to engage more deeply with the context.

The results illustrate the assumption that, on the one hand, the task design is important, but at the same time the teacher's interventions and their expectations and prompts determine the extent to which the students take the context seriously and perceive it as a learning opportunity for ESD.

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[9.] Assessing Portfolios in the Context of Mathematical Modelling – Perceived Expectancy and Values of Pre-Service Teachers

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The importance of mathematical modelling has been discussed for decades. However, teachers do not often ask students to solve modelling problems in the classroom (Greefrath & Maaß, 2020). One explanation is that difficulties arise in integrating authentic modelling tasks in traditional written tests. According to Frejd (2013) portfolios have the potential to be a suitable form of examination for mathematical modelling, because they enable assessing entire modelling processes. The assessment of mathematical modelling, however, can be challenging, especially for unexperienced teachers (Borromeo Ferri, 2022; Krall, 2023). There is evidence that teachers perceive assessing portfolios as time-consuming, which can discourage them from implementing portfolios in their teaching practice (Krall, 2023). From the perspective of the situated expectancy-value theory (SEVT), the time required to assess portfolios can be perceived as an effort cost that affects teachers' choice to use portfolios assessment negatively (Eccles & Wigfield, 2020). According to the expectancy-value model of achievement choices, motivation arises when a person values an action or a task positively and also expects to perform it successfully. The value of a task comprises four components: intrinsic, utility and attainment value as well as the perceived costs (Eccles, 2007). As pre-service teachers tend to identify with reform-oriented teaching methods (Gainsburg, 2012), they might value portfolio assessment positively. Given the assumption that teachers' motivation impacts the integration of innovations into classrooms (Dreer et al., 2017), it is essential to investigate motivational aspects during the assessment process that influence (pre-service) teachers' willingness to implement portfolios in modelling.

This study examines the following research question: *What do pre-service teachers expect and value with respect to the assessment of portfolios in mathematical modelling?* To investigate this research question, a qualitative guided interview study is conducted with eight pre-service teachers. The participants took part in a portfolio project for modelling with ninth graders. The pre-service teachers supported the students in the classroom and were responsible for the assessment of modelling competencies. An assessment form was developed to scaffold the pre-service teachers' assessment processes. In conclusion, the pre-service teachers were questioned in an open interview about how they perceived the assessment, how they got along with it and what chances and challenges they realized. The interviews will be analyzed with a qualitative content analysis with the aim to reveal the participants' expectancy and values during the assessment of portfolios. The interviews will be analyzed inductively-deductively, using the categories of the SEVT framework.

The results will provide insights about the process of assessing portfolios from a (pre-service) teachers' perspective, which can enrich discussions about the potential and the design of portfolio assessment for mathematical modelling in teaching practice.

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[10.] Adapting Mathematical Modelling Tasks to a Variety of Levels for Teaching in a Heterogeneous Classroom

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Mathematical modelling (MM) fosters deep understanding of the mathematics taught in schools by connecting its application to real-world problems (Brown et al., 2024; Siller et al., 2023), specifically in STEM contexts (Kohen & Orenstein, 2021). However, despite growing interest in MM, its implementation in the classroom remains challenging due to the cognitive demands of the tasks and obstacles in each phase of the MM process (Borromeo Ferri et al., 2023; Klock & Siller, 2020). It is mostly evident in heterogeneous classrooms, where the teacher must adapt a MM task to meet the needs of a wide range of students (Wess & Greefrath, 2019). This highlights the need for a more systematic understanding of how MM tasks can be designed to support learners at various competency levels.

This study will illustrate how MM tasks in STEM contexts can be modified to demonstrate various levels, while referring to different phases of the MM process. Specifically, the levels are based on the nature and amount of scaffolding provided for progress in the MM process. To assess MM task, an assessment rubric (Kohen & Gharra-Badran, 2023) is used which includes criteria to differentiate between different levels of three main phases of the MM process, namely mathematising, working mathematically, and interpretation. These phases were chosen since for the MM tasks which are at the focus of this study, a more intense scaffolding is provided for the first two phases of the MM process, i.e. understanding and simplifying (Kohen, 2025). Assessment of the validation phase is also excluded since this phase seeks to ensure that the mathematical results align with real-world phenomena, a process that can be particularly challenging in complex STEM contexts.

Participants are four experts in MM, two of whom were actively involved in developing the MM tasks, and for this study, they further refined multiple versions of these tasks. The other two experts, who constructed the assessment rubric, evaluated and ranked the modelling levels for the various versions for validity purposes. A process of about three rounds of refinement of the versions was carried out between the experts until a satisfactory consensus was reached.

Focusing on an MM task in a STEM context called "YouTube", as a case study, which presents an engineering challenge that allows direct and continuous viewing of videos, resolving using motion problem (Kohen & Nitzan-Tamar, 2022), the study presents a formative evaluation method for modifying the task for learners into three modelling levels (low/medium/high) for each of the three main MM phases. The amount of scaffolding provided varies between versions, which affects the difficulty level of each version. For example, scaffolding for the mathematising phase may involve either providing explicit modelling hints (low level) or requiring complex representation shifts (high level). Thus, nine different versions of the task can be edited.

This study introduces a distinctive approach to adapting MM tasks to different learning needs. Specifically, a new theoretical concept – *modelling levels* – is proposed for an existing MM task. As a methodological contribution, the study presents a method for adjusting the difficulty level

of an MM task at each phase of the MM process, based on an existing assessment rubric. Finally, the study offers a practical contribution by enhancing the teaching of MM in heterogeneous classrooms.

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[11.] Can we use modeling cycles to predict which instructor scaffolding moves elicit specific modeling competencies?

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A pressing instructional and research problem in mathematical modelling is understanding the influence of teachers' scaffolding moves on students' modelling processes. In the literature, these processes are usually characterized by a mathematical modelling cycle (MMC), like (Blum & Leiß, 2007). Naturally, research focused on developing, implementing, and training teachers to scaffold students' modelling processes tends to target supporting those processes (Siller et al., 2023; Stender & Kaiser, 2015). From our cognitive constructivist perspective, scaffolding should be aimed at assisting learners in building and refining their mathematical representations. Thus, in this study, we focus on a specific kind of scaffolding, *contingent scaffolding*, which is responsive and calibrated support (van de Pol et al., 2010) to a modeler's ongoing process and interim models. We systematically examined the structural properties of a contingent scaffolding framework for modelling and then examined the influence of the moves for promoting or suppressing the sub-processes in the MMC.

We used mixed methods: qualitative methods to generate a set of instructor-student scaffolding interactions and quantitative methods to examine any associations between scaffolding moves and elicited modelling processes. The sample were 24 tertiary STEM majors familiar with differential equations who participated in 10 hour-long individual task-based interviews. The tasks asked participants to engage in *modelling from first principles* to create differential equations for dynamic scenarios (Smith et al., 1997) The data for this study are a subset of 51 videos containing interviewer-participant interactions during tasks that the interviewer only used *contingent scaffolding* (as opposed to fixed supports). We analyzed the videos qualitatively according to a modified Scaffolding Moves for Modeling Framework (mSMMF) (Czocher et al., 2024; Stender & Kaiser, 2015) and an observational rubric for the mathematical modeling cycle (Czocher, 2016) and then *quantized* the resulting analysis (Sandelowski et al., 2009). We then used cluster analyses (Aitchison, 2011) to group similarly-functioning moves together and logistic regression to estimate the likelihood that a given scaffolding move would elicit each modeling competency.

We found that the contingent scaffolding moves in the mSMMF can be clustered into three groups: *Reasoning Quantitatively* (moves that support students' quantitative reasoning, see Thompson, 2011), *Representing* (moves that support students aligning their real-world conceptions with mathematical representations), and *Reflecting & Revising* (moves that prompt students to analyze and refine their models). Also, moves that appear similar can have differing effects on students' modeling processes depending on which Stage they reference. Further, while some scaffolding moves were consistently associated with eliciting (or suppressing) specific modelling processes, the log odds further imply that there is not a straightforward mapping between moves in the mSMMF and processes from the MMC. Our findings refute naïve hypotheses that assume a direct correspondence between moves targeting specific modeling

phases and student responses. The disappointing predictive utility casts doubt on the capacity of using MMCs as tools for creating powerful instructional moves aimed at teaching modeling skills rather than improving students' performance on specific tasks.

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[12.] Second Graders' Moves in Emergent Modeling Through a Puzzle Block Game

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Recent research underscores the significance of fostering spatial reasoning in elementary education (Pinilla, 2024), particularly through interactive activities like touchscreen games that enhance strategic thinking and problem-solving (Bruce et al., 2017). This study builds on prior work by examining how second-grade students engage in emerging modeling to develop spatial reasoning while playing a Puzzle Block Game.

We employed two conceptual frameworks to guide our research. Pinilla's (2024) framework on spatial reasoning skills informed our analysis of students' moves in the modeling process, while Gravemeijer's (1999) framework helped us categorize students' levels of model use in supporting spatial reasoning. Our exploratory qualitative study involved 12 second-grade students in an afterschool program at a public elementary school in the Midwestern United States. Working in six pairs, students participated in 45-minute interview sessions where they used a touchscreen iPad to play a Block Puzzle Game.

Data included four and a half hours of video-recorded interviews and transcriptions. Using thematic analysis (Clarke & Braun, 2013), our preliminary findings identified two move types: Exploratory (five sub-moves) and Strategic (seven sub-moves). Exploratory moves were prevalent at the situational level, while Strategic moves emerged as students optimized grid areas and spatial organization. At the formal level, students recognized patterns, partitioned rectangles, and explained equal groups, laying the groundwork for multiplication concept. The study supports Gravemeijer's (2007) argument that emergent models can precede formal mathematics and offers insights for designing purposeful learning experiences.

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[13.] Teachers' Perspectives on Mathematical Modelling: A European View on Characteristics, Benefits and Challenges

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Modelling plays a crucial role in mathematics education, particularly in addressing interdisciplinary STEM problems (Hallström & Schönborn, 2019). From a school-practical perspective, modelling is of great interest, both from an atomistic and a holistic perspective (cf. Brand, 2014; Cevikbas, 2022), and is firmly embedded in curricular guidelines (cf. UNESCO, 2020). However, there appear to be considerable differences in how teachers conceptualize mathematical modelling, as well as in how they implement modelling tasks in their classroom practice. This contribution aims to empirically investigate this (subjective) impression by comparing European teachers' perspectives on mathematical modelling at the lower and upper secondary levels.

The data was gathered in an online survey distributed in early 2024 across five European countries, namely Germany, Greece, Portugal, Spain, and Sweden. A total of 130 teachers participated. In the survey, they were asked to (1) give a personal definition of mathematical modelling, (2) classify tasks with a reference to real-world contexts as modelling tasks (task and items taken from the ABCmaths project; cf. Kuntze et al., 2013), (3) assess potential benefits and challenges of modelling (cf. Borromeo Ferri & Blum, 2010; Schmidt, 2011), (4) describe their use of digital tools to support modelling activities, and (5) evaluate of the role of modelling in integrating sustainability into mathematics education.

Regarding (1), (2), and (3), key findings are sketched below. Most teachers define modelling tasks based on their real-world applicability, authenticity, and openness. However, when teachers should describe a self-designed modelling task or classifying predefined tasks, differing interpretations of these characteristics become evident. Teachers employ diverse justifications to distinguish between modelling tasks and word problems with real-world references, which indicates different ideas of mathematical modelling for their lessons. A notable example is the role of estimation and assumptions and the question whether these activities are sufficient to characterize a task to be a modelling task. Possibly related to this, Fermi problems, which inherently rely on assumptions and estimations, are significantly more frequently classified as modelling tasks by lower secondary teachers (73%) compared to upper secondary teachers (52%).

With respect to the perceived benefits of mathematical modelling, only a few teachers perceive mathematical modelling as beneficial in preparing students for final examinations (19%) or in fulfilling the curriculum (37%). This reveals a gap between modelling in the curriculum and

teachers' awareness – most teachers do not justify modelling formatively, i.e. based on its embedding as a modelling competence in the curriculum (cf. Blum, 2015). Rather, the teachers argue on a pragmatic level: They recognize the value of modelling for applying mathematics (96%, multiple choice), creative working methods (84%), and dealing with open-ended tasks (79%).

In the presentation, we present teachers' perspectives on (3) challenges, (4) the use of digital tools and (5) sustainability in mathematical modelling. By synthesizing these perspectives, we aim to delineate a comprehensive picture of how European mathematics teachers at different educational levels conceptualize and implement mathematical modelling, offering implications for future professional development and curriculum design.

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[14.] Development and Evaluation of a Beginner-Level Workshop to Improve Modelling Lessons for In-Service Secondary Mathematics Teachers with No Prior Modelling Experience

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The purpose of this study is to develop and implement a beginner-level workshop aimed at improving modelling lessons for in-service junior high school mathematics teachers with no prior experience in teaching modelling, and to evaluate its effectiveness. I presented a study on improving modelling lessons for beginning junior high school mathematics teachers in Japan at ICTMA21, held in 2023 in Japan. Based on this presentation, I authored a paper (accepted), in which I analyzed how beginning junior high school mathematics teachers improved their lessons from dealing with “word problems” to conducting modelling lessons. This analysis followed the cycle of “expansive learning” proposed by Engeström, Y. (2010), and examined the competencies these teachers developed during the process. Building on these findings, I developed a beginner-level workshop to improve modelling lessons for in-service junior high school mathematics teachers with no prior modelling experience. The workshop consisted of 4 sessions: Session 1: Clarification of Challenges, Session 2: Modelling Experience, Session 3: Lesson Plan, Session 4: Implementation and Results

The workshop was conducted with a group of 4 participants. The results were as follows: Session 1: Participants identified the main challenge as their previous lessons on “word problems,” which involved merely explaining the meaning of texts and solving mathematical problems without engaging with real-world contexts. Session 2: An analysis revealed that their previous lessons lacked the process of idealization, where students set assumptions and conditions starting from a real situation. Session 3: Participants developed a lesson plan based on a modelling task that involved determining “how many bowls of ramen needed to be sold daily for a family of 4 to sustain a living after starting a ramen shop”. Session 4: During implementation, students followed the modelling cycle and solved the modelling task.

The effectiveness of the workshop was evaluated using the 4-level model proposed by Kirkpatrick, D. and Kirkpatrick, J. (2006), which is widely used for evaluating the effectiveness of training programs across various fields. The 4 levels are: Level 1: Reaction – Participants expressed a high level of satisfaction across all 4 sessions. They provided positive feedback, such as expressing a desire to develop various types of modelling tasks after experiencing actual modelling. Level 2: Learning – The learning outcomes were evaluated based on the “4 dimensions of competence in teaching modelling” proposed by Blum & Borromeo Ferri (2009). Partial but notable improvements were observed. Level 3: Behaviour – Approximately 8 months after the workshop, participants independently developed and implemented a different modelling lesson, demonstrating a change in their instructional behaviour. Level 4: Results – In the lessons conducted after the workshop, students were observed to engage in the modelling process effectively, particularly in the process of idealization where they set assumptions and solved the

modelling problems. Some students even demonstrated the ability to assess the validity of their solutions.

The results demonstrated effectiveness across all 4 levels of the Kirkpatrick model, leading to the conclusion that the workshop was beneficial for improving modelling lessons conducted by in-service junior high school mathematics teachers with no prior modelling experience.

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[18.] **Integration of Artificial Intelligence in Mathematical Modeling: A Qualitative Approach for Engineering Students in Multivariable Calculus**

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Teaching Multivariable Calculus to engineering students poses challenges in demonstrating its real-world relevance. Mathematical modeling bridges theoretical concepts with engineering applications. The integration of artificial intelligence (AI) provides new opportunities to enhance learning by simplifying complex processes and encouraging deeper conceptual understanding. This summary explores AI's potential to improve modeling by focusing on the authenticity of exercises, stages of the modeling process, diverse technological applications, and qualitative insights into student learning.

This ongoing study examines how AI tools influence engineering students' modeling processes in a Multivariable Calculus course. We investigate how an AI application supports reflection on different modeling stages. While AI can automate solutions for differential and integral calculus tasks, this study emphasizes its role in helping students create meaningful connections between mathematical concepts and real-world problems relevant to their personal and professional interests.

The research involves 100 second-semester engineering students, with tasks designed to gauge AI's impact on engagement and comprehension. A qualitative perspective guides the analysis, focusing on how students interact with each modeling stage following Rodríguez's framework (2015).

Authenticity in Modeling Exercises

Authentic modeling tasks are crucial for fostering student engagement. Incorporating real engineering scenarios—such as optimizing resources or designing efficient systems—enhances perceived relevance. AI contributes by:

- Generating realistic data sets.
- Simulating physical phenomena.
- Personalizing problems according to student interests.

Stages of the Modeling Process: A Qualitative Perspective

AI supports each stage of mathematical modeling, with qualitative analysis highlighting student engagement and interpretation:

- **Understanding the Problem:** AI summarizes key information, aiding conceptual clarity. The focus is on how this understanding shapes student approaches.
- **Model Formulation:** AI tools help identify equations and constraints. Qualitative analysis examines how students justify these formulations and build conceptual meaning.

- **Analysis and Solution:** AI assists in solving complex models. The study explores how students interpret AI-generated results and link them to original problems. For example, tools like Mathematica / Wolfram Alpha/ MATLAB enable dynamic simulations of abstract calculus concepts. It is possible to examine how visualizations aid understanding.
- **Validation:** AI compares results with empirical data. Research focuses on student strategies and challenges in validating models.
- **Interpretation of Results:** AI-powered language tools explain solutions in accessible terms. The analysis investigates how students construct coherent narratives from these explanations.

Conclusion

AI integration in Multivariable Calculus modeling enhances authenticity, supports personalized learning, and streamlines complex processes. Qualitative findings suggest AI fosters deeper, contextualized understanding by encouraging reflection at each modeling stage. Effective implementation requires thoughtful pedagogical strategies to ensure AI complements skill development essential for future engineers.

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[19.] Studying the Epistemic Value of Generative AI in Modelling Activities

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Prior research, when examining the epistemic value of digital resources (Jessen, 2022) such as dynamic geometry systems or use of the internet and CAS, indicates the lever potential of these regarding modelling activities (Siller et al., 2022). Generative AI (GenAI) has become an active tool in students' educational practice, including modelling activities (Carlsen & Jessen, 2025). When students employ GenAI, it serves as a dynamic media that supports the inquiry process of modelling and introduces new knowledge to be explored (Carlsen & Jessen, 2025). In contrast, we have static media such as textbooks, webpages and videos. The inclusion of dynamic media into modelling activities suggests new possibilities for task design that foster rich modelling and learning activities (Carlsen & Jessen, forthcoming). This early research on students' use of GenAI indicates that students use dynamic media to study the modelling process more generally while using static media for knowledge of specific techniques.

Our study is situated in a statistics course. However, recent research has pointed to the close relation and potential benefits between statistics (including statistical modelling) and mathematical modelling. In a systematic study on the connections Ärleback & Kawakami (2023) conclude, amongst others, that there are numerous examples of a mutual beneficial relation between the two entities, also in the light of various frameworks.

This study investigates the students' use of dynamic media and what type of knowledge the dynamic media is used for. We consider a context of data science students in Denmark, where students engage with modelling in data science during the early weeks of a statistics course, where they are asked, "Which exam format is best?". The students are thus still unfamiliar with standard models and traditions of statistical analysis. Specifically, we examine the students' use of GenAI when students are asked to both model and conduct a statistical analysis. We therefore ask: What specific knowledge do students seek using dynamic media? How does dynamic media support students' development of statistical and modelling knowledge?

To address these questions, we analyse students' logbooks containing GenAI prompts and responses, links to all digital resources consulted, as well as written project reports that were handed in. Our work is guided by the view of modelling from the perspective of the Anthropological Theory of Didactic, which is a process initiated by a generating question which the students cannot immediately answer but must engage in a process of posing derived questions which requires a study and research process where new knowledge is sought and researched in the context of the generating question (e.g. Jessen and Kjeldsen, 2023). The analysis is based on the framework Praxeology (Chaachoua, et al., 2019) that lets us explicit knowledge into the categories of *type of tasks*, *technique*, *technology* (discourse) and *theory* (validation of technology). This framework allows us to explicitly study the knowledge studied using dynamic media and consider the relations and disconnects between the elements.

Our analysis shows that GenAI can contribute knowledge on all levels, including norms and traditions for conducting statistical analysis. Furthermore, the students also used GenAI to develop derived questions to pursue in the modelling process, as well as how to achieve answers to these questions.

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[20.]

[21.] Finance-specific modelling tasks in mathematics instruction – Opportunities and challenges from the perspective of students and teachers

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The Organisation for Economic Co-operation and Development describes financial literacy as a key factor in strengthening individual and societal financial resilience (OECD, 2021). In schools, various subject disciplines can help to promote financial literacy in the classroom. Mathematical modelling, recognised globally as a process-related competence and incorporated into many school curricula (Schukajlow et al., 2023), allows finance-specific topics to be systematically integrated into mathematics teaching (Greefrath & Kirsten, accepted). However, little is known about how the use of finance-specific modelling tasks in mathematics teaching is perceived by students and teachers, which is why the following research question arises: *What opportunities and challenges do students and teachers perceive in the use of finance-specific modelling tasks in mathematics teaching?* To answer the research question, a qualitative interview study was conducted with seven students and six teachers, and the data collected was evaluated using a qualitative content analysis. Both students and teachers highlighted, for example, the relevance for life, motivation and interest as opportunities. The central challenges mentioned by both groups included the complexity of the topics as well as emotional and social barriers. In addition to these common categories, there were also differences in the perceived opportunities and challenges between teachers and students. For example, students identified a lack of financial literacy among teachers as a key challenge, while teachers pointed to insufficient teacher training in financial education. Another opportunity mentioned exclusively by teachers was interdisciplinarity, as they saw potential in connecting financial topics with various subjects such as social sciences and economics. The results are discussed in the context of existing theories in order to derive possible implications for school practice and teacher training.

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[22.] Supporting Prospective Elementary Teachers' Use of Mathematical Modeling through Real-World Societal Issues

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Elementary mathematics education should embrace the everyday mathematics found in real-world experiences to include issues and events for the society they live in (Beard, 2021; Felton-Koesler et al., 2017; Kretz, 2023; Litster et al., 2018; Xenofontos, 2020). Teachers are key to making these connections through mathematical modeling. There are different levels of integration with lower levels focusing on thematic applications to situate the relevance of mathematics within the world and higher levels focusing on modeling authentic applications of mathematics that reflect the world students live in (Litster et al., 2023). Mathematical modeling uses mathematics to answer “big, messy, reality-based questions” (Bliss & Livertini, 2016, p. 7). This interdisciplinary conjunction of mathematics and other knowledge can help students engage in effective learning opportunities, meet the diverse needs of students, and engage in increased social awareness (Litster et al., 2023; Williams & Roth, 2019).

Thanheiser and Sugimoto (2020) propose that preservice teachers (PSTs) can and should be developing a joint understanding of both mathematical knowledge and social issues using real numbers and data sources in teacher preparation courses to support this effort as PSTs move into their own classrooms. Technology can be a great way to add in those numbers and provide a verified data source to extend mathematical understanding to real-world and social justice contexts, as long as it is being used effectively to support learning (Kolb, 2017).

This research explored impacts of modifications to a mathematical modelling of real-world societal issues assignment in an elementary math methods course for 157 prospective elementary teachers across a period of 3 years. Specifically, it looked at how the changes in the assignment impacted prospective teachers' choices in social issues and digital resources as well as how those choices encourage or discouraged higher levels of integration and mathematical modelling for Grade K-5 students (ages 6-11).

Results showed PSTs who focused on cultural social topics generally used qualitative digital sources such as images, articles or videos to set a tone or theme for the math tasks rather than encouraging modeling to explore the topic (low level of integration or modelling). In contrast, PSTs who explored social issues that presented statistical data from one or more online sources were more likely to support math modeling. Several aspects of early iterations of the assignment that were associated with lower levels of integration and modelling included requiring prospective teachers to use non-statistical categories of math (e.g., geometry) or focus on younger grade levels (e.g., Grade 1). Aspects of later assignment criteria that were more likely to encourage mathematical modeling included focusing on upper grade levels, focusing on mathematical domains such as operations or data analysis, and requiring the use of virtual manipulatives.

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[23.] Mathematical modeling through a Socio-ecological perspective: Promoting sustainable development in mathematics education

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Mathematics education provides opportunities to transform students into practical users of mathematics who can apply mathematics concepts in real-life contexts. Geiger et al., (2023) also emphasized the importance of connecting mathematics education with emerging phenomena to cultivate responsible citizenship. Moreover, there has been an increasing need for mathematics education to expand its scope by responding to social and environmental issues such as climate change, pollution, water and food insecurity, and biodiversity loss (Coles et al., 2024). Mathematics through counting, measuring, estimating, and modeling provides powerful ways to understand the earth (Coles et al., 2013). Mathematical modeling—as a cyclical and multistep process of problem-solving activities—provides opportunities for students to engage in their reasoning and critical thinking skills when confronting such challenges, fostering their sense of agency. Previous research in mathematics education has explored various social and environmental issues, including climate change (Barwell, 2013), sea-level rise (Basu & Panorkou, 2020), biodiversity loss (Yanagimoto & Yoshimura, 2013), and environmental concerns specific to a certain location (Solares-Rojas et al., 2022). Additionally, studies have explored mathematics education and social injustices and inequalities (Gutstein, 2012; Jung & Brand, 2021). Socio-ecological perspectives align closely with critical mathematics education (Skovsmose, 2023), which emphasizes the interconnectedness of environmental and social justice. This approach highlights how these issues can be meaningfully explored in the mathematics classroom with a focus on future generations and sustainability. Both social and environmental sustainability are key issues within critical mathematics education, requiring careful reflection. In this context, mathematical modeling is not only essential for advanced applications of mathematics but also highly relevant for classroom activities that promote critical inquiry. Additionally, mathematical modelling activities can also become a tool for implementing education for sustainable development (Wiegand & Borromeo Ferri, 2023). This study explores the connection between mathematical modeling and a socio-ecological perspective in mathematics education, highlighting how modeling provides students an opportunity to critically analyze and respond to real-world social and environmental issues and promoting sustainable development. This study draws on a mathematical modeling project that explores the context of a mudflow disaster in a specific region of East Java, Indonesia. This phenomenon raises multisectoral issues for local people particularly and Indonesia in the broader

context. Thousands of families in twelve villages have lost their homes and their sources of income permanently (Schiller et al., 2008). The accumulation of mud severely damaged the environment by contaminating water sources, destroying local ecosystems, and harmful human health (McMichael, 2009). Moreover, it also leads to social issues, such as securing housing and alternative livelihoods for the victims, as well as injustice compensation that impacts women's rights (Tridewiyanti et al., 2025). The findings suggest that integrating socio-ecological perspectives and mathematical modeling can serve as meaningful approaches to teaching and learning mathematics while also promoting sustainability and responsible citizenship within mathematics education. The study examines preservice teachers' engagement with mathematical modeling tasks through the Lumpur Sidoarjo (LuSi) Project, which investigates their ideas, problem-solving strategies, and perspectives in response to the ongoing mudflow disaster in East Java, Indonesia.

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[24.] **Modelling tasks from written comparison tests in Germany**

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Introduction

Drawing on established criteria for modelling tasks (Maaß, 2010), this study examines modelling tasks presented in written comparison tests for 8th grade. The central inquiry focuses on the representation of mathematical modelling conveyed to teachers through these assessments. Preliminary findings suggest that the depiction of mathematical modelling is somewhat restricted, as numerous criteria commonly associated with modelling tasks are not fulfilled. These limitations can be partially attributed to the overarching requirements for the construction of diagnostic tasks and assessments (Drüke-Noe, 2012). Considering these constraints, good-practice examples are utilised to propose recommendations for tasks designed to assess “sub-competencies of mathematical modelling” (Cevikbas et al., 2022, p. 209).

Background

The nationwide administration of written comparison tests is a key component of the overarching educational monitoring strategy established by the German Standing Conference of the Ministers of Education and Cultural Affairs (KMK, 2006). These tests aim to empirically assess students’ achievement of educational standards (Blum et al., 2005). As in many other countries, mathematical modelling is recognised as a key objective of mathematics education in Germany (Kaiser, 2020; KMK, 2004). Consequently, these tests serve as a reference framework, with tasks classified as modelling tasks shaping teachers’ understanding of mathematical modelling. However, research on modelling tasks in Austrian and German central examinations indicates that they often fail to meet the criteria for high-quality modelling tasks—such as authenticity, real-world relevance, and different possible solutions (Greefrath et al., 2017; Siller & Greefrath, 2020).

Research questions

In this context, the following questions emerge:

1. To what extent do the analysed tasks provide teachers with an accurate representation of mathematical modelling?
2. How can these tasks be optimised to align with the criteria for high-quality modelling tasks?

Methods

Among the 474 publicly available tasks from the German mathematics comparison tests for 8th grade, 168 contain at least one subtask designed to assess modelling competence. These 168 tasks, which include a total of 310 subtasks—266 of which focus on modelling—are analysed using a classification scheme for modelling tasks (Maaß, 2010). Each criterion specified in this scheme is evaluated through binary coding by two independent raters, while a substantial portion of the tasks underwent double coding.

Results

With respect to research question (1), the analysis reveals that German comparison tests offer a limited perspective on mathematical modelling:

- Several key steps in modelling—such as understanding, simplifying and structuring, as well as interpreting and validating—are rarely represented.
- Only a small proportion of tasks address the issue of missing data.
- Less than half of all tasks can be considered authentic.
- Scientific contexts are largely absent.
- Only a minimal number of tasks require normative modelling.
- Visual elements—such as photographs and images—often serve a decorative rather than an essential function in solving the respective tasks.

With respect to research question (2), the analysis also identifies examples demonstrating how modelling competencies can be effectively assessed within written comparison tests. These good-practice examples will be examined in greater detail throughout the article.

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[25.] The importance of experimental experiences that activate data-driven mathematization in high school chemistry

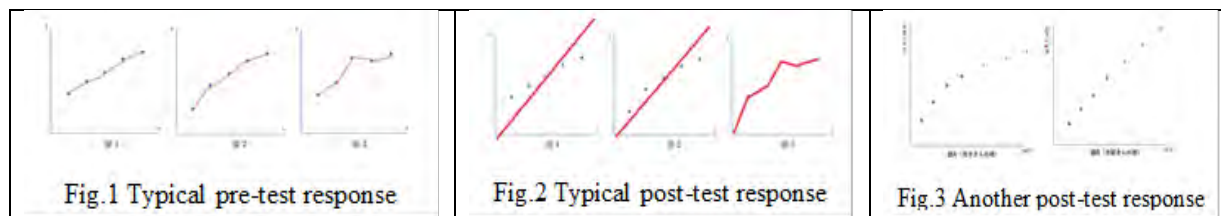
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In Japan, the unit on chemical reaction rates is presented in chemistry textbooks for Year 11. The subject of the experiment is the relationship between the amount of oxygen produced when a catalyst is added to hydrogen peroxide solution and time, and the rate constant of the reaction. However, there are very few classes in which this experiment is carried out. The class of chemical reaction rates requires students to engage in model-eliciting activities of MMP through data-driven mathematization from experiments. Therefore, a 350-minutes chemical reaction rate class including two experiments was conducted in which students independently analyzed and discussed the results of the experiments in January 2025. The first experiment was a follow-up experiment as described in the textbook, while the second experiment was a student-led experiment in which the amount of catalyst, concentration or temperature of the hydrogen peroxide solution was changed.

In the class there was a pre-survey and a post-survey. The common question was to ask the students to draw a graph of any function for each of the three graph sheets shown. In addition, there was another question in the post-survey to set the scene so that students could specifically recall the classroom experiments. This was a question about plotting more plots, recalling the experiment, on a graph sheet that already had 4-plot graphs.



In the pre-survey, line graphs were very common (Fig.1). However, in the post-survey, where two experiments were carried out, there was a change in the range of domain and more regression line descriptions (Fig.2). In another post-survey question, students continued to plot on the two 4-plot graphs, many mathematical models were plotted as curves with a boundary line at the top on the left graph and as straight lines on the right graph (Fig.3). From these student responses, the students who responded as shown in Fig.1 are also creating a mathematical model that captures the chemical experimental event through their experimental experience.

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[26.] Effect of reading literacy on performance and confidence solving modelling problems with missing or excess data

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Modelling problems are mathematical problems strongly connected to reality, so the context provides crucial information to develop the solution (Palm, 2008; Tran & Dougherty, 2014), being essential to understanding, interpreting and using the information provided in the statement (Chang et al., 2020; Peng et al., 2020). These problems can be classified by the amount of information presented in the statement. In modelling problems involving missing data, it is essential to notice this and complete the statement information by making realistic assumptions, identifying variables and quantifying them (Ferrando et al., 2024; Krawitz et al., 2018). In contrast, in modelling problems with excess data, it is essential to notice that there is mutually exclusive or superfluous information, and that decisions must be made about the real situation that involve discarding unnecessary data (Krawitz & Schukajlow, 2018).

Segura and Ferrando (2023) define performance in modelling problems as success in developing an adequate mathematical model and working mathematically without errors to find the solution. The results from Ferrando et al. (2020) suggest that the amount of information and data in the modelling-problem statement affects performance, and this can be related to reading literacy. Reading literacy is “understanding, using, evaluating, reflecting on and engaging with texts in order to achieve one’s goals” (OECD, 2019, p. 28). Maña (2014) highlights the use of information to answer questions among the different aspects of reading literacy. However, few studies emphasise the effect of reading literacy on performance in solving modelling problems (Krawitz et al., 2017).

In addition, there are affective aspects related to performance that could also be influenced by problem type and reading literacy, such as confidence (Foster, 2016; Stankov et al., 2012). Confidence can be defined as “a state of certainty about the success of a given behavioural act” (Stankov et al., 2012, p. 747). Following this definition, as stated by Foster (2016), we understand confidence as the certainty that students have that their solution is correct.

In this study, we posed the following research questions:

(RQ1) Does students’ performance and confidence in solving modelling problems depend on the amount of information presented in the statement?

(RQ 2) Does reading literacy influence secondary students’ performance and confidence in solving modelling problems with excess or missing data?

To answer these questions, we have conducted a study with 188 secondary students. First, the students completed a reading literacy test (Llorens-Tatay et al., 2011) and from their scores we established 3 levels using a cluster analysis. In another session, students solved a sequence of 4 modelling problems, 2 with missing data and 2 with excess data. After solving each problem, students were asked to report their confidence using a 5-point Likert scale. To assess performance, the researchers followed Segura and Ferrando’s (2023) coding for problems with missing data and agreed on a reliable coding for problems with excess data.

Using a mixed ANOVA test, we measured the effect of the amount of information in the modelling problems (intra-subject) and reading literacy (inter-subject) on students' performance and confidence, as well as the interaction effect between the two factors. Our findings show that students perform better on problems with missing data than on problems with excess data, although their confidence is higher on the latter (RQ1). We also find that higher reading literacy implies better performance and confidence of students in solving modelling problems, with an interaction effect on performance, as students with low reading literacy have similar (very low) performance on both types of problems, but no interaction effect on confidence (RQ2). These results are important for the design of sequences that effectively develop students' modelling competence, taking into account different factors affecting cognitive and affective aspects.

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[27.] Different mathematical competencies afforded by tasks: An analytical framework for determining modeling competency

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Much research has been done on specifying frameworks within which mathematical competencies related to modeling competency can be assessed on tasks to reflect students' mastery or progress. Besides the previous frameworks (e.g., Turner et al., 2015; Lithner et al., 2010) focusing on generally describing mathematical practices to complete a task, very little attention is paid to the dimensions of competencies afforded by tasks (Yunus, under review). Niss and Højgaard's (2019) competency model is multidimensional (with degree of coverage, radius of action, and technical level) and offers possibilities to assess the implicit cognitive structure dimensions of modeling competency that is closely related and entangled with other competencies.

Given the lack of frameworks that would allow us to capture the implicit dimensions of competencies evoked by tasks to align with the fundamental notion of competency, this study explores the potential of a framework in this respect. It does so by focusing on the following research question based on competencies as a multidimensional construct: What are the potentials and challenges of a three-dimensional approach to assessing mathematical competencies evoked or afforded by tasks?

By utilizing Niss and Højgaard's (2019) competency model, which established the three dimensions of a person's competency mastery, we propose a framework for determining which competencies are afforded by tasks. The competencies afforded or evoked by the task are illustrated by focusing on mathematical modeling competency in relation to the three dimensions, which concern respectively the degree of coverage of the competence, i.e., which aspects of the competency come into play on task; the mathematical technical level in the use of the competency; and in relation to the domain of situations where the competency is activated on task.

After presenting two concrete examples of existing tasks on modeling, the study revealed different degrees of activating competencies on tasks that align with students' progression of competency mastery at a point in time. Two challenges form the basis of the analysis: how reasonable are tasks inherently activate competencies and what would generally be considered the solution to a task.

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[28.] Developing modeling competencies of mathematics preservice teachers: Problem-based approach

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Mathematical modeling has increasingly been recognized as an important part of mathematics education, serving to bridge the often-perceived gap between abstract theoretical concepts and their real-world applications (Blum & Niss, 1991; Lesh & Doerr, 2003). Research highlights the critical role of modeling in developing students' problem-solving abilities, deepening their conceptual understanding, and fostering a more authentic appreciation for the relevance of mathematics across diverse disciplines (Izard et al., 2003; Kaiser & Sriraman, 2006). The integration of modeling activities empowers students to engage with complex, open-ended problems, promoting critical thinking, effective communication, and collaborative skills (Brown & Stillman, 2024). Furthermore, studies emphasize the importance of contextualizing mathematical concepts within real-world scenarios, thereby enhancing students' motivation and engagement (Palm, 2008; Verschaffel, Greer, & De Corte, 2000). The development of modeling competencies not only equips students with essential skills for future academic and professional pursuits but also cultivates their ability to navigate and interpret the quantitative aspects of their environment. To effectively foster these competencies in students, mathematics educators themselves must possess a deep understanding of mathematical modeling and its applications. This paper presents a comprehensive analysis of the design and implementation of a graduate-level course specifically tailored for mathematics educators, focusing on the principles and applications of mathematical modeling.

The course employs a problem-based learning (PBL) approach, immersing students in authentic scenarios that necessitate the application of diverse modeling techniques, including discrete, continuous, stochastic, and data-driven methodologies. PBL, as a pedagogical strategy, has been shown to enhance students' problem-solving skills, promote collaborative learning, and foster deep conceptual understanding (Hmelo-Silver, 2004; Savery & Duffy, 1996). By engaging with real-world problems, students are encouraged to actively construct their own knowledge, develop critical thinking skills, and apply mathematical concepts in meaningful contexts (Torp & Sage, 2002). The primary objectives of the course include cultivating students' abilities to identify appropriate modeling strategies, formulate mathematical models, analyze their behavior, utilize computational tools for simulation and forecasting, and interpret results within real-world contexts.

This paper will present findings from an action research study that focused on assessing and analyzing student learning and engagement with mathematical modeling in a graduate level course for mathematics educators. The course was offered during three semesters at a private university in the Northeast USA. The analysis is based on the data collected from the students as part of the coursework, which includes critical analysis of mathematical models presented in published papers, student solutions to a series of challenging modeling problems designed to simulate real-world scenarios and peer reviews of these solutions, written papers in which

students formulated and analyzed their own modeling problems, and final presentations of these problems to the class. Total of 33 students agreed to have their work included in the study.

The critical analysis of published mathematical models aimed to develop students' ability to deconstruct complex models, identify underlying assumptions, and evaluate the validity of conclusions drawn from the models. By engaging with real-world applications of modeling, students gained a deeper appreciation for the power and versatility of these tools. The challenging modeling problems designed to simulate real-world scenarios required students to apply the modeling techniques learned in class to novel situations, demanding creativity, critical thinking, and problem-solving skills. The analysis of student solutions to these problems provided valuable insights into their ability to translate real-world situations into mathematical frameworks and to effectively utilize modeling tools. Students were required to provide constructive feedback on their peers' solutions, focusing on the clarity of model formulation, the appropriateness of solution strategies, and the accuracy of interpretations. This process not only enhanced students' analytical skills but also fostered a collaborative learning environment where they could learn from each other's approaches and insights. The final project of the course involved students in formulating and solving their own modeling problems. This task required students to identify relevant real-world scenarios, construct appropriate mathematical models, and interpret the results in the context of the original problem. The analysis of these student-generated problems revealed their evolving understanding of the modeling process and their ability to apply it to diverse contexts. This task also allowed the students to explore areas of personal interest, and to delve deeper into specific applications of modeling. As part of the final project students presented their modeling problems to the class, providing an opportunity to articulate their modeling process, explain their findings, and respond to questions from their peers and instructors, and receive peer feedback. The analysis of these presentations shed light on students' ability to synthesize complex information and communicate it effectively.

The analysis of the student work provides a well-rounded picture of the learning that occurred within the course. The varied nature of the assignments allowed the students to engage with modeling in multiple ways, from theoretical analysis of published models, to practical application of modeling skills to solve given problems, to the creation of their own modeling problems and solutions. This multi-faceted approach allowed for a deeper understanding of the modeling process, and a greater appreciation of its power and utility. The findings of this action research study could have significant implications for mathematics education, highlighting the importance of integrating authentic problem-solving experiences, student-driven inquiry, and collaborative learning into the teacher education curriculum. The paper will also discuss the implications of the findings on the design of future courses,

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[29.] AI-Powered Learning: Exploring Secondary School Students' Use of ChatGPT for Solving Mathematical Modelling Problems

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Large language models, with their generative abilities and innovative nature, are currently being explored for potential applications in education (Noster et al., 2024). To equip students with essential 21st-century skills and prepare them for evolving global challenges, it's crucial to adopt new learning methods that effectively integrate technology and Artificial Intelligence (AI) (Bani-Hamad & Al-Kalbani, 2024; Cevikbas et al., 2023). As noted by Caldarini et al. (2022), a key benefit of AI in education is its ability to enrich students' learning experiences, offer tailored assistance, and boost their academic outcomes.

Generative AI has the potential to play a crucial role in supporting students with problem-solving, particularly in activities related to mathematical modelling. By providing tailored assistance and enhancing students' ability to approach complex problems, it can significantly contribute to the effective implementation of these practices in the classroom. This use of AI can help create a more interactive and personalized learning environment, improving both student engagement and understanding (Gallart et al., 2025).

Although Gallart et al. (2025) and Noster et al. (2024) have investigated the use of ChatGPT by pre-service teachers, the use of ChatGPT by students, particularly in mathematics, remains unexplored. Students widely use ChatGPT as both a tool for completing tasks and a source of knowledge, raising concerns about how they interact with the tool. To address this, our study aims to focus on how students explore ChatGPT, how they structure their prompts, and the actions they take during mathematical modelling process. The study design is specifically chosen to allow students to engage with ChatGPT, providing insights into their use and prompting behaviour. In this regard, this qualitative study seeks to address the following research questions:

1. How do students use ChatGPT when solving mathematical modelling tasks, and which prompting strategies do they use?
2. To what extent do students find the answers provided by ChatGPT reliable, and if/how do they verify the correctness of these answers?

Data collection occurs during a two-hour class session, supervised by the researchers. At the start, participants are introduced to mathematical modelling and the modelling cycle with a problem they solve. A questionnaire follows to explore students' use of AI tools, their purposes, frequency of use, and trust in AI. Prior to the session, researchers will create ChatGPT accounts, and participants will log in using school computers. Working in pairs, students solve the Moving House Problem (Borromeo Ferri, 2018) by prompting ChatGPT. The chat interactions and audio records will be analyzed using qualitative content analysis (Mayring, 2022).

We are currently collecting data from around 10 9th/10th grade students at a public secondary school. A pilot study with two 9th-grade students was conducted to refine the main study and identify gaps. According to the initial data obtained from the pilot study, students used the Few-shot (Snell et al., 2017) technique, asking questions about problem components before requesting

solutions. The bot suggested two answers, which students recorded without questioning. This shows that students have focused on the result rather than the stages in the modelling cycle. The main study will provide insights into students' use of ChatGPT, including their prompting behaviour and construction of mathematical knowledge.

This study is part of the project “‘Teachers leave their kids alone?!’ – Attitudes and cooperative skills of prospective and practising maths teachers regarding the use and handling of AI at school (KIMAKO)” and is funded by the Klaus Tschira Foundation.

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[30.] Solving Mathematical Modelling Problems Benefitting From Generative AI: Affordances, Challenges, and Reliability

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Mathematical modelling is a critical component of essential 21st century competencies such as critical thinking and interdisciplinary reasoning (Blum, 2015). Despite its curricular prominence, modelling remains a complex and often unpredictable process (Stillman et al., 2013; Borromeo Ferri, 2018). This challenge extends to pre-service teachers (PSTs), who should develop strong modelling competencies to effectively support their future students. While digital tools have demonstrated some potential in facilitating modelling processes, the role of emerging Generative Artificial Intelligence (GAI) technologies remains largely underexplored in modelling education (Cevikbas et al., 2023). This study investigates how PSTs engage with two GAI tools—ChatGPT 4o and Thetawise—when solving modelling problems. It explores both the opportunities and challenges of AI-assisted modelling as well as PST’s perceptions of tool reliability within educational contexts.

A total of 150 PSTs studying to become primary teachers from a large German university participated in the study. Working in small groups, they tackled three established modelling problems using both GAI tools and paper-based methods. They engaged in collaborative discussions to evaluate the accuracy, plausibility, and pedagogical coherence of the AI-supported solutions. Of the total participants, 44 PSTs also responded to an online survey, and five participated in follow-up interviews to further explore their experiences and perspectives. Data were collected through modelling worksheets, AI interaction logs, surveys with open-ended questions, and interviews. Employing qualitative content analysis (Mayring, 2014), the study examined how PSTs interacted with GAI tools and how these tools influenced their modelling approaches and strategies.

Findings indicate that PSTs predominantly utilized GAI during the mathematization, simplification, and validation phases of the modelling cycle. While participants appreciated the tools’ capacity to generate initial models, identify variables, and explore alternative solution pathways, they expressed reservations about the logical consistency, contextual adequacy, and mathematical correctness of some AI-generated responses. Crucially, PSTs did not exhibit uncritical reliance on the tools. Instead, they adopted a reflective and inquiry-oriented stance—questioning, verifying, and sometimes rejecting AI outputs based on their own mathematical judgment and collaborative discussion.

These results align with broader research in AI-in-education that cautions against overdependence on algorithmic solutions and highlights the importance of preserving learner agency, interpretive autonomy, and critical evaluation (Luckin & Holmes, 2016). In conclusion, this study suggests that GAI tools can serve as valuable yet non-authoritative supports in modelling—particularly in the early and evaluative stages of the modelling cycle—when their use is embedded within pedagogies that emphasize critical engagement,

collaborative reasoning, and epistemic vigilance. Future research should explore how these tools can be further integrated into teacher education curricula to scaffold modelling competencies without undermining conceptual understanding or problem-solving autonomy. This work contributes to an ongoing discourse on the responsible, purposeful use of AI in mathematics teacher education opening new avenues for research on human–AI collaboration in complex contexts.

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[31.] The role of mathematical models in the lives of ‘non-mathematical’ citizens

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Modern societies are more and more data driven. This entails that large numbers of data are collected, stored and used for decision making. In this decision making, mathematical models play a role (O’Neil, 2017, Vos, 2024). The aim of this paper is to describe, analyse, and theorize how adults in society engage with such mathematical models in their everyday lives.

In many countries, the mathematics curricula aim to prepare students for a professional life, in which they can competently engage in mathematical modelling activities to solve problems for governments or industries, in medical research, in engineering, and so forth (Frejd & Bergsten, 2016). The modelling activities by these professionals are well described and analysed by the so-called *modelling cycle* (Niss & Blum, 2020). The mathematical professionals create new models, but also adapt existing ones to new situations. These mathematical models are often expressed in symbolic forms (formula, algorithms, code, tables, etc.).

However, not all students become professional mathematical modellers. In fact, quite some students struggle later in life with mathematical information; about one in four adults score low in numeracy (OECD, 2024). Nevertheless, they may encounter mathematical models. The study described in this paper analyses and theorizes how mathematical models play a role in adults’ everyday lives. Our aim is to identify and characterize modelling activities by adults in everyday contexts that are different from those of professional mathematical modellers. A second aim is to describe competences that all adults need, including those with low numeracy levels, in order to understand the underlying mathematical models. The theoretical framework used for analysis is based on Frejd et al. (2025), Frejd and Vos (forthcoming), Niss and Blum (2020), Skovsmose (2005) and Vos (2024).

We selected three contexts, in which adults engage with mathematical models, without actively engaging in the modelling activities described in the modelling cycle (understanding the problem, simplifying, structuring, making assumptions, mathematizing, working mathematically, translating, validating, etc.). The contexts relate to personal health, personal finances, and climate. More concretely, we selected three infographics that require adults to read (1) information in an invitation for a breast cancer screening, (2) interactive information on mortgages, and (3) climate information on rising national temperatures. In all three cases, the information reading had the purpose to inform a reflected decision for further actions (Hasan, 1996), such as (1) participating in the breast cancer screening, (2) deciding on taking a mortgage, or (3) contributing to decreasing CO₂ emissions.

The analysis of the infographics reveals that they are based on much quantitative information and the mathematical modelling thereof, but that the resulting visualizations give as little as possible symbolic mathematics. The underlying mathematical models (formula, equations, algorithmic codes, etc.) remain hidden behind colourful, numeric graphs or diagrams. The data sources are sometimes explained in small font, but sources to verify the underlying data and

modelling processes often remain opaque. We describe competences for reading the infographics and reflecting on the data sources and underlying mathematical modelling processes. These relate to a qualitative understanding *about* mathematical modelling (Vos, 2022). Also, we describe how the infographics aim to prompt adults to undertake activities, which are captured in new data and, again, processed in mathematical models. For instance, participating in a health screening leads to new data, and likely to adapted mathematical models. We conclude the paper with recommendations for mathematical modelling education including its assessment, and in particular with an eye on those students who are mathematically vulnerable.

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[32.] The use of toolkits for socio-critical modelling in Norway and South Africa

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In this paper we focus on Norwegian and South-African pre-service teachers (PTs) engaging in socio-critical modelling. We investigate their discussions and concerns after having participated in a teaching session with a toolkit about water consumption.

The socio-critical modelling perspective emphasizes, according to Kaiser and Sriraman (2006), critical reflections about the role of mathematics and mathematical modelling in society and about the nature of mathematical models. Barbosa (2006) describes socio-critical modelling as focusing on relations between mathematics, mathematical models and society and considers the models to be a simplification of the world (Barbosa, 2006), which is in line with Skovsmose (2014) who called models as just one way of “re-presenting” reality. Barbosa (2006) further highlights socio-critical modelling as connected to the use of reflexive discussions, which is concerned with the nature of a model, and the criteria used for making the model as well as the consequences of these criteria.

This qualitative case study presented here is part of a larger study, “Mathematical toolkit design North South”. A toolkit is defined as a compilation of readings, lectures, activities and questions for discussion etc., around a social topic and is used in the mathematics teacher education program (Kacerja et al., 2024). In this study, 30 PTs from Norway and 20 from South Africa worked with one toolkit in the context of a sustainability goal about clean water. To facilitate the PTs’ discussions, they were engaged in different elements of the toolkits: they watched videos about water scarcity issues and water infrastructure, accompanied by lectures about mathematical modelling, before working on a discussion task where they could choose to explore one issue about the topic. The PTs’ discussions were video or audio recorded. Here, we focus on one Norwegian and one South-African group of PTs, chosen because they present opportunities to discuss reflexive discussions as described by Barbosa (2006). The transcribed data were scrutinized by the first author, with a particular focus on identifying elements of reflexive discussions.

Preliminary findings suggest that the toolkits can facilitate PTs’ reflexive discussions in both countries. In Norway, three boys focus on the total water usage of various countries and critically discuss what assumptions are taken into consideration when making a mathematical model and what is *not* shown in the model. This indicates that the PTs were engaging with the nature of the mathematical models and their assumptions, which is one of the aims of the socio-critical modelling. In South-Africa three girls discussed various economic models for paying for water consumption (e.g., prepaying for water, paying for consumption, monthly payments) and the fairness of these, that wasting water leads to higher prices, or technical solutions (e.g. drilling for water). These PTs were therefore engaging with the consequences of the use of mathematical models for the people in the different areas of the city, and the factors that influence the models. In conclusion, these results highlight the potential of engaging PTs

with toolkits and socio-critical modelling, contributing to a deeper understanding of the societal implications of mathematical models and the interplay between mathematics and complex real-world problems.

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[33.] **Understanding the role of beliefs in pre-service primary teachers' learning to teach mathematical modelling**

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The integration of mathematical modelling into primary mathematics education can serve as a crucial bridge between informal mathematical experiences in early childhood and the more complex approaches to modelling in secondary school (Greer et al., 2007). Mathematical modelling is explicitly positioned as a key competency in national curriculum standards from primary school onwards in many countries around the world (since 2001 in Singapore, 2004 in Germany, 2020 in Norway; only to name some examples on the timeline). The successful integration of mathematical modelling activities in the primary classrooms poses challenges and opportunities on multiple levels. Implementing the reformed educational goals depends not only on the availability of suitable task material. But also - and crucially - on the knowledge, skills and willingness of teachers to teach mathematical modelling. These personal requirements can be subsumed under the concept of professional competence to teach mathematical modelling in primary schools (Arndt et al., 2018; Baumert et al., 2013; Wess, 2020). In response to these challenges, the present study aims to gain an in-depth understanding of how teachers learn to teach mathematical modelling. It examines the interplay between different facets of professional competence - in particular beliefs and pedagogical content knowledge - of 38 pre-service primary teachers enrolled in a modelling course.

Building on a differentiated understanding of teacher beliefs (e.g. Fives & Buehl, 2016; Richardson, 2003; Törner, 2002), we focus on three key dimensions: (1) professional self-efficacy beliefs (Bandura, 1997), (2) conceptions of mathematics as a discipline (Grigutsch et al., 1998), and beliefs on to the teaching and learning of mathematical modelling (Ärlebäck, 2009; Staub & Stern, 2002; Voss et al., 2013). The modelling specific pedagogical content knowledge is conceptualised as the knowledge that cognitively enables teachers to support students in engaging with real-world modelling tasks effectively (Hill et al., 2008; Shulman, 1987). Our framework also integrates findings from secondary teacher research (Borromeo Ferri, 2021; Borromeo Ferri & Blum, 2010; Greefrath et al., 2022) and adapts them to the primary context (Nehrkorn et al., 2023).

To assess these concepts, existing test scales (Wess et al., 2021) are combined with newly developed primary specific test scales (Nehrkorn et al., 2023). Data collection took place at three points during the course of study, allowing us to test the predictive relationships between the variables longitudinally. We used a cross-lagged panel design to model the complex relationships of the developing aspect of competence (Reinders, 2006). Our results reveal a significant effect of learning-related beliefs on knowledge development. Constructivist beliefs supported the knowledge acquisition, while transmissive beliefs inhibited it, confirming prior findings (Kuntze & Zöttl, 2008; Maaß, 2011). In contrast, no significant predictive relationships were found for self-efficacy beliefs and conceptions of mathematics. These findings point to the relevance of explicitly addressing beliefs about teaching and learning in teacher education – especially when aiming to foster innovative content areas such as mathematical modelling in

primary education. Embedding activities that challenge transmissive beliefs and foster constructivist orientations could enhance the effectiveness of modelling courses. (Bönke et al., 2024; Mewborn, 1999). In the presentation, we will relate these findings to hypotheses derived from the literature and discuss further implications.

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[34.] Making the Invisible Visible: Using Mathematics Modeling to Understand Systemic Inequity

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This paper explores two iterations of a university-level senior capstone course at a US-American university designed to make systemic racism visible through mathematics modeling (Gutierrez, 2018; Gutstein, 2006; Martin, 2009), responding to the International Conference on the Teaching of Mathematical Modelling and Applications (ICTMA) call to use real-world contexts for deepening students' modeling competencies (Kaiser, 2007). The paper investigates how engaging with modeling tasks, that connect a deep conceptual understanding of mathematics (e.g., ratio, rate, slope) and systemic inequities, influences students' understanding of mathematics and their awareness of systemic racism.

The findings reveal a significant broadening of students' views of mathematics, shifting from a narrow, procedural focus to recognizing mathematics as a tool for understanding and critiquing the world (Boaler, 2002; Leonard et al., 2010; Thanheiser, 2023). This pedagogical approach presents an opportunity to make mathematics more relevant and engaging for students by connecting abstract concepts to real-world contexts (Kaiser & Blum, 2007; Niss & Blum, 2020). Simultaneously, students developed a more nuanced understanding of systemic racism, moving from viewing it as an individual bias to recognizing systemic entrenchment and the often invisible ways inequality is perpetuated (Ladson-Billings, 1997; Tate, 1994). This highlights the potential of mathematical modeling education to foster critical citizenship (Stinson & Bullock, 2012).

The study also reveals challenges in students' (including mathematics majors) initial abilities to interpret mathematical models in context, such as the slope and y-intercept of trendlines, when analyzing real-world data on racial disparities (Frankenstein, 1983; Wager & Stinson, 2012; Thanheiser & Sugimoto, 2020). Connecting mathematics to real-world contexts enhances students' ability to apply it meaningfully while also affirming their identities and experiences (Aguirre, Mayfield-Ingram, & Martin, 2019). The course addressed this through deliberate connections of mathematics to the social and historical context (Gutstein, 2003).

Furthermore, the paper emphasizes the crucial role of community-building activities in creating a supportive learning environment conducive to discussing sensitive topics (Sugimoto & Thanheiser, 2020; White, 2016). This underscores the opportunity to enhance mathematical modeling education through inclusive pedagogical practices (Gutiérrez, 2013; Skovsmose, 2012).

This research contributes to theorizing the teaching and learning of mathematical modeling by demonstrating its power in exploring social justice issues (Stinson, 2004; Thanheiser, 2023). It offers insights into the opportunities and challenges of implementing such approaches in educational settings, particularly in fostering deeper mathematical understanding alongside critical awareness of societal inequities (Gutiérrez, 2013; Ladson-Billings, 1997; Leonard et al., 2010). The use of data analysis (e.g., in Excel or Google Sheets) to examine real-world

disparities also touches upon the challenges and opportunities of integrating digital tools in mathematical modeling education (Geiger & Goos, 2015). By making the invisible visible through mathematics, this work suggests pathways for mathematical modeling education to address critical societal challenges (Gutiérrez, 2013; Skovsmose, 2012; Thanheiser, 2009).

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[35.] An instructional proposal to address the intra-mathematisation of mathematical modelling in teacher education

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Mathematical modelling ability plays a primary role in the teaching and learning of mathematics, to the extent that it has been included as one of the goals of the high school curriculum internationally. If mathematical modelling has a prominent place at this educational level, can teacher education afford to overlook it? What potential challenges and opportunities arise from its integration? Yang et al. (2025) highlight teacher-related obstacles to the implementation of mathematical modelling, with teacher education emerging as one of the main challenges. One in-service teacher remarks:

When I was in school and university, I did not study mathematical modelling; we did not have the opportunity to learn such content at that time, so for us teachers with a certain length of teaching experience, we barely know this (mathematical modelling). We need to learn, if we do not have the basic knowledge, how can we teach? (Data analysis section, para. 2)

Faced with this learning opportunity, as pointed out by teachers, the Anthropological Theory of the Didactic (ATD) has led to the development of specific didactic devices for teacher education (Barquero et al., 2018), known as Study and Research Paths for Teacher Education (SRP-TE). These are framed within the new paradigm of *questioning the world* (Chevallard, 2013), which highlights the institutional constraints encountered by teachers when implementing modelling activities. The SRP-TE represents a didactic design focused on mathematical modelling education, in which teachers have the opportunity to experience an SRP based on a modelling activity and subsequently adapt a similar activity for implementation in school settings.

This communication presents the design of an SRP-TE entitled “Characterisation of the electronic components of a defibrillator”, proposed to address a ‘classic’ issue: *Klein’s second discontinuity* (Klein, 2016/1933), in relation to the study of differential equations in the teacher education of Honduras (Paz-Corrales et al., 2024). This issue is analysed from a theoretical approach that considers two complementary dimensions: institutional and cognitive (Gascón, 2024; Moreno-Armella & Harel, 2024). Klein’s second discontinuity is presented as a specific didactic phenomenon (Gascón, 2013), referred to as the *intra-mathematisation of modelling*. This phenomenon refers to the tendency to treat a modelling activity based on an extra-mathematical situation as if it were purely intra-mathematical. As a result, the extra-mathematical context becomes incidental; moreover, teachers take on the role of users of mathematical models; and the emphasis is on technical analytical proficiency. Such an approach could eventually be adopted in high school mathematics education (Frejd, 2013). This SRP-TE consists of three phases: (1) using differential equations to describe the behaviour of a phenomenon; (2) interacting with the model by assigning values to components; (3) simulation and validation. In turn, it is structured into five modules (Barquero & Ferrando, 2024), considering that the role assumed by the teacher in each module (e.g., teacher role, student role, didactic analyst role, designer role) will lead to the emergence of institutional constraints. Furthermore, it reflects on how the study of a learning process (SRP), similar to what might occur in high school classrooms, is based on a reference epistemological model (REM, Florensa

et al., 2020), from which three fundamental ideas regarding differential equations as mathematical models are derived.

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[36.] Developing preservice teachers' creativity through a mathematical modelling week

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The South African mathematics school curriculum advocates mathematical modelling (MM) as a vehicle to equip learners with 21st-century skills. However, mathematics modelling is not taught at the school level due to teachers' unpreparedness to deal with it. Many teacher education programmes offer preservice teachers limited opportunities to enhance their creativity. Therefore, the study aimed to develop preservice teachers' creativity through an MM week. Specifically, preservice teachers' fluency, flexibility, and originality in solving mathematics problems and establishing the usefulness of such solutions (Wessels, 2017) were established. These indicators are associated with divergent production abilities, signifying the generation of ideas in solving mathematics problems suggested by the model of creativity coined by (Guilford, 1967), which theoretically underpinned the study.

Twenty-eight fourth-year mathematics education students from a particular university were divided into seven groups consisting of four members during a five-day MM week. Each group had to solve a case study addressing sustainable development goals like poverty, quality education, health, and renewable energy.

Qualitative and quantitative research methods were used to collect data through a survey, observation sheet, and focus-group interviews. The survey consisted of five open-ended questions and 21 Likert-scale items. The observation sheet aligned with the seven steps of the MM process (Niss & Blum, 2020) and comprised the four indicators

of creativity (Wessels, 2017), which were measured on five levels and varied from not viable to mostly appropriate. Both these instruments were adapted from those of Bicer et al. (2022), focusing on pre-service teachers' perceptions towards research-based instructional practices to foster students' mathematical creativity. The focus-group interviews also focused on the participants' perceived experiences regarding the four creativity indicators (Wessels, 2017).

The survey was administered before and after the MM week, while observations and focus-group interviews were done at the end of each day during the MM week. The quantitative data were analysed employing descriptive statistics, and the qualitative data were analysed through thematic analysis.

The tentative findings revealed that groups who attempted to understand the problem through research and could relate the case studies to their own lives showed more fluency in solving their case studies. Other groups who initially perceived their case studies as easy and were solution-orientated without engaging in a process were less fluent. Thus, preservice teachers' fluency in defining the problem and relating various variables to the problem can be related to their understanding of the problem, thus, the first step of MM.

Two groups who moved too quickly to mathematics calculations expressed frustration and showed limited fluency. However, one group showed some flexibility when changing their

strategy from using mathematics calculations to identifying variables to simplify their case study. Three other groups showed notable flexibility by reducing variables and structuring their case study using various approaches to derive a mathematics equation and testing hypotheses, which aligned with Steps 2 and 3 of the MM process, namely, mathematising and working mathematically.

All the groups signified the usefulness of MM, implying using mathematics to solve real-world problems to make sense, which links to Step 5 of the MM process, namely, interpreting. Most groups revealed originality by referring to new approaches associated with validating and exposing the case solution to real-life solutions (Steps 6-7 of the MM process). Therefore, the findings suggest that providing preservice teachers opportunities to think critically during an MM process could foster their creativity.

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[37.] The Impact of Chinese Prospective Teachers' Previous Experiences on self-efficacy in the Domain of Mathematical Modelling: the Mediating Role of Pedagogical Content Knowledge and Beliefs

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Mathematical modelling is a crucial skill for students in many countries (Niss & Højgaard, 2019; Schukajlow et al., 2018), yet its integration into teaching is challenging (Barquero et al., 2018; Niss & Blum, 2020). Self-efficacy can influence teachers' willingness to confront teaching challenges and their professional behaviour, thereby further affecting student outcomes (Ikeda et al., 2025).

Self-efficacy is more modifiable during initial teacher education but becomes less so post-employment (Leavy et al., 2023). Cultural and educational contexts, epistemological beliefs, and pedagogical content knowledge (PCK) influence pre-service teachers' (PSTs) self-efficacy in mathematical modelling (Ikeda et al., 2025). Learning experiences also shape general mathematical self-efficacy (Leavy et al., 2023). Given these influences, exploring factors affecting PSTs' self-efficacy in mathematical modelling within Chinese cultural and educational context is crucial. This study focuses on the following research questions:

1. What is the current status of Chinese PSTs' previous experiences and professional competence (including PCK, beliefs, and self-efficacy) in mathematical modelling be described?
2. How are the various components of PSTs' previous experiences, PCK, and beliefs related to self-efficacy in the domain of mathematical modelling?
3. Do the various components of PSTs' previous experience, PCK and beliefs have a direct impact on self-efficacy for mathematical modelling?
4. Do PCK and beliefs mediate the impact of PSTs' previous experience on self-efficacy regarding modelling?

To address these questions, this study conducted a survey of approximately 350 PSTs from six normal universities in China. The participants took part in the survey at the end of their junior year. The participants were asked to individually complete the Chinese translation of the test developed by Wess et al. (2021), which includes previous experiences, beliefs about mathematical modelling, self-efficacy for mathematical modelling, knowledge of concepts, aims and perspectives, knowledge of interventions, knowledge of modelling processes and knowledge of modelling tasks.

To analyze the data, we used the Rasch model to measure teachers' PCK. Confirmatory factor analysis assessed the validity of PSTs' previous experiences, beliefs, and self-efficacy in modelling, and cronbach's alpha coefficient examined their reliability. Subsequently, correlation analysis was conducted to explore how the various sub-dimensions of teachers' previous experiences, PCK, and beliefs are correlated with self-efficacy. We estimated two groups of regression models: one group with PSTs' modelling self-efficacy for mathematical modelling as

the dependent variable, and the other group with self-efficacy for working mathematically as the dependent variable. Path analysis was employed to uncover the underlying mechanisms through which PSTs' previous experience, PCK and beliefs influence their self-efficacy in the domain of mathematical modelling.

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[38.] Collective Planning and Executive Functions in Teacher Education for Teaching Using Mathematical Modelling

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Although teacher education in mathematical modelling is a recurring theme in research within the field of Mathematics Education and one of the main interests of ICTMA members, there are still concerns about “what is needed in terms of teacher preparation and support” (Galbraith, 2024, p. 138). The literature presents different frameworks for both initial and continuing teacher education. One such framework considers a teacher education model based on three axes: *learning about modelling*, *learning by doing modelling*, and *teaching using mathematical modelling* (Dalto, Borssoi, & Silva, 2024).

We understand that teachers who engage with the axes of *learning about* and *learning by doing modelling* may be better prepared to implement practices within the axis of *teaching using modelling*. Since 2016, a strategy we have implemented in the course *Mathematical Modelling from the Teaching Perspective*, part of a Master's programme at a Brazilian university, is collective planning. In this process, teachers share and discuss ideas for a problem-situation to be addressed with a Basic Education class. They identify the phases of the modelling cycle that may be followed under their guidance and the materials required to implement the activity, resulting in a lesson plan. Throughout this process, Executive Functions are mobilised.

Executive Functions (EF) are self-regulatory cognitive abilities that enable individuals to control impulses, adapt behaviours, make decisions, and solve problems with goal-directed focus (Diamond, 2013). Although various theoretical models exist, there is consensus regarding their multidimensional nature, with three central components: inhibitory control, working memory, and cognitive flexibility (Diamond, 2013). Inhibitory control involves the ability to suppress impulses, emotions, and inappropriate responses; working memory refers to the temporary storage and manipulation of information; and cognitive flexibility relates to the ability to shift perspectives or strategies in response to varying cognitive demands. These three components underpin other executive abilities such as reasoning, planning, and problem-solving (Diamond, 2013).

In this paper, we aim to investigate: which core Executive Functions are manifested by in-service teachers during the planning of mathematical modelling activities for Basic Education? To this end, we adopted a qualitative approach grounded in the Tree of Association of Ideas (Spink, 2013), based on data generated through audio and video recordings of a group of eight teachers who took the course *Mathematical Modelling from the Teaching Perspective* in 2024.

The results show that, during discussions, the three core executive functions were mobilised at various moments, especially when suggestions and clarifications were offered. These findings reinforce the idea that collective planning can support the implementation of modelling practices in the classroom and, by engaging executive functions at different stages, may also contribute to their development.

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[39.] Secondary Teachers' Attention During the Presentation of Solving Mathematical Modelling Tasks

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This study aims to analyze the aspects that secondary mathematics teachers focus on when presenting their solutions to mathematical modelling tasks. While previous research has explored the cognitive and affective domains involved in mathematical modelling, further discussion is needed on how these tasks are presented after solving (Brady & Jung, 2022). Given that solving mathematical modelling tasks typically involves finding pathways through discussion in social contexts, the learning that occurs during the presentation phase is equally significant. Presenting solutions to mathematical modelling tasks differs from presenting solutions to formal mathematical problems, as it often entails not only outlining the solving process but also engaging in validation work. In everyday mathematical modelling classrooms, time constraints require presenters to prioritize and focus their presentations. This study illuminates the meaning of sharing in social contexts by analyzing presenters' attention to specific aspects of their modelling task solutions.

This study was conducted within a graduate-level course at a university in Korea. Participants were 21 (prospective) secondary mathematics teachers enrolled in the course. The course involved weekly activities including reading, presenting, and discussing articles on mathematical modelling, as well as group-based problem solving and presentations on modelling tasks. Data were collected from the solutions to mathematical modelling tasks completed during a semester-long course, along with video recordings of participants' presentations. The participants' solutions and their explanations during the presentations were analyzed based on the stages of the mathematical modelling cycle (Blum & Borromeo Ferri, 2009) to identify the focal points emphasized in their presentations.

The analysis revealed that participants primarily focused their presentations on sharing the results of their investigations into realistic contexts and the assumptions they formulated during the simplifying/structuring and mathematising stages of the modelling process. Rather than presenting detailed mathematical work, participants emphasized how they explored real-world situations and made assumptions to simplify complex phenomena into manageable mathematical models. This focus suggests that participants regarded contextual understanding and sharing of assumptions as central to effectively communicating their problem-solving processes in mathematical modelling tasks.

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[40.] *The Empty Modelling Cycle* as a tool for students to learn about mathematical modelling

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Like many other countries, Norway has a two-dimensional curriculum where one dimension is content-oriented (algebra, geometry, statistics etc.) and the other is process-oriented (problem solving, reasoning, modelling etc.) (Højgaard & Sølberg, 2019). One of the processes called for, is modelling and applications, and the legislation describes that the mathematical content-oriented aims should be assessed “in terms of” the processes. What this means is not further described, but there have been examples where the nation-wide exam for grade 10 explicitly asks for modelling. Hence, Norwegian students need to both manage modelling tasks and know the nature of what they do. I find that this coincides with the *insight* and the *action* of the definition of a competency by Niss and Højgaard (2019); ‘A *mathematical competency* is someone’s insightful readiness to act appropriately in response to a *specific sort* of mathematical *challenge* in given situations.’ (italics in original, p.14). By relating *insight* to a holistic meta-perspective, I know from Maaß (2006, p. 139) that for the students to gain this insight could be a long-term endeavor. However, Beckschulte (2020) and Schukajlow (2015) have had promising results for accelerating the learning by using a solutions plan for the students to use while modelling. I wanted the students to use their own experience and words to make meaning of a solution plan for modelling, and since modelling processes do not necessarily flow in a neat cycle (Blum & Borromeo Ferri, 2009), I designed the *Empty Modelling Cycle* (EMC) to a posteriori organize students’ contributions in accordance with the plan. The EMC is a simplified four-steps cycle with no descriptions other than a heading for each step. To what extent can an EMC support students’ development of modelling competency?

While students were modelling in classroom, I created time-outs, asked what students were doing and they made their actions into content of the EMC (Figure 1). A short classroom dialogue was needed to agree on where in the cycle each action belonged. The lessons included four principles considered important. Let the students decide directions of their activity and give them time to do this (1). Use the students’ contributions to create flow forward and sense of community (2). Allow and exploit differences (3). And (4), use the modelling cycle to sequence the consolidating part at the end of the lesson.



Figure 1 Student writing in the EMC

I tested the use of the EMC in grade 8, 9 and 10 with three different activities, and in all grades the students managed to connect their actions to the cycle. The older students used more general words to describe their actions, and the younger used words more closely related to the activity at hand.

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[41.] The Model Interpretation Capacities of Undergraduate Students

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With each passing day, society is becoming more and more reliant on mathematical models to make decisions and understand the world around us. And yet, as discussed by Jablonka (2010), to most of the public mathematical models are black boxes. The responsibility of interpreting model results in light of the real world is left to a small group of people with the skills to peer inside the boxes and tell the rest of us what is going on. Undergraduate mathematics students are likely candidates for this job. But are we confident that they are graduating with the capacities necessary to interpret modelling results? Interpretive capacities can include but are not limited to: the ability to match mathematical answers, properties, and behaviours to real-world phenomena (Alpers, 2011), communication of results in non-mathematical language (Caron & Bélair, 2007), generalizing solutions made for specific contexts (Ramírez-Montes et al., 2021), and, given the current interest in ethical mathematics education (see e.g. Sumpter et al., 2024), recognition of the real-world ethical implications of modelling results. Further, model interpretations are intricately tied to model assumptions. Do students recognize this fact?

Prior research into this topic at the undergraduate level is sparse and sometimes contradictory. The study by Klymchuk et al. (2011) showed that when students were able to use their prior knowledge of the real-world context, they could make sense of differences in prediction from models of disease spread. On the other hand, other studies have shown that model interpretation can certainly be a stumbling point for students (Camacho-Machín & Guerrero-Ortiz, 2015; Galbraith & Stillman, 2006; Rowland & Jovanoski, 2004).

In this talk, I will present the results of a case study that investigates the mathematical model interpretation capacities of undergraduate differential equations students. The guiding research questions are the following: What capacity to interpret mathematical modelling results do undergraduate students demonstrate? Further, how do they demonstrate understanding of the connection between interpretation and assumption? The case study involves a small group of undergraduate students at The University of Calgary, all of whom completed an introductory differential equations course in the Fall of 2024. Study participants complete two interviews. The first, a task-based interview which is completed in pairs, and includes both a problem designed to engage the students in the full modelling process and another problem that specifically asks participants to interpret the results of a model. Second, participants complete a semi-structured interview, with the researcher, where they are asked to expand on interpretations that emerged in the task-based interview and are asked specific questions about model interpretation.

Results presented will focus on categories of analysis such as, the level of prompting required to complete the interpretive tasks, the relation between task type and level of interpretive capacity, and the degree to which assumptions are considered when interpreting. Using the data on undergraduates' current interpretive capacities as a reference point, I will highlight the strengths

and weaknesses of current course structures for preparing students to take on the responsibility of model interpretation in the 21st century.

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[42.] Cognitive processes of students while solving mathematical modelling tasks revealed with the help of eye tracking

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The modelling cycle by Blum and Leiß (2005) illustrates the process of mathematical modelling. Its first three steps—understanding, simplifying/structuring, and mathematizing—lead to three stages of development: the situation model, the real model, and the mathematical model. Understanding students' modelling processes is a central goal in mathematics education research and there is an ongoing need to investigate the cognitive processes due to the fact, that there is still limited insight into the processes and strategies students use during the different stages of model construction (Blum & Leiß, 2007; Schukajlow et al., 2023).

Also, there is a growing interest in and increasing use of eye tracking in mathematics education research (Schindler, 2021). The interpretation of the eye tracking data often relies on the *Eye-Mind Hypothesis* (Just & Carpenter, 1980), which originates from research in reading. This is why its assumptions cannot be uncritically transferred to the context of learning mathematics (Schindler, 2021). Findings from the geometry study by Schindler and Lilienthal (2019) indicate that the *Eye-Mind Hypothesis* largely holds – meaning that what is being visually fixated is, in most cases, also being cognitively processed in a mathematical way. The combination of eye tracking and stimulated recall interviews therefore proved to be particularly beneficial (Schindler, 2021).

This pilot study aims to investigate the cognitive processes of students engaged in solving mathematical modelling tasks outdoors by integrating mobile eye-tracking data with stimulated recall interviews. Three secondary school students (age 15) participated in the study, each working individually on three of four different modelling tasks situated in real-world contexts. Three tasks were solved twice and one even three times. During task processing, students wore a mobile eye-tracking device that recorded their gaze behaviour as they interacted with real-world objects in an outdoor setting.

Immediately after completing the three tasks, the students were shown the video sequences of their own gaze paths and asked to comment on their thoughts during the solving processes. These interviews were transcribed and analysed with a focus on the connection between verbalised cognitive strategies and corresponding gaze data.

The analysis revealed that the phases found in the analysis correspond to the modelling stages of the modelling cycle by Blum and Leiß (2005). Each phase was associated with characteristic patterns of eye movements. Additionally, we expect that different patterns of eye movements represent different solution approaches.

Furthermore, we also discovered that the stimulated recall interviews not only served a reflective purpose but also triggered new cognitive engagement. The students often developed additional solution ideas upon viewing their own gaze paths, sometimes identifying overlooked elements or re-evaluating previous assumptions.

In conclusion, the pilot study supports the rich potential of combining eye tracking with stimulated recall interviews to uncover students' cognitive processes during modelling. The findings provide preliminary evidence that visual patterns can act as indicators of underlying problem-solving strategies and success.

The presentation and paper will focus on describing the results of the pilot study and giving visual examples of how the different stages of the models developed.

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[43.] **How can AI win *Race to Twenty*? Exploring mathematics, strategy and machine learning**

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As artificial intelligence becomes more embedded in daily life, the need for a basic understanding of computational thinking grows. Researchers have long noted the strong connections between computational thinking and mathematics (Papert, 1980), and recent studies show that programming can enhance students' grasp of mathematical concepts (Benton et al., 2018), while mathematics can support understanding of AI's foundations and implications (Tamborg et al., 2022). This study contributes to that discussion by examining how the strategic game *Race to Twenty* (Brousseau, 2002) can be used to model and explore AI as a concept. The game, which fosters early algebraic reasoning, changes character when played by an AI: the mathematics shifts from algebraic strategy to statistical pattern recognition. We ask: How can the game *Race to Twenty* be used to model and understand the underlying processes of artificial intelligence?

In our analysis, we draw on the Anthropological Theory of the Didactic (ATD), particularly its conceptualization of modelling as a Study and Research Path (SRP) situated within the Paradigm of Questioning the World (Chevallard, 2006; 2015). Within ATD, mathematical activity is understood as inherently a modelling endeavour—not simply as a translation between intra- and extra-mathematical domains, but as an inquiry-driven process of constructing answers based on prior knowledge, data, and emerging insights (García et al., 2005). An SRP begins with a generating question, Q_0 , and the subsequent works (w), drawn on to attempt to answer this question, are not predefined. Rather, they must be identified as relevant in response to Q_0 and the derived questions that emerge ($Q_1, Q_2, \dots, Q_n$). Crucially, SRPs are not linear sequences; they evolve along multiple, often unpredictable trajectories (Bosch & Winsløw, 2015).

The study adopts Didactic Engineering (DE) as its research methodology (Artigue, 2008), which consists of four interrelated phases: (1) preliminary analyses, (2) design and a priori analysis, (3) implementation with observation and data collection, and (4) a posteriori analysis (Barquero & Bosch, 2015). This paper focuses on the final phase—a posteriori analysis—by examining the question-and-answer dialectics that emerged during the SRP. These interactions are represented using a Q&A diagram (Østergaard, 2022), allowing for a structured reconstruction of the inquiry process. The empirical context of the study involves 20 first-year master's students enrolled in a mathematical modelling course. Most of the participants had professional teaching experience at the primary or lower secondary level but limited formal training in mathematics.

The analysis of questions and answers begin with the question: “How can one win the game *Race to Twenty*?” This led students to model the game algebraically and explore strategic reasoning. As the SRP progressed, new questions emerged: “How does an AI learn to play?” and “What is the logic or code behind it?” These questions necessitated a shift from algebraic to probabilistic thinking, which prompted reflections on the contrast between human reasoning and machine learning. The Q&A trajectory captured in the a posteriori analysis illustrates how

students navigated between mathematical modelling and computational thinking—revealing key insights into both domains.

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[44.] **How do pre-service mathematics teachers solve a modelling problem using AI?**

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As technology is evolving rapidly and AI-based tools become easily accessible to the general public, mathematics education is also subject to major changes. Using AI-based tools for the teaching and learning of mathematics can offer many benefits such as supporting self-regulated learning and eliciting critical thinking as well as creativity, but also poses challenges concerning, for example, a reliance on technology that limits the understanding of fundamental concepts (Pepin et al., 2025). Studies assessing the ability of generative AI technologies such as ChatGPT to solve mathematical problems have yielded mixed results, where especially more complex problems were sometimes solved insufficiently or incompletely (e.g., Wardat et al., 2023), while the development of more sophisticated versions showed an improvement in its mathematical problem-solving ability regarding the clarity and correctness of answers (Schorcht et al., 2024). In the context of mathematical modelling, the AI-generated solutions were found to be less sufficient or incomplete in terms of modelling sub-competencies for going through each step of the modelling cycle for an increased complexity of context – even though only requiring basic mathematical operations (Spreitzer et al., 2024).

Digital technologies can be used to assist in solving mathematical modelling problems in each phase of the modelling cycle (Greefrath et al., 2018) and support modellers in both extending their capability and gaining extra mathematical-knowledge (Siller et al., 2024). Several studies showed that the use of digital technologies for mathematical modelling offers many benefits such as the development of knowledge and skills but also an emotional and psychological enhancement (Cevikbas et al., 2023). This raises the question of the role of AI as an emerging technology in the context of mathematical modelling.

In this study, a learning environment is designed, in which a version of ChatGPT is used to provide assistance to pre-service mathematics teachers in solving a modelling problem. The participants had only limited experience with mathematical modelling, but were sensitised to the use of AI-driven tools. The chatbot was primed to adhere to didactic principles, to offer modelling-specific support and to refrain from presenting a solution. In order to evaluate the learning environment and analyse the participants' interaction with the chatbot, the following research questions are addressed:

- In which phase(s) of the modelling cycle is ChatGPT used? Is it only used in certain phases?
- How does the use of the AI-based tool support the modelling sub-competencies? What purpose is it used for?
- What is the trigger for its use? At which point do participants become aware of a situation, where the interaction with ChatGPT could support them in proceeding and solving the modelling problem?
- What is the relationship between the use of (meta-)cognitive strategies and the use of ChatGPT?

In order to answer these questions, the participants' chat history as well as their written solutions are analysed using the qualitative content analysis (see Kuckartz, 2014).

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[45.]

[46.] Mathematical modelling in everyday contexts: An analysis of the “Christmas dinner” task in primary schools in Slovenia and Croatia

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As part of the research conducted for a doctoral dissertation, a professional development program for primary school teachers was implemented in Slovenia and Croatia. The program included four mathematical modelling tasks, one of which was the "Christmas Dinner" task, adapted from English (2007), and is the focus of this analysis. The study involved fourth and fifth grade students from seven primary schools, with the primary aim of investigating how students approach solving mathematical modelling tasks.

The research methodology combined qualitative and quantitative analysis of students' written responses. The tasks were completed in small groups during regular school hours, with emphasis placed on group decision-making, collaboration, and creativity. The goal of the "Christmas Dinner" task was to engage students in organizing a complex event—a Christmas dinner—within time, space, and logistical constraints. Students were required to create a detailed schedule for meal preparation, including optimal use of kitchen appliances, task distribution, and spatial organization. Particular focus was placed on quantity relationships, time optimization, and realistic planning, making the task a strong example of the development of combinatorial thinking.

Initially, the collected responses were separated into those from Croatian and those from Slovenian students to allow for comparison of potential similarities or differences. Students' solutions were then categorized based on their level of completeness, i.e., by the number of criteria considered in solving the task. The five criteria were: time scheduling, task delegation, cooking time for each ingredient, additional required activities, and the use of specific kitchen tools and appliances. Responses were divided into two levels. Level one included those that addressed at least four of the criteria, while level two included those that addressed all five. Croatian students reached the second level of solution more frequently (74%), while Slovenian students more often remained at the first level (63%). Students from both countries demonstrated creativity and adaptability by adding several additional criteria that were not required, such as preparation time, decoration, and creating a pleasant atmosphere (English, 2007; Hu et al., 2023).

The analysis shows that tasks such as “Christmas Dinner” enable students to develop key competencies such as collaboration, planning, flexibility, creativity, and the ability to connect mathematics with real-life situations. Most solutions involved basic mathematical operations and simple approaches, while some demonstrated abstract reasoning and deeper data analysis. This confirms the task's potential to foster higher-order mathematical thinking.

This study affirms that mathematical modelling through real-life and culturally relevant tasks can significantly enhance mathematics education in the primary classroom, especially when such tasks promote knowledge integration and the development of essential life skills. As shown by

other studies as well, tasks based on real-life situations not only enhance problem-solving skills but also foster students' creativity and collaboration (Armutcu & Bal, 2023; Akapame, 2022; English & Watters, 2004).

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[47.] **Extra-mathematical and Technological Knowledge in the Mathematical Modelling of Sustainability-related Situations**

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Various sources highlight the need to include authentic problems in mathematics education as a means of developing knowledge and skills that allow students to reflect on real-world situations (Vos, 2011). Authentic problems may involve STEM or sustainability contexts, as also suggested by different curricula worldwide (Wiegand & Borromeo-Ferri, 2024). The incorporation of mathematical modelling in the classroom, considering these contexts, responds to this challenge (Camacho-Machín et al., 2024; Bulut & Borromeo Ferri, 2025). However, doing so requires that teachers possess knowledge of modelling tasks, processes and assessment (Borromeo Ferri, 2018). Furthermore, given the complexity of these contexts, we argue that modellers require an inquiry-based attitude that goes beyond the mathematics in order to frame questions and construct appropriate mathematical models grounded in extra-mathematical knowledge. Likewise, due to the nature and inherent difficulty of sustainability or STEM problems, it is necessary to employ analytical and technological tools to formulate conjectures or solve the proposed situations.

Despite the potential of mathematical modelling to promote critical thinking and its relevance to Education for Sustainable Development, its integration into classrooms is progressing slowly, partly due to teachers' limited knowledge in these areas (Alsina & Vásquez, 2024; Guerrero-Ortiz & Borromeo Ferri, 2022). Therefore, we explore the knowledge that prospective secondary mathematics teachers construct when participating in a modelling seminar aimed to develop critical thinking and the use of technology. The research questions guiding this study are: What are the characteristics of the extra-mathematical knowledge that emerges when modelling sustainability-related problems? and How are digital tools used in these contexts?

Based on a framework that integrates knowledge of technology, mathematics, pedagogy, and extra-mathematical aspects for the teaching of modelling (Castillo & Guerrero-Ortiz, in press), the work produced by 22 participants during a seminar on modelling and critical thinking was qualitatively analysed. The seminar was structured into five phases: exploration of open situations (ecological issues, resource use, etc.), individual and group resolution of a water crisis problem, redesign of a problem, design of tasks using technology, and final design of tasks in sustainability contexts. To evaluate knowledge construction, the participants' work developed progressively across all phases was analysed; this paper presents the results of the analysis of the work produced in the first and fourth phases, completed by three groups.

The first phase allowed us to identify the participants' interests in current issues by analysing the questions they proposed in relation to a situation of their choice. The process they developed to respond to these questions, as well as their use of technological tools, was also examined. The fourth phase enabled a comparison between the knowledge elements that participants included in their own work and those they aimed to promote among their future students.

The results reveal the presence of environmental and sustainability-related knowledge in the analysed processes and tasks, with varying levels of detail. Disciplinary, technical, socio-political, economic, historical, and contextual knowledge differs and deepens depending on each group. The use of digital tools supports both the mathematical process and the understanding of sustainability issues, where extra-mathematical knowledge is essential for a comprehensive and meaningful interpretation of a mathematical modelling. In task design, the use of technology stands out as a valuable aid in the construction of graphs and simulations. Finally, extra-mathematical knowledge proved to be essential in fostering critical thinking regarding the issues addressed in class. Likewise, its relevance stands out in the design of modelling tasks focused on sustainability-related problems. Moreover, it encourages students to take a stance on individual and collective actions and to anticipate future situations.

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[48.] Bridging Critical Thinking and Mathematical Modelling: A Theoretical Framework and Instructor-Based Study

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Critical thinking (CT) is widely recognized as a key skill to develop for the 21st century (Gravemeijer, 2017). In mathematics, CT is often associated with mathematical reasoning and problem-solving. However, as Jablonka (2020) points out, the connection between mathematics and CT is under-theorized. This presentation explores how mathematical modelling can serve as a bridge between the two, allowing us to examine its role in fostering critical thinking within mathematics education.

Drawing on the American Philosophical Association's (APA) Delphi framework (Facione, 1990), which identifies six core CT skills, we developed a theoretical model that maps these skills onto the six phases of Caron's (2008, 2018) modelling cycle. To test the model, we analysed the experiences of eight Calculus instructors engaging with modelling tasks structured around this cycle. This allowed us to identify both productive alignments and points of friction in practice.

Participants naturally engaged in critical thinking during the final phases of modelling—analysis and interpretation-validation. Despite initial discomfort with interdisciplinary content, these phases allowed us to witness deeper reflection. However, many teachers still view them as outside their domain, often preferring to defer responsibility to other subject experts.

Conversely, the phases directly involving mathematical concepts, such as mathematization and mathematical processing posed challenges. Teachers have expressed discomfort or a lack of clarity regarding the contextual application of mathematics, the use of diagrammatic representations to illustrate systems, and the use of concepts like flows to describe components. Some of these difficulties are linked to a lack of training, making it difficult to assess critical thinking during these phases. As a result, teachers tend to focus on the more technical and abstract aspects of the exercise. Thus, while the link between modelling and critical thinking is clear, its application is more challenging in certain phases.

Our study contributes to the theorization of the link between mathematics and critical thinking by conceptualizing how the phases of mathematical modelling align with critical thinking skills. Our findings confirm that this framework is practicable in an educational setting and provides valuable insights for integrating critical thinking into mathematics education. We will point out the limitations of the study, highlighting the need for further investigation into how critical thinking skills are mobilized during transitions between modelling phases.

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[49.] **The Critical Role of Extra-Mathematical Knowledge in STEM-based hands-on mathematical modelling activities**

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Mathematical modelling (MM) plays a pivotal role in showing students the practical applications of mathematics in daily life through real-world context (Kaiser et al., 2011), particularly in STEM real-world workplace scenarios (Kohen & Orenstein, 2021). However, fully implementing MM, especially in STEM contexts, depends not only on executing each phase of the MM process, but also on student's extra-mathematical knowledge (EMK) tied to real-world contexts (Borromeo Ferri, 2006). Despite its importance, especially in specific phases of the MM process, such as the simplifying/structuring phase which demands bridging reality and mathematics (de Almeida, 2018), research regarding its characterization and role in applying different MM phases remains limited.

Research indicates that STEM-based hands-on activities simulating engineers' teamwork enhance MM engagement (Geisler, 2021). Engineering design process parallels MM, emphasizing real-world descriptions via mathematical models, making MM practically applicable (English, 2024). Developing a prototype based on these models gives students hands-on experience, particularly in the early MM phases (Kertil & Gurel, 2016; Baioa & Carreira, 2019). Thus, this study aims to characterize EMK and explore its application as students progress through MM phases in an engineering design project.

Study involved 351 eighth-grade students from 16 schools nationwide, who participated in a 30-hour engineering design project as part of STEM classes. Each class was introduced to a technology chosen by the teachers, either GPS or ultrasonic sensors. The introduction covered basic background, necessity, and operating principles, which led to an initial enrichment of EMK, which students could later apply in the early MM phases.

Students worked in teams of 4–5 members to collaboratively engage in a hands-on MM process, which included exploring the technology to acquire necessary EMK and build a prototype of an engineering product based on it, requiring the application of appropriate mathematical models. The research methodology included analyzing group presentations created by the student teams to document their work throughout the project. Students were instructed to prepare a detailed presentation to be shared with other teams and engineers. These were analyzed using content analysis (Creswell & Poth, 2016) to identify patterns reflecting students' EMK.

Findings reveal three main characteristics of EMK— technology comprehension, product vision and product design – each emphasized differently across MM phases; Technology comprehension was most applied in the understanding phase, product vision in simplifying/structuring phase, and product design in mathematizing phase. Furthermore, findings highlight the hierarchical relationship between these characteristics, with technology

comprehension and product vision serving as a foundational element for obtaining EMK related to practical product design.

This study contributes theoretically by emphasizing the components of EMK and its role in shaping realistic models, particularly in hands-on MM activities. Practically, the findings underscore the importance of balancing theoretical and practical experiences to support EMK development, particularly in early MM phases. The study deepens our understanding of fundamental knowledge development and how it is applied in STEM-based hands-on MM activities, providing insights into designing such activities to promote EMK.

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[50.] Online Assessment of Mathematical Modelling Competency for Grade 6 Students in China

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Educational assessment serves as a crucial mechanism for cultivating students' mathematical modelling competencies (Blum, 2015; Kaiser, 2017; Niss & Blum, 2020). Online assessment of mathematical modelling offers advantages by transcending temporal and spatial constraints while enabling efficient data collection, automated scoring, and suitability for large-scale implementation (Xie, 2021). Following the bottom-up approach, mathematical modelling competency is regarded as a set of cognitive and metacognitive competencies aligned with the mathematical modelling cycle (Niss & Blum, 2020). To develop our online assessment tool (*Mathematical Modelling Competency Online Test* (MMCOT)), we employed the structural model with seven sub-competencies of mathematical modelling (Wang et al., 2023): the cognitive competencies of understanding information, making models, working mathematically, and interpreting and validating; and the metacognitive competencies of orienting and planning, monitoring and regulating, evaluating and improving. We investigated the validity of the MMCOT and Grade 6 students' performance with the following research questions:

- 1: How valid is the MMCOT for Grade 6 students' mathematical modelling competency?
- 2: How do Grade 6 students in China perform in mathematical modelling competency?

Methods

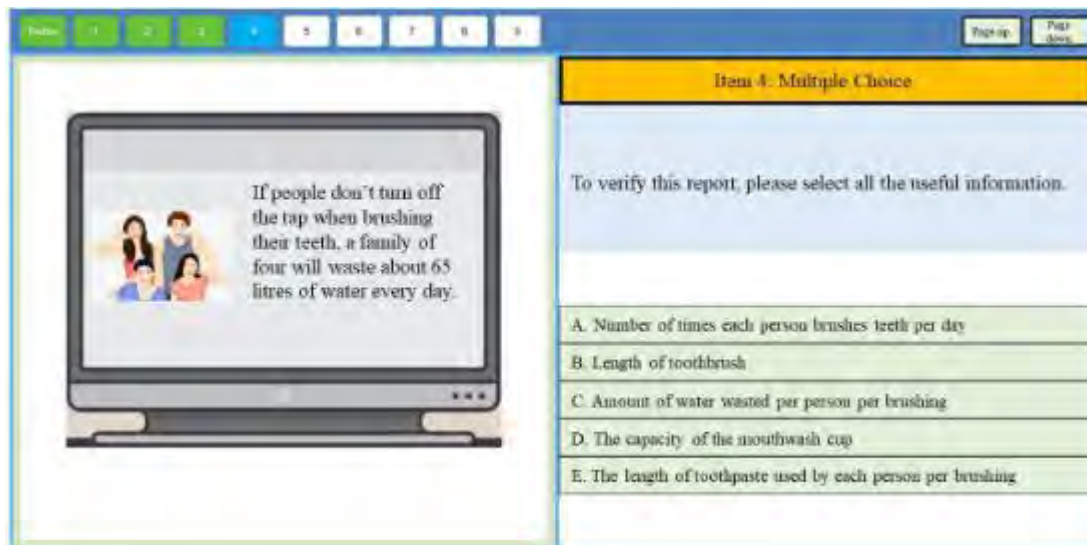
We used validation by expert consultation and latent modelling to evaluate the validity of the MMCOT to assess students' mathematical modelling competency and a survey study to get insight into Grade 6 students' mathematical modelling competency. Six experts specialized in mathematical modelling education and large-scale assessment reviewed the MMCOT. A total of 2,360 Grade 6 students participated in the testing; 1,201 boys (51%) and 1,159 girls (49%) from 19 primary schools across four cities in China. The assessment was conducted online on the Open-CT platform. Students individually completed the MMCOT in their school's computer lab under standardized conditions, with a test duration of 60 minutes. Upon completion, both achievement data and process data were extracted for analysis.

Instrument

The MMCOT consists of a warm-up task and three mathematical modelling tasks. There are 29 items in the three modelling tasks, including multiple choice items and open-ended items. Figure 1 shows an example of a modelling task and an item measuring the sub-competency of *Understanding Information*.

Figure 1

An Example of the Saving Water Task



Results and Conclusions

Content validity for the MMCOT was established via expert evaluation, who confirmed the overall test quality, item-to-framework alignment, and item quality. Internal consistency ($\alpha=0.858$) and excellent confirmatory factor analysis-model fit confirmed the latent structure (CFI=0.944; TLI=0.934; $\chi^2/\text{df} = 1.415$; RMSEA=0.038; SRMR=0.045). The correlation ($r=0.601$, $p<0.01$) between MMCOT and mathematics achievement on the final Grade 6 test showed criterion validity.

The overall mathematical modelling competency of Chinese Grade 6 students is not very high ($M=67\%$, $SD=19$; between 50% to 83% correct). Students especially find the sub-competencies of *Making models* (54% correct) and *Monitoring and regulating* (50% correct) difficult, while the sub-competencies of *Understanding information* (83% correct) and *Orienting and planning* (76% correct) were more successful.

This study confirms the validity of the MMCOT and reveals that Chinese Grade 6 students have a mixed profile of mathematical modelling competencies.

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[51.]

[52.] From an Imprecise Imprecision: Reflections on Culture and Modelling

Daniel C. Orey

In this talk, I share my thoughts in relation to culture and modelling.

My awareness gradually emerged over time beginning while growing up in an extremely diverse west coast of North America, later working in Guatemala, New Mexico, California, and Nepal, and where I live now in Brasil (seasoned with a few Fermi problems and some emic/etic dialogue). As a young and naïve master's student in New Mexico, I participated in an NSF grant that allowed us to travel throughout the Southwestern USA, and then to Guatemala and México. Using LOGO, we observed first contacts with computers, this was where I began to observe interactions between humans, culture and mathematics. Over time, I have developed 4 personal caveats for my work:

Caveat 1: *Math is a language*. Every known language, synthetic or natural, has its own grammar, prose and poetry. Mathematical prose can be expressed through modelling, which is incredibly diverse, and provides beautiful expressions of a modeler's own creativity, values, culture and mathematics.

Caveat 2: *Universality*. All people on this planet “do” mathematics. Bishop said that we can think of mathematics as the numerous and diverse ways in which humans “count, locate, measure, design, play and explain.” Bassanezi and D'Ambrosio added modelling to the mix.

Caveat 3: *Precision vs Imprecision*. There are different and diverse ways to model – some are exact, precise and beautiful in their own right. At the same time, those emerging from informal contexts, often local and historically based, and can be somewhat imprecise. I study the imprecise, that is found in common day-to-day contexts, yet I remain grateful for the precise. Ethnomodellers begin by celebrating different types of modelling. Like D'Ambrosio and Bishop, this is culturally based, where each model pursues its own truth, and context. One type of modelling represents a very necessary scientific-academic modelling, the other represents local-informal, or what I call, “mais ou menos.” From this informality, some interesting ideas emerge, which allows for introduction to formal, academic science/modelling. One of my favorites emerged in the early 1980s where it was found that Aymara (an Andean indigenous language) could be used to machine translate human language. This led to software which magically translated, one word to another, to of late where we have AI translated aspects of whale language into human language!

Caveat 4: *Humility*. Looking at mathematics and the models that do not emerge in western capitalist contexts demands humility, flexibility, and an open mind. I learned this from my dear uncle on his ranch in Northern California, and Mayan merchants, and a rice farmer in Piauí who all asked the same question, “you studied at a university, why don't you know this?”

These important elements form dialogical processes of ethnomodelling that researchers in Brasil, the USA, Ghana, México, Costa Rica, Nepal and Indonesia are “playing with”. My talk will conclude with some ethnomodels produced by researchers in many countries.

[53.] Training primary school teachers to create modelling problems and the impact on their professional knowledge of modelling: A case study in France within an action research project

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Modelling is not widely taught in French primary schools, yet the national curriculum requires primary teachers to propose situations rooted in reality. Such situations tend to lead pupils to use an alternative approach to that required in solving intra-mathematical problems, as they must first be made accessible for mathematical treatment. To address this, a collaborative research project, the Lieu d'Éducation Associé LÉA MaPcV ([Mathematisation of Concrete Problems in Video Format](#)), has been set up, funded by the French Institute of Education at the École Normale Supérieure de Lyon. LÉAs (*Associated Educational Places*) are collaborative structures designed to foster partnerships between educational practitioners and researchers. They promote long-term action research projects combining classroom experimentation, professional development, and co-construction of knowledge. The LÉA MaPcV specifically brings together teachers and trainers, a national education inspector and two researchers in mathematics didactics with the objective of conducting collaborative research over three years to promote the teaching and learning of modelling in primary schools, including reflections on training (Yvain-Prébiski, 2023). Work for the 2023-2024 academic year focused on developing 2 problems in video format, for pupils in grades 3 to 6, to enable a real-life mathematical modelling activity to be completed in class. Proposing a video-based problem on everyday life requires a first step to make it accessible for mathematical processing: the crux of horizontal mathematisation (Yvain-Prébiski, 2021). Also, mathematical processing, the vertical aspect of mathematisation, must draw on the mathematical knowledge accessible to pupils (Baumanns & Rott, 2024). These two aspects of mathematisation, borrowed from Treffers and set out in Realistic Mathematics Education (Freudenthal, 1991) are building blocks of mathematical modelling. In this paper, we examine how situating primary school teachers as designers of such problems develops their professional knowledge of teaching modelling.

We first studied how, in developing an extra-mathematical situation, the dialectical relationships between horizontal and vertical mathematisation are viewed by teachers and trainers in the LÉA MaPcV. Then, we explored the extent to which, by placing teachers in the situation of creating a modelling problem, they develop the professional skills involved in modelling, in line with Hartmann & al. (2023). To do this, we interviewed teachers to measure the impact of the creative activity on their professional modelling knowledge. Our methodology used qualitative analyses to identify potential changes in knowledge about the mathematisation process. These analyses are in progress. Initial results suggest that teachers' modelling knowledge evolves from a procedural to a more conceptual view with, for example, awareness of the need to make choices and simplify assumptions. However, the analyses also show that this awareness has not yet translated into formal learning objectives for pupils, in keeping with previous research on teaching modelling in secondary schools (Yvain-Prébiski & Chesnais, 2019).

So, this paper briefly presents the objectives of the LÉA MaPcV, the problem created, and choices specified by the teachers. We then present our methodology for analysing interviews and the results regarding how creating a modelling problem impacts the professional knowledge of teacher-designers.

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[54.] How do primary mathematics student teachers respond to a modelling unit? Some answers from empirical research

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This paper reports on the responses of primary mathematics student teachers to a mathematical modelling unit developed in the project Studies in Teaching Approaches for Mathematical Modelling at Primary Level (STAMM@prim). The project, which follows from CoSTAMM (Durandt, Blum & Lindl, 2022), has been launched early in 2025 at a public university in Johannesburg and aims to link essential elements of an effective teaching design (particularly the ‘method-integrative’ design) with contextual factors (e.g., key elements of online learning) in the development of student teachers’ modelling competence. This new project differs from previous work in the CoSTAMM studies mainly in three aspects: i) the level of mathematical content was adapted to primary mathematics in the topic area of fractions, ii) an online learning component was incorporated in the design and implementation of the study, and iii) a component was added towards the PCK development of student teachers.

The overarching purpose of the study is to develop primary mathematics student teachers mathematical and modelling competencies, and their pedagogical content knowledge (PCK) regarding mathematical modelling. This purpose is aligned with the educational needs in South Africa, and globally.

Both instructional and constructional elements were appropriately intertwined in the new study, based on a review of the state-of-the art, and a 10-lesson modelling unit were developed that included both face-to-face and online offerings (with both synchronised and asynchronised components). Data were collected from approximately 90 participants, all third-year students, during the first semester through a pre- and post-test. The design of the instruments was informed by guidelines from Larson and Silbey (2015), suitable for teaching mathematics at the primary level. Both pre/post-tests include questions on conceptual aspects, mathematical operations, modelling, and teaching the mathematical topic (fractions) through modelling, and are aligned with the modelling unit. Quantitative data were collected and afterwards analysed via SPSS. In particular, a *t*-test and regression analysis via ANOVA and ANCOVA was applied.

We know an educational intervention can effectively be transferred from one context to another while yielding simultaneously positive learning gains (Lindl, Durandt & Blum, 2025). This promising result, knowing the several similarities between the educational projects (CoSTAMM and STAMM@prim) in terms of instructional and constructional elements, ceded the expectation that students who will be exposed to the new study will respond positively in terms of their learning gains. In the presentation I will report on the essential results, thus how participants responded to the intervention.

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[55.] Integrating computational thinking into mathematical modelling: A conceptual framework with example projects

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While mathematical modelling is already an established field of research in mathematics education, the integration computational thinking (CT) remains a relatively recent development. Looking at CT as a process, Kallia et al. (2021) view CT to be similar to mathematical modelling. Their process model of CT in the context of mathematics is structurally similar to the modelling cycle extended by a computer model proposed by Greefrath (2011). Furthermore, Ang (2021) looked at three different modelling projects and highlighted how CT and computer programming supports the modelling process through aspects like pattern recognition in data or the construction of simulation. Looking at modelling projects without a focus on digital tools, Bayer et al. (2024) recognized multiple different shared aspects of CT and mathematical modelling in the modelling processes carried out by secondary school students.

Despite the promising intersections between CT and mathematical modelling and their potential to support each other, research on the integration of both aspects is limited.. In this contribution, we explicitly address the didactical potentials and challenges arising from integrating CT systematically into mathematical modelling processes. As a basis for further research, we introduce a theoretical framework for fostering CT in mathematical modelling projects and leveraging the potential of CT for the modelling process. We illustrate this integration through prototypical modelling projects focused on aspects such as data analysis, algorithmic problem solving, and computer-based simulations. Finally, we discuss possible criteria to evaluate and reflect the effectiveness of fostering CT within mathematical modelling contexts at the secondary school level.

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[56.] Dialoguing with AI: A Case Study of Peer-like Collaboration Between ChatGPT and a Pre-service Teacher in a Mathematical Modelling Task

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The integration of generative artificial intelligence (AI) in education offers new avenues for enhancing mathematical modelling (MM), particularly in supporting student reasoning and problem-solving. This study investigates how a pre-trained AI language model, ChatGPT, can function as a peer collaborator in the context of two complex university-level modelling tasks. Focusing on a single-case study of a fourth-year pre-service mathematics teacher, the research explores the dialogic interaction between human and AI, with an emphasis on co-construction of understanding and mutual engagement in problem solving.

Framed within the theoretical lens of Cultural-Historical Activity Theory (CHAT), the study conceptualizes ChatGPT not merely as a tool but as a semi-autonomous participant within the activity system. CHAT enables the analysis of how subject, object, mediating artifacts, rules, community, and division of labor interact dynamically during modelling activities (Engeström, 2001).

The study is also informed by a cognitive perspective on mathematical modelling, drawing “modelling routes” to examine the pathways and decisions taken during problem solving (Borromeo Ferri, 2007, p.265).

The research addresses the following questions: (1) How does a pre-service teacher engage with ChatGPT during a mathematical modelling task? (2) In what ways does ChatGPT function as a peer-like collaborator, including its ability to ask questions and guide the process? and (3) What types of modelling routes emerge in the interaction between the AI and the pre-service teacher? To investigate these questions, the researchers used two open-ended modelling problems (Erbaş et al., 2016) that required the use of technological tools such as GeoGebra, Excel, or Desmos. The pre-service teacher was asked to solve both problems while interacting with a specially trained version of ChatGPT, designed to function as a pedagogical peer. ChatGPT was capable of asking questions, initiating dialogue, and offering reflective prompts. Data sources included screen recordings of the interactions, think-aloud protocols, and a post-task interview. Qualitative analysis was guided by CHAT and cognitive modelling perspective, focusing on interactional dynamics and the identification of modelling routes.

Findings show that the student–AI interaction reflected key aspects of peer collaboration, such as shared reasoning, negotiation of ideas, and questioning. ChatGPT contributed to the modelling process by proposing assumptions, generating representations, and guiding validation steps. The pre-service teacher engaged in exploratory reasoning, sometimes leading the discussion and at other times following AI prompts.

Analysis of both modelling sessions revealed distinct modelling routes, reflecting different starting points and sequences through the modelling phases (e.g., understanding, mathematizing, interpreting), which were then compared with those reconstructed in earlier studies (Borromeo Ferri, 2007).

This study contributes to the discourse on AI in education by demonstrating how large language models can act as dialogic partners in mathematical modelling tasks. By combining CHAT and cognitive perspectives, it provides insights into how peer-like AI collaboration can shape modelling activity systems and influence cognitive modelling processes. The case opens new possibilities for the design of modelling tasks and teacher preparation in AI-supported learning environments.

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[57.] Pre-service teachers discussions of “best” economic model for water consumption and sustainability – in Norway and South Africa

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Economic models influence both our lives and society, and teachers can play an important role in broadening students’ understanding of how different models can impact sustainability issues. In this paper, we discuss how pre-service teachers (PTs) explored and presented their investigations of water usage and sustainability through the lens of mathematical modelling. Julie and Mudaly (2007) highlighted the concept of mathematical modelling as content, which can involve constructing an economic model without prescribing specific mathematical concepts or procedures as outcomes of the modelling process. This perspective serves as a theoretical lens for examining how future mathematics educators can integrate modelling strategies into their teaching practice. We focus on the following question:

What did South African and Norwegian PTs consider when discussing and modelling the ‘best’ economic model for water consumption and sustainability?

Starting from the sustainability goals, the researchers designed a toolkit (Kacerja, forthcoming) consisting of literature, lectures, tasks, videos, contextual information, prompts, and student-active learning activities. An intervention study focusing on PTs was conducted in two classes in Norway and one class in South Africa. In this paper, we focus on the work—presentations and discussions—of selected PTs who specifically explored economic models for water payment.

Preliminary findings showed that the PTs discussed various models for paying for water. They compared pre-paid water systems with different post-use billing models, highlighting that while pre-paid systems could “encourage financial management” and “prevent debts,” the latter carried a “high possibility of debt” and placed “less focus on saving water.”

Some PTs concluded that “prepaid systems seem BEST.” Others noted that when the cost of water is included in apartment rent, “one is less attentive to usage.” They also emphasized the importance of having a “social responsibility” towards “communities who cannot afford” to pay for water. Some groups outlined different payment models for different regions. For example, in South Africa: “Soweto metropolitan: no payment,” “Karoo Hoogland: 6000 L free, pay for the rest,” “Witzenberg: R1425/month,” and “Kraaifontein: R2500 pm.” In Norway, one group created a comparison table showing water costs across three municipalities (see Figure 1).

Østre Toten: 3780,- + 27,80/1000 L + 3938,- + 35,00/1000 L (for awlep)	Østre Toten	Porsgrunn	Aseral
Porsgrunn : 1873,- + 29,00/1000 L + 2125,- + 43,00/1000 L (for awlep)	100 m ²	13998,-	10335,-
Aseral : 545,- + 21,00/1000 L (for awlep) + 2090,- = 56,00/1000 L (for awlep) drainage	200 m ²	20278,-	18035,-
	400 m ²	32838,-	33435,-

Figure 1 PTs list of cost from three municipalities in Norway, based on volume and size of apartment.

Two PTs from Norway presented their local water payment models to PTs from South Africa. They lived in two different municipalities, each with a distinct payment system (see Figure 2). One student paid for water based on the area of the apartment, while the other followed a model that included two components: a fixed amount independent of water usage, and a variable amount based on actual consumption.

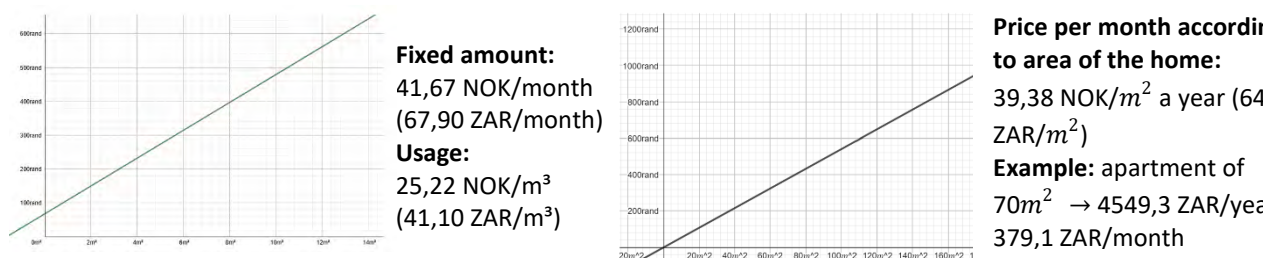


Figure 2 Norwegian PTs present two models for payment. **Left:** one fixed amount (not depending on consumption), and an amount on usage. **Right:** A model where you pay depending on your apartments size (square meters) – not depending on usage at all.

Economic payment models for water consumption vary across countries. Through mathematical modelling, PTs in Norway and South Africa examined various systems, including pre-payment schemes, mixed consumption-based models, pricing based on the size of one's home, and regional differences in pricing. They discussed these models and their impact on families—financially, in terms of fairness, and with regard to sustainability.

While the specific challenges in Norway and South Africa differ, there were notable similarities. Both groups emphasised the need for water conservation due to supply challenges, the critical role of clean water for public health, discrepancies in municipal water rates and the fairness of these (e.g., higher rates in the City of Cape Town), and the costs associated with technological solutions such as boreholes, JoJo tanks, rainwater harvesting, and desalination.

They also considered differences in infrastructure between rural and urban areas, governance issues, the effects of overpopulation, and the demands placed on water resources by different sectors—including agriculture, industry, and households.

Ultimately, mathematical modelling proved to be more than an academic exercise; it empowered PTs to contextualise economic models and engage in meaningful discussions about complex sustainability issues within their own communities.

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[58.] The concept of model in mathematics education and science education

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Models and modelling play an essential role in both mathematics and science, also from an educational standpoint. Integrating modelling into teaching can foster more meaningful learning experiences for students (Justi et al., 2009) and provide opportunities for active participation in scientific investigations of natural phenomena (Taber & Akpan, 2017). Nevertheless, the interpretation of the concepts of *model* and *modelling* can vary significantly across STEM subjects.

In mathematics education research, there are several perspectives on modelling, such as model-eliciting activities, through the anthropological theory of the didactics, and with different versions of modelling cycles (Barquero & Jessen, 2020). In this paper, we consider mathematical modelling from the modelling cycle perspective. This typically involves translating a real-world phenomenon into mathematical terms, e.g., by using an equation and establishing a model that offers a solution to a problem. Then a mathematical model can be described as “a representation of aspects of an extra-mathematical domain by means of some mathematical entities and relations between them” (Niss & Blum, 2020, p. 6). In science, the concept of models is often defined as simplified *representations* of phenomena (objects, systems, events, or processes) (e.g., Justi et al., 2009). These are made for different purposes, such as simplification, depiction, description, prediction or design of experiments. Thus, models “are not simply knowledge representations of the world, they are epistemic tools for making sense of the world” (Gouvea & Passmore, 2017, p. 56). Representation is but one of the purposes of models and the activity of modelling is connected to the construction of new knowledge (Gilbert and Justi, 2016).

Addressing the differences of the concept of models and their purposes is the focus of a broader design-based research study on the variation of model and modelling concepts in upper secondary STEM education. Hence, determining how the concept of model is characterised in the individual subjects and comparing differences and similarities is a prerequisite for this work.

In this paper, we address the ambiguity of models by investigating the following research questions: *How does the perspective of a mathematical model as output of a mathematical modelling cycle align with concepts of model in science education research? And what challenges and opportunities might this give rise to in transdisciplinary STEM education?*

To investigate this issue, we present prevailing definitions of the concept of model found in science education research. As mathematics is often an essential part of scientific models, we direct a special focus on the varying role that mathematics plays. The findings indicate that for the students, mathematical models (as results of mathematical modelling), mathematical models in science, and scientific models utilising mathematics constitute three different concepts of

models. In mathematics education, working with mathematical models is connected to active modelling (Niss & Blum, 2020). In science education, however, it seems possible to study a scientific model without addressing the modelling process. This might be a challenge in a transdisciplinary collaboration between science and mathematics, but there is certainly also a potential in these differences between the subjects—especially if teachers and educators are aware of them and use them purposefully.

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[59.] Conceptualising and Generating a Typology for Evidentiary Practices for Students' Work on Application Problems Related to Sustainability

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The role of mathematical modelling is key to understanding and making predictions about the impact of disruptive phenomena on the health of the planet, its people, and all other living things (Coles, 2023; Geiger 2024). Current and potential impacts of these phenomena, such as those related to aspects of global sustainability (see United Nations General Assembly, 2015), are reported in the media on a daily basis. Such reports, however, can vary in their descriptions of the severity of disruptive phenomena and their likely effects, even when based on reliable sources. This raises the question of what citizens should believe and do (Fischer, 2000) in situations where credible but conflicting claims are made on the basis of reliable data sources. We argue that such situations should be resolved through the employment of evidentiary practices (e.g., Unshelm et al., in press). While such practices have been identified in fields such as medicine and science education (e.g., Duncan et al., 2018), to our knowledge, they have yet to be developed within the context of mathematics education.

In this presentation we track the process of developing a typology of evidentiary practices as an aspect of an international collaborative study supported by the Australian Universities-German DAAD Joint Research Cooperation Scheme. This study was conducted in both Australia and Germany, involving one Year 9 and then one Year 10 class in each country over two successive years, as well as their teachers. Students were engaged with two application tasks related to sustainability issues, the first presented conflicting reports about atmospheric CO₂ omissions, and the second referred to different results in media items about adult poverty. Students were provided with up to four large data sets, specific to the tasks, which they were expected to use when evaluating claims in the media reports. Collected data methods included screen cast recordings of students as they worked with computers on the tasks and semi-structured interviews of student focus groups.

An outcome of the study was a typology of evidentiary practices in mathematics education. The typology was first conceptualised via a synthesis of relevant research literature in which core practices were identified and then refined and substantiated through the use of transcriptions of student screen cast recordings and interviews. Refinement and substantiation were based on a deductive approach to analysis using the initial typology as a lens. Data was initially coded in each country by one researcher, followed by cross coding of 20% of the data set to establish comparability. Where misalignments occurred between coders, these were discussed until consensus was achieved. The resulting typology consists of five core practices: exploring context; selecting data; handling data; reasoning and argumentation; and evaluating. Each core practice is further defined by elaborations relating to the specifics of each core practice.

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[60.] Logarithms in biology education: A content analysis of YouTube videos on survivorship curves

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Biology is the scientific study of life and living systems, driving progress in healthcare, environmental conservation and biotechnology to address global challenges and improve quality of life. Mathematical models help biologists analyse data, generate hypotheses, predict outcomes, and communicate findings (Otto & Day, 2007). Specifically, in ecology, models like the survivorship curve (Figure 1) illustrate survival patterns across age groups and species (Relya & Ricklefs, 2018).

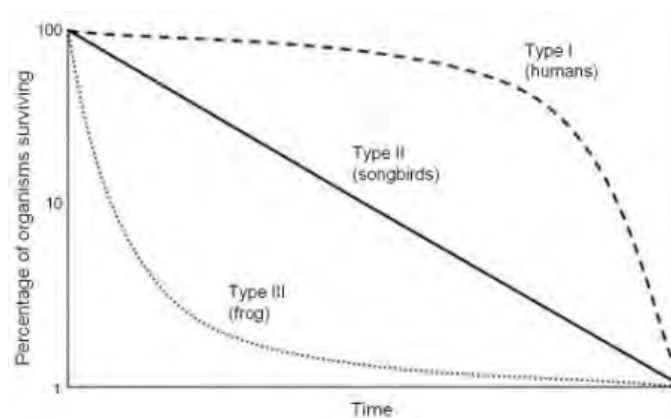


Figure 1. Survivorship curve diagram. (Husthwaite, 2009)

The diagram displays three curves representing type I, type II and type III species, where type I (humans, whales) experience high mortality late in life, type II (birds, squirrels) display a relatively constant mortality rate throughout life, and type III (plants, insects, frogs) have high mortality early in life as many seeds, eggs, and larvae do not survive infancy (Relya & Ricklefs, 2018). This diagram employs a linear scale on the x-axis and a logarithmic scale on the y-axis. Logarithmic scales are commonly used in ecological data visualization, and hence biologists are likely to frequently encounter these types of diagrams (Menge et al., 2018). These scales use the logarithmic ability to transform multiplicative relationships into additive ones, enabling the display of data across several orders of magnitude in compact space. Furthermore, exponential relationships are linearised. However, interpreting data presented in logarithmic scale formats requires the reader to accurately understand the logarithmic elements presented in the graphs. Menge et al. (2018) showed that ecologists are more likely to misinterpret graphs displayed on log-log diagrams than on linear-linear diagrams. The fallacies related mainly to the *linear extrapolation error* (c.f. Matz, 1980), where the log notation is treated as a variable that can be cancelled out (Menge et al., 2018). While this error has been noted in algebraic contexts (e.g. Chua & Wood, 2005; Weber, 2002), Menge et al. (2018) extended it to graphical misinterpretations – such as assuming a straight line in log-log space remains straight in linear space. These findings highlight a critical issue: despite the importance of mathematical models in ecology, many biologists struggle to interpret them correctly. This underscores the need for

better educational support in mathematical concepts for biologists, especially when interpreting log-scaled data in ecological research.

Since the Covid-19 pandemic, students increasingly use online resources to support learning subjects such as biology. In Europe, 25% of EU internet users engage with online learning (Eurostat, 2025). In Sweden, internet users seeking knowledge often prefer YouTube (Internetstiftelsen, 2023), a global video-sharing platform and the world's second most visited website (Similarweb, 2025). For this study, it is assumed that students may use YouTube to understand mathematical models such as the survivorship curve.

This paper will present a study addressing the logarithmic knowledge treated within the instructional content of YouTube videos about the survivorship curve. Using qualitative content analysis (Elo & Kyngäs, 2008), I examine which logarithmic facts are embedded in the models and how explicitly instructors present them. These facts may be assumed, explained, or partially addressed, potentially influencing how well viewers understand logarithmic graphs. The analysis explores this variation, and the results are presented and discussed in the paper.

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[61.] **Structured-to-Directed Iterative Modelling: Fostering Mathematical Modelling Competencies through Task-Based Scaffolding**

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Mathematical modelling (MM) serves as a vehicle to foster students' competencies in solving real-world problems (Kaiser, 2005, 2017; Maaß, 2006). Its adaptability across diverse contexts, particularly in STEM, supports the integration of science, technology, engineering, and mathematics, bringing practical relevance into school mathematics (Maass et al., 2019; Siller et al., 2023). Despite its promise, MM poses challenges due to the complex coordination across iterative phases (Blum & Borromeo Ferri, 2009; Durandt et al., 2024; Galbraith & Stillman, 2006; Niss & Blum, 2020). Scaffolding has addressed these challenges either through strategies that instruct linear progression across modelling phases (Beckschulte, 2020; Blum, 2011; Schukajlow et al., 2015), or through structured tasks that intensively support early phases and guide transitions between subsequent ones (Kohen, 2025). However, scaffolding that prepares students for iterative modelling remains scarce.

Introducing a Structured-to-Directed-Iterative Modelling approach, this study focuses on an iterative task route requiring application of MM sub-competencies across phases, following prior engagement with a structured MM task guiding linear progression situated in the same real-world context.

Participants were 2,000 ninth-grade students across four academic years (2020–2024). The research tool was an online quiz comprising closed- and open-ended tasks that direct students through an iterative modelling route to assess their application of sub-competencies. The focus is on two STEM-related real-world quiz contexts of: enabling direct streaming of YouTube videos and preventing car collisions using the Mobileye system.

Each sub-competency was assessed on a 0-1 success ratio scale, with mean scores indicating students' capacity to retain a complete modelling route. Success ratios, for both sub-competencies and complete modelling route retention, were categorized into five application levels from 0 (0=incorrect or irrelevant) to 4 (1=correct), and intermediate levels 1 to 3 in specific scoring ranges. ANOVA was used to compare the application of sub-competencies and levels of complete route retention.

Most students applied the MM sub-competencies correctly, with notable differences within and between STEM contexts. In the YouTube context, where four sub-competencies were measured, interpreting and understanding were applied correctly by more students than validating and solving mathematically. Within the three intermediate levels of complete route retention, level 1 students performed better in understanding the problem despite low overall scores, level 2 students were strongest in interpreting, and level 3 students excelled in both interpreting and understanding to a similar extent. In the Mobileye context, where three sub-competencies were

measured, solving mathematically and understanding were applied correctly by more students than validating. Within the three intermediate levels of complete route retention, students in levels 1 and 2 performed better in solving mathematically, while level 3 students achieved similarly high scores in both solving mathematically and understanding.

The study contributes theoretically by proposing iterative scaffolding as a supplement to linear scaffolding, methodologically by offering a way to assess MM competencies, and practically by demonstrating how structured-to-iterative task design can foster students' application of multiple MM competencies.

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[62.] Modelling the concept of proportion through the Brousseau's puzzle: a synergic approach in an integrated STEM environment

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In the context of STEM education, integrated didactics plays a crucial role in fostering meaningful learning across disciplinary boundaries.

This contribution is framed within the theoretical approach of the Theory of Semiotic Mediation (TSM) (Bartolini Bussi & Mariotti, 2008) and the Project-Based Learning (PBL) (Johnson & Lamb, 2007). The TSM emphasizes the importance of artefacts and collective discussion in the construction of mathematical meanings. The PBL is a teaching method that places the student's investigative process at the centre, aiming to develop a product with real-life connections and applications.

In this context, we refer to the concept of artefact synergy (Faggiano et al., 2018), which underscores how the synergy of multiple artefacts—manipulative, digital, and symbolic—can support a richer and more flexible modelling process. The didactic path presented here focuses on the concept of proportion, a fundamental mathematical idea that underlies both geometric reasoning and technological modelling.

The core of this contribution is a teaching experiment conducted in an Italian lower secondary school participating in a STEM-oriented curriculum innovation project aimed at constructing the proportion concept.

The sequence began with the Brousseau Puzzle (Brousseau, 1997, Figure 1), a classic problem involving proportional reasoning. Students first engaged with the puzzle using paper folding and cutting techniques, constructing and decomposing shapes while exploring invariant relationships. After this, the students were asked to reproduce the same puzzle but with different measures, in particular only the following instruction was given: “The 4 cm segment becomes 7 cm.”. To construct the enlarged puzzle, the students had to determine the measurements of all the other sides, correctly using proportional reasoning.

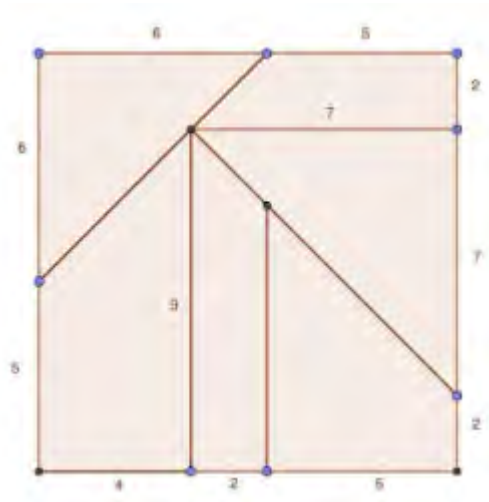


Figure 1. The Brousseau's puzzle

Subsequently, the activity moved to the digital environment of Tinkercad, a 3D design software. Students recreated the puzzle digitally, applying their proportional reasoning to adjust dimensions and ensure consistency across parts. The modelling process culminated in the production of physical 3D puzzle pieces using a 3D printer (see Figure 2). This phase completed the didactic cycle synergically involving manipulative and digital artefacts to construct the characteristics of proportional meaning.

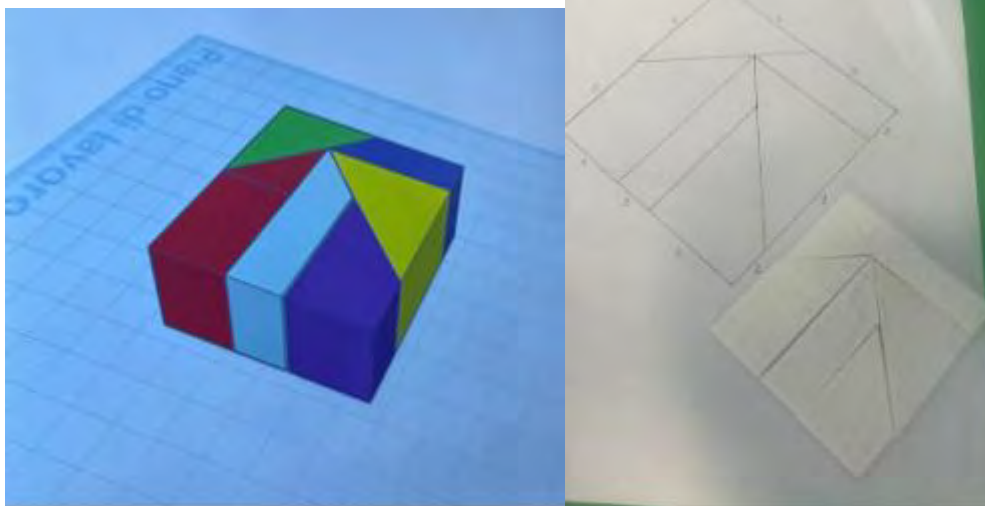


Figure 2. 3d project and model of the Brousseau's puzzle

Throughout the activity, students' modelling competences were fostered through the iterative design and the semiotic potential of manipulative and digital artefacts, encouraging critical reflection on mathematical properties by means of collective discussions (Bartolini Bussi, 1998). The synergy among artefacts supported a dynamic approach to proportional reasoning, integrating geometry, algebra, and digital design within a STEM framework.

This research offers insights into the design of interdisciplinary modelling tasks and highlights the opportunities and challenges of implementing such activities within school curricula. Finally, it showcases how in an integrated education environment, artefacts assume a fundamental role for the learning of complex mathematical concepts through modelling in authentic STEM contexts.

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[63.] Attitudes of student teachers towards teaching and learning mathematical modelling

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Mathematical modelling is firmly anchored as a process-related competence in the German educational standards for school mathematics teaching and is equally relevant for all pupils, including, for example, pupils with special educational needs (KMK, 2022; Greefrath et al., 2013). Current research findings by Borromeo Ferri et al. (2023) indicate that modelling tasks are well suited for the use in inclusive mathematics lessons, particularly due to their high potential for self-differentiation.

In school practice, mathematical modelling has so far received little support (Bommerro-Ferri, 2018). One of the reasons for this is that teachers often have relatively few skills in teaching mathematical modelling and are often inadequately qualified to do so (Hagena, 2019; Borromeo Ferri 2018; 2017). The demand to realise inclusive education and to enable high-quality mathematics learning for a diverse student body represents an additional challenge that is often perceived as extremely challenging (Viernmann, 2022). Accordingly, it is important to professionalise future teachers for teaching and learning mathematical modelling in inclusive lessons right from the first phase of teacher training. The research project ‘Reality-based tasks in inclusive mathematics education (RAIM)’ addresses this issue.

In the project, student teachers of special needs education with mathematics as a teaching subject are trained in the use of modelling tasks in inclusive mathematics lessons. The seminar design is based on a four-dimensional model for fostering the skills that teachers need to teach and learn mathematical modelling (theoretical dimension, task-related dimension, teaching-related dimension & diagnostic dimension) by Borromeo Ferri and Blum (2010). Taking the results of Hwang & Han (2025) into account, the project focuses on the task-related dimension in order to promote not only the students' theoretical knowledge but also their practical competencies for developing modelling tasks. After a theoretical introduction to the teaching and learning of mathematical modelling, the student teachers either develop modelling tasks themselves on the basis of a theoretical framework for modelling tasks for inclusive mathematics lessons (Schlüter et al., i. p.) (Group A) or solve the tasks and analyse them with reference to the framework (Group B). The students in both groups test the tasks subsequently in school practice and evaluate their suitability for inclusive mathematics teaching.

Using Ipad-supported group discussions (Schäffer, 2018; Abrams & Gaiser, 2017), a qualitative pre/post design is used to research the attitudes and self-assessment of the participating students with regard to teaching and learning mathematical modelling and the extent to which both are influenced by the task development (group A) or the task processing (group B) as well as the subsequent practical phase. Researching the attitudes of student teachers towards teaching and learning of mathematical modelling is especially interesting for teacher training because, from a

biographical perspective on professionalization (Terhart, 2022), it influences how student teachers will teach mathematics in the future.

The contribution presents first results on the research questions mentioned and derives implications for the professionalisation of future teachers in the first phase of teacher education with regarding the teaching and learning of mathematical modelling.

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[64.] Designing an instrument for measuring mathematics teacher educators' Rationale for Mathematical Modelling in Teacher education (RMMT) – a methodological exploration

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In many countries, mathematical modelling is now part of the formal mathematics curriculum, and hence, also is taught in mathematics teacher education. For conducting research on mathematical modelling competences, there are instruments to measure the competences of students (e.g., OECD, 2022) and of pre-service teachers (Czocher et al., 2021; Klock & Wess, 2018). However, to my knowledge, there do not yet exist instruments for measuring the competences of mathematics teacher educators (MTEs). My research aims to fill this gap. It is a methodological study on designing an instrument aiming to measure MTEs' Rationale for key factors, standards and priorities for Mathematical Modelling in Teacher education (RMMT). Although the final result of the RMMT research will be a questionnaire to be used in quantitative research, the exploratory phase of the RMMT research is qualitative. This first phase is validated through a literature review and semi-structured interviews with five Norwegian MTEs, selected as sample of convenience for the study. To also ensure the reliability of the questions, additional probing questions were developed to uncover potential ambiguities. The RMMT research seeks to build on a robust theoretical framework, as called for by Frejd and Vos (2022).

Klock and Wess (2018) adopted a methodological approach when developing a measurement instrument aimed at diagnosing professional modelling competencies among teachers in Germany. To capture these competencies, they structured these into self-efficacy, beliefs, and professional knowledge in mathematical modelling, and developed a test instrument to quantitatively measure teachers' professional knowledge in modelling. Similarly, Czocher et al. (2021) designed and validated two test instruments to quantitatively measure preservice teachers' professional competence and self-efficacy in mathematical modelling. In both studies, the test instruments were developed not only to measure competences but also to capture the respondents' underlying reasoning and thought processes behind their performance. The RMMT research adopts a similar methodological approach to draw on insights from design theories to develop tests instruments. I want to uncover contrasts between espoused beliefs (what they say) and enacted beliefs (what they do).

Aim of the instrument is to uncover MTE's rationale behind their priorities in preparing preservice teachers for their future professions. In the RMMT research, the concept of rationale is grounded in Discourse Theory, offering an epistemological perspective on the rationales for how MTEs can facilitate mathematical modelling teacher competencies among preservice teachers. From a discursive perspective, mathematical modelling teaching competencies are conceptualized in diverse ways, including considerations on how to prioritize their cultivation. Following Blömeke et al. (2015), I define competencies as an analytical construct that exists on a continuum, ranging from those that are observable and dichotomously measurable through clinical tests to those required by experts in complex, holistic, real-life situations. The following research questions have been formulated:

RQ: what are specific features of a survey instrument measuring key factors, standards and priorities for mathematical modelling in teacher education?

At ICTMA22, I will present the first two design versions of the questionnaire to measure the Rationale for Mathematical Modeling Teacher Education (RMMT), and give preliminary results based on the pilot study with five MTEs.

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[65.] Exploring the Roles Between a Teacher and Researcher in Designing and Implementing a Mathematical Modeling Unit

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A “research-practice gap” exists in mathematics education where the processes employed by researchers and teachers to improve teaching and learning are disconnected (Goose, 2008). To address this concern researcher-practitioner cooperation can be used, which allows both parties to contribute their expertise towards addressing educational issues (Bickel & Hatrup, 1995; Wagner, 1997). Teacher-researcher collaborations, however, remain uncommon in the United States (Goos, 2008; Herrenkohl et al., 2010) and when such opportunities do arise, they are often dictated by the agendas of researchers who generally hold institutional power and resource control (Goos, 2008). This dynamic is problematic as meaningful teacher participation is necessary for generating research questions and findings that are applicable for schools (Edwards & Jones, 2003; Wagner, 1997). It is therefore important to advance the field with research on teacher-researcher collaboration, documenting the challenges and successes that occurred to motivate more studies to explore these relationships. This study contributes to this effort by sharing stories of one teacher-researcher collaboration on implementing a mathematical modeling unit in secondary education.

Drawing on Wegner’s (1998) theories of communities of practice and the meta-didactical transposition model (Arzarello et al. 2014) that acknowledge the evolving factors and roles of teacher–researcher collaborations, this study aims to add to the understanding of how teachers and researchers exchange and share roles and knowledge for implementing mathematical modeling tasks. Specifically, the purpose of this research is to explore the dynamics of teacher-researcher cooperation, with a particular focus on how they navigate and leverage each other’s expertise, negotiate roles and responsibilities, and build collaborative relationships throughout the research process. The study will follow an ethnographic framework (Hammersley & Atkinson, 2019) that will occur in two major phases: (1) co-planning modeling lessons and (2) co-teaching and debriefing implemented lessons. Data analysis will draw from lesson planning and debriefing sessions, as well as classroom implementation where we will focus on identifying themes related to teacher-researcher collaboration, instructional decision-making, and perspective about the modeling tasks. These aspects will be analyzed through the lens of meta-didactical transposition and in connection to communities of practice theories. The research question guiding this study is: *How do a teacher and researcher establish roles and construct relationships as they co-design and co-implement mathematical modeling tasks in secondary classrooms?* By addressing this question, the study will offer valuable insights for educators and researchers seeking to conduct similar explorations. Additionally, it will shed light on the challenges and opportunities of such a collaboration within a professional development program.

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[66.] Can Fermi graphs boost problem-solving performance? Comparing student approaches across educational levels

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Mathematical modelling has become an increasingly prominent component of the mathematics curriculum in many countries. Its inclusion is often motivated by goals such as demonstrating the capacity of mathematics to describe complex phenomena and fostering students' critical thinking skills (Niss & Blum, 2020). However, despite this growing emphasis, mathematical modelling has not yet been fully integrated as a regular part of mathematics instruction in most countries, particularly in Spain (Borromeo Ferri, 2021).

Fermi problems are a type of open-ended, small-scale modelling tasks (Ärleback, 2009) centred on identifying relevant quantities and estimating numerical values necessary for problem resolution. Methodologically, Fermi problems offer a more accessible approach to engaging with key features of mathematical modelling, effectively capturing essential aspects of the modelling process (Albarracín & Gorgorió, 2014; Ärleback, 2009). In previous research, we have already studied the evolution of the models produced by students from grades 2 to 10 when facing the same Fermi problem consisting of estimating the number of elements in a rectangular bounded enclosure (Ferrando & Albarracín, 2019). Also, Ferrando et al. (2021) identified a relationship between the contextual features of these Fermi Problems and the strategies while solving a sequence of Fermi Problems.

Albarracín and Ärleback (2025) present a tool called Fermi Problem Activity Template; this tool is useful to visually structure the solution process of a Fermi problem. Jablonski et al. (2025) use similar representations (called Fermi Graphs) in their study with primary and secondary students. Fermi graphs are presented as a visual tool to help students structure their thinking, organise relevant information and represent relationships between data. In addition, as the solution process for this type of problem is more visible when students use graphs, these visual representations support analysing students' productions.

This exploratory study aims to analyse the role of Fermi graphs in the solution of such problems by a sample of students from different educational levels (grades 3, 5 and 8). As far as we know, the use of graphical representations has not been used to study the evolution of students' performance when facing a sequence of Fermi Problems with different contextual features.

Specifically, the study seeks to:

1. Explore differences in the ability to construct and use Fermi graphs depending on the students' educational level.
2. Analyse the influence of the use of Fermi graphs on students' performance
3. Examine the relationship between the completeness of the constructed graphs and the models and strategies applied during the problem-solving process.

To this aim, we conducted a quasi-experimental study that involved a sample of N=60 students. Firstly, an introductory activity was implemented, which was solved with the collaboration of the whole class group and using a Fermi graph. Then, the data were collected through a

questionnaire consisting of a sequence of three Fermi problems, similar to some of the problems used by Ferrando et al. (2021).

The analysis will focus on students' written productions, paying attention to the quality and clarity of the constructed graphs, as well as the strategies used in each case and the grade of completeness of the Fermi graphs constructed by the students. Our results are interpreted in terms of the potential for the use of Fermi graphs for teaching modelling in school.

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[67.] Fostering and supporting high school students' productive struggles during mathematical modelling and its applications to South African national curriculum

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This study offers a theoretical perspective on how to foster and support high school students' productive struggles during mathematical modelling and its applications within the South Africa context. The integration of mathematical modelling into national mathematical curricula has increasingly highlighted the importance of students to engage productively in complex and often unfamiliar model-eliciting activities (MEAs). Mathematical modelling, through solving the MEAs, is a cognitively demanding real world task for students, which serves as a “didactical vehicle for developing modelling competency and enhancing students' conceptual learning of mathematics” (Blomhøj & Kjeldsen, 2013, p. 151). It is common during mathematical modelling for students to expend time and effort making sense of, and mathematising, the MEAs with the aim of figuring out a solution that is not immediately apparent (Warshauer, 2015; Zeybek, 2016). In other words, students struggle to find solutions for MEAs. A struggle to solve MEAs is not necessarily a bad thing, provided the struggle is productive (Young et al., 2024). In the context of this study, students encounter productive struggles during the mathematical modelling cycle when they experience difficulties with solving MEAs which are within their cognitive [or grade] levels. These MEAs primarily serve as an interface between mathematics as defined and taught in the classroom and mathematical concepts as they are applied in the real-world context (Ang, 2010; Huang et al., 2021).

This study seeks to address the following research questions:

1. *What types of productive struggles are encountered by high school mathematics students during mathematical modelling and its applications?*
2. *How can students' productive struggles be supported?*

A systematic review of mathematical modelling education in South Africa between 2003 and 2022 shows that research in this area is increasing (Jakobsen & Mhakure, 2024). However, there is limited integration of mathematical modelling in classrooms, because it is considered difficult by both teachers and students. Furthermore, the rigid high-stakes national examination does not assess mathematical modelling competencies. This study foregrounds the view that high school students engaging in mathematical modelling and its applications experience various productive struggles, and that they need to be supported when solving MEAs. These productive struggles include, but are not limited to: (a) navigating the modelling demands of the national curriculum and the expectations of the high-stakes examinations; (b) establishing and nurturing modelling as a content versus modelling as a vehicle for learning mathematical concepts; (c) understanding didactical modelling – referring to how students develop modelling competencies when solving MEAs; (d) understanding conceptual modelling – referring to the belief among students that modelling should enhance the learning of mathematics; and (e) understanding cognitive modelling – referring to cognitions and emotions that occur in students' modelling process

(Huang et al., 2021). Using practical examples, this study discusses these productive struggles and gives suggestions for how students can be supported.

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[68.] Aspects of relevance to the students in a modelling project

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Modelling is a bridge between mathematics and the real world, and modelling lessons provide an effective way for the students to apply their mathematical competencies to understand the world around them. To get a better understanding about how modelling projects are received by the students is important for both lesson planning and learning outcomes. In this proposed article, we want to find out what aspects of modelling activity are perceived as relevant by the students in middle school. Two classes of students (38 in total) from 9th grade participated in a transdisciplinary modelling project which lasted about five weeks. In this project, the students were supposed to learn about bogs and to find out how much carbon is stored in bogs in a suburban area outside Bergen and in the whole municipality. They were required to present their findings both orally and in a poster in the end of the project. After this modelling project the students were given a questionnaire to reflect and self-evaluate on their experience.

To answer our research question, we conducted a thematic analysis on the students' responses to the questionnaires, using a framework for relevance of mathematics activities proposed by Hernandez-Martinez and Vos (2018). They highlight four aspects of relevance, that is, relevance to whom, relevance according to whom, relevance of what, and relevance to what end. Here we take the students' perspective and consider what is relevant to them according to themselves. We analyze what aspects of the activity the students consider as relevant (relevance of what) and what aims the students have with what is considered relevant (relevance to what end). In our preliminary findings, we identified four groups of students. One group of students considered working with the modelling process and find out how much carbon is stored inside the bogs to be relevant. The second group of students brought forward the aspect of working in groups and cooperating with others as important. The third group emphasized the practical elements where they went to the peatland to find out the depths of the bogs. These aspects can be related to the different elements, that is, object (modelling process), division of labor (the group cooperation), tools (practical measurements) in the cultural and historical activity theory (CHAT) (Engeström, 1999). The fourth group of students wrote that they did not like any parts of the activity, without providing explanations.

These preliminary results indicate how different aspects of the activity can play important roles depending on the students in question. They also indicate how modelling activities in general have the potential to satisfy several groups of students based on their preferences. Understanding what parts of a modelling activity that are considered as relevant for the students can in return give us indications about how such modelling projects can be designed in the future. In particular, our results suggest that good group cooperation can play an important role as quality of the cooperation can directly affect their feelings about the project. It has previously been noted that that student-student relationships are effective in reducing negative attitudes toward complex tasks (Mottet et al., 2005) and in increasing student engagement (Xerri et al., 2018). This may indicate that to promote social competency and practice collaboration in groups can increase the relevance of a collaborative modelling projective to some students as social

competency is an essential part of modelling competencies (Cevikbas et al., 2022). Further, understanding why the fourth group of students did not show enjoyment in this activity could also be necessary and rewarding, as it may reveal which aspects of the activity need adjustment to better relevance for them.

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[69.] Teaching modelling with GeoGebra and affective characteristics towards digital tools

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The integration of digital tools in mathematics, and in mathematical modelling in particular, has become established in mathematics education (Koyunkaya & Dede, 2024; Weigand et al., 2024). There are several studies which highlight the advantages and challenges for the use of digital tools in mathematical modelling (Cevikbas et al., 2023). These research findings show a wide range of possible applications for digital tools in the modelling process, from experimentation to visualization and control. To be able to use digital tools in different stages of mathematical modelling, and especially in digital modelling tasks, it is necessary for students to become familiar with the use of digital tools and to acquire digital tool skills. Program-related self-efficacy and attitudes towards the software might also affect the acquisition of modelling competencies. A study on geometric modelling with GeoGebra has shown a positive correlation between program-related self-efficacy and the competence of mathematising (Greefrath et al., 2018). Despite the importance of affective characteristics for modelling, there is a lack of studies on the effects of affective characteristics regarding digital tools on modelling competencies. This leads to the questions of whether teaching modelling with a digital tool can increase affective characteristics towards the digital tool, and whether affective characteristics towards digital tools can predict performance not only in the sub-competency of mathematising but also in mathematical modelling in general.

To investigate these questions, German eighth and ninth graders from six different high schools (N=200) were asked to participate in a quantitative intervention study in a pre-post design. Before the modelling intervention, the students were tested on program-related self-efficacy and attitudes towards GeoGebra. After passing the modelling unit, the students were tested again on program-related self-efficacy and attitudes towards GeoGebra and on their modelling competencies. To analyse the students' modelling competencies, we developed a modelling test which included a holistic modelling task and separate tasks on the competence of mathematising. We collected the affective characteristics towards GeoGebra using a questionnaire. The modelling unit was implemented via a digital learning environment and was designed using GeoGebra and GeoGebra Classroom. It lasts 90 minutes and is conceptualized as individual work or partner work.

In the talk, we will illustrate the changes on affective characteristics and the results of the two parts of the modelling test. First results show an increase in program-related self-efficacy, but no changes in attitudes. The results also suggest that program-related self-efficacy and attitudes towards GeoGebra are significant predictors of modelling competencies. The talk concludes with a discussion of the results.

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[70.] Perspectives on the implementation of modelling with digital tools

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Generally, the opportunity to use technology is essential in mathematical modelling processes (Greefrath, 2011). Reflecting on how mathematical modelling is framed in education, Galbraith (2012) criticised the tendency to inadequately differentiate between the perspectives of "modelling as content" and "modelling as a vehicle," as suggested by Julie (2002). Research indicates that inviting pre-service teachers to engage with technology enriches their experience with "modelling as content" (e.g., Villarreal et al., 2018).

In this study, we began by presenting a class of pre-service teachers with examples of how spreadsheets and mathematical software tools could be purposefully used in educational modelling. We also invited the pre-service teachers to reflect on the two modelling metaphors mentioned earlier. Subsequently, based on this instruction, we assigned the pre-service teachers a task to design a mathematical modelling assignment for secondary school students that incorporated digital tools. To work on the assignment, the pre-service teachers were divided into groups of three to four individuals. Groups could freely choose and utilise digital tools, including the internet, spreadsheets, mathematical software, and programming languages. Additionally, the groups were asked to propose two potential learning paths for students working on the task. These paths should include specific steps and be mathematically detailed. Examples of calculations using the digital tools or mathematical computations that students might naturally perform between steps should be provided. Furthermore, the pre-service teachers in each group were asked to write individual reflections on what they believed students might learn from the task regarding digital tools and mathematics.

Our study focuses on how the pre-service teachers suggested that students would engage with the modelling task, particularly the benefits they perceived from using the digital tools. We are also interested in the pre-service teachers' meta-reflections on how working on the task could potentially deepen students' understanding of both mathematics ("modelling as vehicle") and the application of digital tools in mathematical modelling ("modelling as content"). We have formulated our inquiry as two preliminary research questions: 1. How did the groups suggest students would engage with mathematics throughout the modelling activity? 2. How did the pre-service teachers propose students would utilise digital tools in the modelling activity?

To explore these questions, we will conduct a thematic analysis of the content of the documents containing the pre-service teacher group's responses to the assignment. The documents included group reflections on the two previously discussed perspectives, a proposed modelling task, learning paths, and individual reflections from the pre-service teachers. To categorise the content, we will work inductively but also deductively, by searching the research literature on modelling with digital tools to generate an adequate framework. The Norwegian Agency for Shared Services in Education and Research (SIKT) has approved the project.

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[71.]

[72.] Enhancing Sustainable Critical Thinking through Mathematical Modelling: Utilizing Textbook Tasks in Grade 6 Education

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Introduction

In the study on education for sustainable development (ESD) by Lundegård and Caiman (2019), which highlights five forms of democratic participation, critical thinking is identified as a vital element. Solving modelling problems serves as a method to practice critical thinking, as they provide opportunities to evaluate different mathematical models in order to find a model, appropriate for the given problem. Fujimura (2001) has shown that a way of developing students' proportional reasoning is through tasks that have a modelling perspective, an everyday context and that the models in a chain of tasks are isomorphic and can be handled by students. The class of textbook problems about mixture of substances, particularly in different aggregation forms is well suited in this respect (as is the class of plane geometric pattern problems). The models in chains of tasks should advance from being linear (proportional) to being related but slightly more complicated (Lundberg, 2024).

RQ: How can a task from a mathematics textbook about mixture of substances be utilised in modelling activities, and what potential opportunities for sustainable critical thinking do these activities offer within mathematics education?

Method

Data for this analysis comes from a larger set of video recordings collected as part of an international project (VIDEOMAT cf. Kilhamn & Säljö, 2019). In the VIDEOMAT project, several teachers were video recorded during four consecutive algebra lessons without any intervention. In the Swedish segment, four qualified teachers taught grade 6 students, aged 12–13 years. These teachers were recruited for the study due to their interest in participating for professional development purposes. The teacher in question has 10 years of teaching experience. Three cameras were used during the recordings to capture the teacher, a focus group, and the entire class. Additionally, written materials were collected, and a brief post-lesson interview was conducted after each lesson. The analysed four lessons from one grade 6 classroom form a single case study. The tasks handled in the analysed lessons (see Figure 1) were found in a textbook by Carlsson et al. (2004).

47. You want to mix "Lemon Squash". You have 5 dl lemon juice, $x = 5$ dl. How much do you need of:

a) Water?

b) Sugar?

48*. How much lemon juice is there in 7dl

mixed "Lemon Squash"?

"Lemon Squash"

x lemon juice

$2x$ water

$x/2$ sugar

Figure 1. Task 47 & 48 from the textbook (Carlsson et al., 2004, p. 103).

Results

The teacher analyses the tasks 47a, 47b, and 48 according to the model from Borromeo Ferri (2006): *understanding, simplifying/structuring, mathematising, working mathematically* with final steps *interpreting, validating and presenting the solution*. For the tasks 47a and b she uses the recipe correctly to conclude that a proportionality model is applicable. Task 48 has a Fermi character due to the lack of sorts and the undefined aggregation form of sugar in the recipe. It requires modelling competency. The teacher initially proposes that the added sugar does not affect the volume., but realising that this contradicts everyday experiences, the teacher after some hesitation decides to apply a proportionality model also here, which in fact is only approximately correct since it does not take the decrease in volume in account, when sugar crystals form a saturated solution

Discussion

The task that the teacher solved ad hoc contains excellent opportunities for discussion of the range of applicability of different models as well as practical modelling. It turns out that the task provides a valuable opportunity to practice critical thinking and discuss real-life relationships to mathematical models, e.g. perform an experiment with the data the teacher's initial model and the proportionality model gave. An objection is that the exact model for task 48 is not isomorphic to the model for task 47 (although the approximate model is). This is compensated by its potential to stimulate students' critical thinking. To ensure that teachers do not miss these types of opportunities, teacher guides should include instructions on how teachers can support students in deepening their modelling competence.

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[73.] “The Cat in the Hat”: children's literature as a context for mathematical modelling

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Within mathematics education, mathematical modeling is a valuable instructional strategy for fostering conceptual understanding and connecting mathematics to real-world contexts (Blum et al., 2007). Modeling engages students in processes such as representing, conjecturing, justifying, and quantifying, which support deeper comprehension and show the relevance of mathematics in science, technology, and daily life (English & Watters, 2005; Kaiser & Schwarz, 2010). Lesh and Doerr (2003) argue that modeling tasks aim to help students develop skills for solving real-world problems, unlike traditional tasks that focus on applying known procedures in abstract settings.

Despite its curricular relevance, modeling is often absent from classroom practice (Maaß, 2010), partly due to the challenges involved in designing meaningful tasks and the difficulties they present for both teachers and students (Blum & Borromeo, 2009). Frequently, modeling is reduced to a mere application of procedures, rather than a space for exploration and knowledge construction (Young & Marroquin, 2006). The context in which modeling occurs is key to transforming it into a genuine learning experience.

Contexts for modeling vary according to students' level, interests, and educational goals. Research highlights the use of real-life, professional, and engineering contexts (Park, 2023). However, one underexplored context is children's literature—a “realm of fantasy” that can support early modeling skills by offering rich, imaginative settings (Maaß, 2010). Literature has the potential to stimulate creativity and improve attitudes toward mathematics (Zacarías & Faria, 2005; Ewing et al., 2018).

This paper presents a modeling activity based on *The Cat in the Hat Comes Back* by Dr. Seuss. After reading the book, students are challenged to determine the size of Cat Z, who emerges from the hat of Cat Y, and so on up to Cat A—nested cats with no given dimensions. Participants propose strategies to estimate the height of each cat and its hat, aiming to find a pattern. This promotes reasoning based on previous knowledge and invites diverse representations.

A qualitative methodological approach has been utilized to implement the situation across various educational levels, including both pre-service and in-service teachers. The results indicate that the situation proved to be of interest to the participants, who engaged with the task and proposed a variety of representations and solution strategies. Elementary-level students struggled with expressing the recursive relationship algebraically, yet their drawings and reasoning demonstrated understanding. Higher-level students modelled the problem with exponential functions. The process reflected key modelling phases: mathematization, analysis, and validation.

Results emphasize the importance of diversifying modelling contexts. Children's literature, though rarely used in this way, offers opportunities for designing meaningful and imaginative modelling tasks that foster problem-solving and mathematical reasoning.

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[74.] How to integrate mathematic modelling into interdisciplinary STEM teaching

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Introduction

STEM education (Science, Technology, Engineering, and Mathematics) has become a central framework for equipping students with problem-solving skills relevant to future societal needs (Maass et al., 2019). Within this framework, mathematical modelling plays a key role connecting disciplines. While the value of STEM integration is recognized, its practical implementation remains challenging (Bybee, 2013; Larsen et al., 2022; Svendsen et al., 2025). Teachers face difficulties aligning subject goals, covering curricula, and navigating institutional constraints (Dare et al., 2018; Margot & Kettler, 2019).

This study examines how upper secondary teachers perceive and implement mathematical modelling for interdisciplinary STEM, using integration strategies defined by Bennett and Ruchti (2022).

Context and Methods

The LabSTEM project developed ten STEM activities focused on mathematical modelling, four of which were implemented in upper secondary education. All were videorecorded, and three were selected for in-depth analysis; one was excluded due to limited mathematical content. A post-participation teacher questionnaire with open and closed questions (Creswell & Guetterman, 2019) was distributed and is ongoing. It explores whether teachers are aware of how they integrate disciplines, based on four strategies by Bennett and Ruchti (2022):

1. Simultaneous Infusion - integrating practices concurrently
2. Accentuate Interdisciplinary Constructs - using one discipline to deepen understanding in another
3. Temporary Shift in Discipline - temporarily focusing on a second discipline
4. Lateral Concept Connections - developing concepts in parallel

Findings

The first STEM activity involved a visit to a wastewater facility. Students investigated bacterial growth via exponential models, combined with biological and chemical analysis. This was a Lateral Concept Connections, as students developed conceptual knowledge in parallel.

The second activity focused on programming robots to navigate obstacle using angle calculations and basic coding. Mathematical and technological thinking were embedded concurrently, making it Simultaneous Infusion as students developed conceptual knowledge in parallel across disciplines before synthesizing insights in a final presentation

The third activity explored allele combinations. Students examined their blood to understand patterns and used combinatorics to calculate genotypes and phenotypes. They conducted a χ^2 test to compare class data with the Hardy-Weinberg expectations. This was an Accentuate Interdisciplinary Constructs. Students worked intensively and concluded with a competition, showing applied modelling.

Survey responses (currently $n=9$) indicate that Simultaneous Infusion was rated most important by the teachers (86% "very important"). Lateral Concept Connections was seen as least important. Findings are based on a small sample size and may evolve.

Conclusion

Teachers applied different integration strategies in their teaching, either intentionally or intuitively. Most teachers favoured Simultaneous Infusion in the questionnaire, though they acknowledged the value of other strategies. One teacher noted that integration strategies was not a conscious part of her planning, which instead focused on the more subject-specific goals. We argue that increased awareness of integration strategies can strengthen planning and the meaningful inclusion of mathematical modelling in STEM.

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[75.] Statistical and Mathematical Models of Bivariate Data from Japanese Junior High School Science Textbooks: A Textbook Analysis

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The significance of integrated STEM education has gained increasing recognition. As Watson et al. (2020) argue, statistics has the potential to serve as a central component of STEM education, particularly because it enables learners to engage with data and its variation. However, previous studies have pointed out that within the context of integrated STEM education, the role of mathematics tends to be underestimated. To address this issue, it is essential to examine how mathematics functions within STEM subjects other than mathematics. Such an investigation can serve as a first step toward realizing truly integrated STEM education.

This study investigates how statistical and mathematical models are treated in Japanese junior high school science textbooks, with a particular focus on bivariate data. Compared to elementary science, junior high school science involves more diverse and complex representations of graphs and data. However, little is known about how modeling—particularly involving bivariate data—is addressed in this context. In Japan, teachers are expected to follow textbooks closely, and classroom practice tends to closely reflect textbook content. Therefore, textbook analysis provides valuable insight into how mathematical models are handled in junior high school science.

Six government-authorized science textbooks (three per publisher) from two major Japanese publishers were analyzed. Drawing on existing literature (e.g., Dvir & Ben-Zvi, 2023; Stillman & Brown, 2021), a theoretical framework was employed that distinguishes three types of models: (1) theoretical mathematical models, which are not data-driven; (2) empirical mathematical models, which are based on data but ignore variation; and (3) statistical models, which account for both the signal and the noise in data.

The analysis revealed that statistical models were most clearly described in first-year textbooks, particularly in sections that taught students how to draw graphs. These sections explained how to account for measurement error when fitting smooth curves or lines to scattered data points. In contrast, in the second- and third-year textbooks, although some instances suggested the use of statistical models, there were no explicit references to data variation. Instead, many models were based on idealized data without noise, and empirical mathematical models were frequently used. This suggests that, although the data presented often contain variation, such variation is not consistently acknowledged or addressed. Given that statistical concepts such as regression lines are part of the senior high school mathematics curriculum, their treatment in junior high school science remains necessarily informal. This reveals a curricular gap and highlights the need for better articulation between school subjects and educational stages.

These findings underscore the importance of paying closer attention to how mathematical and statistical models are addressed in science education. This is particularly relevant for advancing

integrated STEM education, where the contribution of mathematics—especially informal statistical modeling—should be more explicitly recognized and systematically developed.

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[76.] Constructive Resources in Mathematical Modeling

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Within the cognitive perspective of modelling, the ICTMA community has made significant progress in developing accounts of student thinking as students construct mathematical models of real-world scenarios. Specifically, the ICTMA community has produced various types of modelling cycles (e.g., Blum & Leiß, 2007) and used those modelling cycles as analytical tools to describe students' modelling routes (Borromeo-Ferri, 2007), decision making strategies (e.g., Wickstrom & Jung, 2024), typologies of validation (Czocher, 2018) and to diagnose student challenges in modelling (e.g., Jankvist & Niss, 2020). While modelling cycles and the research on student thinking grounded in them serve as powerful descriptive schemes, they sacrifice explanatory power that attends to modelers' fine-grained thinking (Czocher, in press). In her plenary talk at ICTMA-21, Czocher (in press) emphasized the importance of accurately diagnosing students' representational conventions through an anti-deficit lens (Adiredja, 2019), framing what students know as resources students' draw upon as they construct models.

Consistent with this call, we found that networking theories (Radford, 2008) from quantitative reasoning (Thomson, 2011) and resources (Hammer, 2000) have been beneficial in describing beyond *what* students did, to *why* they did so (e.g., Roan et al., 2025). Following Sherin (2000), we propose a constructive resource for mathematical modeling (CRMM) framework with potential to develop second-order theories of students' modeling activities (Steffe & Thompson, 2000). We operationalize CRMM as the resources—consisting of *quantification processes* and *cognitive resources*—that contribute to students' construction of mathematical models (Lee, n.d.). *Quantification processes* are the mental operations involved in conceiving a real-world situation consisting of quantities and relationships among quantities (Thompson, 2011). According to Thompson (2011), a quantity is a mental construct consisting of an object, attribute, and a conceived measurement process. Relationships between quantities can be conceived through operating on quantities or reasoning about variations among quantities. *Cognitive resources* are fine grained knowledge elements that are networked together to make sense of a given situation (Hammer 2000). Literature has identified three types of sense-making resources students draw upon when making decisions as they work on scientific tasks (diSessa, 1993; Hammer, 2000; Swanson, 2025). These include conceptual, epistemological, and intuitive resources. Conceptual resources are knowledge elements that learners have acquired regarding mathematical and scientific concepts. Epistemological resources can be characterized as learners' knowledge elements for making sense of the provenance of acquired knowledge (Hammer & Elby, 2002). Finally, by intuitive resources, we refer to the knowledge elements acquired through experiences that may contribute to the activation of certain quantification processes.

In this talk, we will elaborate on the CRMM theoretical framework using illustrative vignettes from two pairs of pre-service teachers' (PSTs) work on the Laundry and Basket tasks.

Specifically, we address the question: *what constructive resources do PSTs draw upon as they construct models for real-world situations?* Finally, we will make conjectures about the

CRMM framework's capacity to prescribe pedagogical intervention that responds to student thinking.

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[77.] Modelling using Numerical Methods and analysis of error sources

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Numerical Methods (NM) are used in mathematics and in the modelling of various phenomena. In this context, research on students' learning of these contents becomes increasingly relevant, as several studies have reported challenges. These include difficulties in applying NM to solve contextual problems (Lappas & Kritikos, 2018), challenges related to the integration of digital technologies (DT) and limited preparation for their effective use (Granados Ospina, 2015). Other difficulties relate to predominant teaching approaches (Marinelli, 2023), as well as to students' lack of knowledge to analyse errors when engaging in modelling processes involving NM (Wang & Han, 2021).

To support modelling learning, several tools such as GeoGebra, Tracker, and VidAnalysis have been implemented. In particular, Tracker has been employed to analyse various types of motion (Garzón & Villate, 2022; Wee et al., 2012). However, the potential errors introduced by this tool during modelling processes are rarely addressed in the literature.

In light of this, the present study aims to explore how modelling processes unfold in experimental activities that incorporate NM and use Tracker as a data collection tool. Specifically, it seeks to answer the following questions:

What numerical procedures support students' progress in modelling projectile motion? And what types of errors arise in the modelling process, and how do they influence students' interpretations?

To address these questions, this study adopts a conceptual framework that considers the different roles of digital technologies in modelling processes (Guerrero-Ortiz, 2025), complemented by a perspective that incorporates elements specific elements of NM and the analysis of various sources of errors (Álvarez-Blanco et al., 2009).

The research was conducted in a Numerical Methods and Differential Equations course for future secondary mathematics teachers (31 participants). The activity involved one student throwing a ball while another recorded the motion using a mobile device. The students were asked to determine the maximum height reached by the ball. To do so, they used Tracker and applied numerical methods to solve fitted theoretical models. Data were collected over two 70-minute classes and included written work, board screenshots, and audio recordings. The data were analysed using content analysis methodology (Bardin, 1991), based on the conceptual framework established in the study.

The results show that most student groups successfully constructed the Bisection Method as part of their modelling process. The analysis in Tracker involved various errors that led to models with small differences, an aspect that was not recognized by the students. These findings highlight the need to further examine the pedagogical use of such tools as aids in mathematical modelling. Finally, modelling and measurement errors were identified during the activity; however, the students showed no evidence of having identified or made sense of these errors.

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[78.] Unpacking the modelling mindset in teacher education: A behind-the-scenes journey to empower future mathematics teachers

Serife Sevinc

Mathematical modelling is a powerful interdisciplinary tool that opens up rich learning opportunities for problem solvers of all ages, from students to teachers and teacher candidates. Research have shown that mathematical modelling plays a vital role in developing insights into real-world problems, which is more crucial than ever in today's rapidly changing world. The global challenges of the COVID-19 pandemic, climate change, economic instability, and food insecurity during wartime have highlighted the urgent need for a modelling mindset – one that not only helps us understand and navigate current issues but also equips us to anticipate future crises. Mathematical modelling allows individuals to understand, explain, and interpret complex phenomena, predict potential outcomes, and develop proactive solutions, which makes it an essential mindset for the world citizens of the 21st century.

But how do we cultivate this mindset, especially in the next generation? How can we ensure that students develop the ability to think mathematically about the world? And, more critically, do teachers themselves possess this modelling mindset? Further, how can we support them in understanding mathematical modelling and overcoming the challenges of this task? Indeed, what does this modelling mindset entail?

These are the questions that have driven my research on mathematical modelling education throughout my academic career. This talk will share insights primarily from two longitudinal design-based research projects with prospective mathematics teachers – projects that highlight the rewarding opportunities in supporting teachers to develop their modelling knowledge as well as the behind-the-scenes challenges.

The core premise of these projects is that teachers are not simply curriculum implementers but active knowledge seekers. Therefore, investigating prospective teachers' potential to identify, design, and organize meaningful modelling problems constituted a momentous research path. A shared goal of these studies is to empower future teachers to solve and pose modelling problems through cognitive, communal, reflective, and performance-based practices. The former design study incorporated all of these practices, while the latter concentrated on posing modelling problems and different types of reflection. In addition, the wide range of professional practices prompted deeper theorizing about the nature of teacher knowledge in mathematical modelling, highlighting its pedagogical, curricular, relational, and interdisciplinary dimensions.

Building upon these insights, this plenary will spotlight the meaningful opportunities that prospective teachers experienced through carefully crafted training sessions – opportunities that supported the development of a modelling mindset through practice, reflection, and collaboration. At the same time, it will offer a candid look behind the scenes of this work, shedding light on the complexities, uncertainties, and iterative challenges that shaped these efforts. Join me as we step backstage into the mathematical modelling research, where the

process of cultivating a modelling mindset in teachers comes into focus – with all its accomplishments, tensions, and the layered nature of what this mindset entails.

[79.] A reflection fika is meant to feed our minds and stomachs. Three senior researchers, **Mogens Niss, Peter Galbraith and Susana Carreira**, will share their impressions and reflections on the day's sessions and put these reflections into different perspectives. The floor will then be open to the audience, who can ask questions, expand on their reflections, dream, criticise or just watch.

[80.] Leveraging the concept of relationship to knowledges to understand mathematical modelling's presence in teaching practices

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The presence of mathematical modelling is uneven in postsecondary mathematics education (Blum, 1993; Stillman, 2019), notably due to a lack of resources and support for instructors who guide students through their learning processes (Blum, 1993; Barquero et al., 2018; Maass et al., 2019; Gutiérrez et Gallegos, 2019). The dimensions of mathematical practice that modelling requires—such as transferring mathematical concepts to real-life problems and simulation—have played a marginal role in teaching, both at the level of students and the instructors who train them (Barquero et al., 2018). This situation reflects a worrying trend in higher education, where the importance of a understanding of the modelling process is often neglected.

In this study, we explore eight Calculus instructors' perceptions and practices regarding modelling. Through semi-structured interviews and participatory systems dynamics modelling activities, our goals are to (1) investigate the relationship between instructors' approaches to mathematics and modelling and (2) identify areas of success and challenges in integrating modelling in Calculus courses.

To answer our first research objective, we apply to mathematics and mathematical modelling the theoretical concept of relationship to knowledges, defined by Therriault (2018) as the meaning given to an object of knowledge. Based on this framework, we identify two ideal types (Weber, 1965) of instructors regarding their approach to modelling. In particular, the framework helps to explain specific difficulties encountered during modelling activities, even among instructors who value its pedagogical potential.

In response to the second objective, the participants selected a modelling activity aligned with didactical principles (Vaithyanathasarma et al., 2023) and a modelling cycle (Caron, 2018). The modelling cycle allowed us to identify moments of discomfort and opportunity as experienced by the instructors. Our findings suggest that instructors acknowledge the value of modelling in exploring the limitations of mathematical models, examining approximations, and making explicit the assumptions that underpin real-world applications.

While many viewed modelling as a means of enhancing conceptual understanding, its advantages over more traditional methods were not always evident. Instructors recognized the need to be more explicit in guiding students through the mathematization phase, but many were unsure how to do so effectively. A lack of training and confidence—especially regarding unfamiliar concepts such as stock-flow—exemplified instructors' discomfort. Some instructors viewed the initial phase of simplifying real-world scenarios as belonging to other disciplines rather than mathematics; at the same time, they expressed interest in collaborating with colleagues from other disciplines to support modelling's integration.

This work builds on prior studies exploring STEM instructors' relationships to knowledges (Capiello & Venturini, 2011; Chenevotot et al., 2019). To the best of our knowledge, it is the first study to examine how relationships to mathematics and modelling jointly shape postsecondary teaching practices. Our work contributes to a more nuanced understanding of the conditions necessary for sustainable integration of modelling in Calculus mathematics curricula.

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[81.] The relationship between mathematics curriculum and mathematical modelling research in Japan

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Previous research on mathematical modelling (MM) education has mainly focussed on the theoretical and cognitive aspects of MM, but few studies have addressed the context of mathematical education, such as the curriculum implemented, educational institutions, classrooms and lessons (Niss & Blum, 2020). The latter is clearly embedded in the unique educational culture of each country or region, reflected in aspects such as curriculum goals, objectives and agendas, so there are significant differences between countries or regions (e.g. Hanleln, 2020). Hence, it is very important for the ICTMA community to be aware of how MM research in each country is situated in the context of that country's mathematics education system.

Today, international perspectives on research and practice in MM education are flourishing, blending practical (application-oriented) with scientific and humanistic (structure-oriented) perspectives (Kaiser, 1991), which has led to the emergence of new and diverse perspectives (Kaiser & Sriraman, 2006). Therefore, we adopted Abassian et al.'s (2020) five perspectives as an international perspective to identify the characteristics of MM education research in Japan: (a) *realistic*, (b) *pedagogical*, (c) *models and modelling*, (d) *socially critical*, and (e) *epistemological*. The aim of this study is (1) to identify the characteristics of MM educational research in Japan from the five perspectives of MM, and (2) to explore the relationship between the characteristics and the Japanese mathematics education curriculum.

In the current Japanese course of study, a schematic diagram of mathematical activities is presented consistently across elementary, junior high, and high schools (e.g. MEXT, 2017). This diagram combines processes similar to MM, beginning with the mathematisation of real-world phenomena and integrating the development and refinement of mathematical concepts. It can be inferred that this background is influenced by the mathematical activity diagram proposed by Shimada (1977, English translation: Becker & Shimada, 1997). However, the curriculum guidelines do not explicitly specify how such mathematical activities should be positioned in textbooks. The authorised textbooks produced based on the course of study do not explicitly focus on areas devoted exclusively to mathematical activities, including MM. Instead, the content of each area is organised as a unit within which mathematical activities are embedded.

When interpreting this curriculum structure through five MM perspectives, it seems to be guided by an epistemological perspective that aims to integrate the teaching of MM with the formation of mathematical concepts. In practice, during lesson study meetings in Japan, there are frequent discussions on how to emphasise the process of forming mathematical concepts through MM. When analysing MM research in Japan, it is crucial to first clarify this unique cultural background and then consider the insights suggested by MM research. Accordingly, the following research questions are posed:

RQ1: *What are the characteristics of MM research in Japan in terms of the five perspectives of MM?*

RQ2: *How do the characteristics of Japanese MM research relate to the structure of the Japanese mathematics curriculum?*

To obtain answers to the research questions, this study adopted the *Scoping review* type of literature review (Grant & Booth, 2009). From the papers published by Japanese researchers as first author in international journals related to MM research and in the ICTMA book series etc. after 1977, when Shimada published his mathematical activity diagram in Japan, 54 papers were selected. The results of the analysis and discussion will be presented on the day of the presentation. This study will identify the relationship between the background of the Japanese mathematics curriculum structure and MM research. The findings will provide useful insights for MM education and research under the curriculum of other countries, as well as implications for the next curriculum revision in Japan.

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[82.] How do the computer results output by generative AI affect mathematical models of modeller?

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The first author focus on the difference result outputted by generative AI to mathematical problems (Kikuchi, 2024). Human inputs content(s) to generative AI, and generative AI outputs result(s) in response to those contents. The output results are expressed as a text with embedded mathematical results. In the modelling cycle with added computer model (Greefrath, 2011), the computer results from output from generative AI have inherent mathematical results. So human is needed to require to interpret the mathematical results from computer results.

We conducted a modelling workshop using generative AI for 17 undergraduate Japanese students. The students solved a problem using generative AI. The problem is “Given a four-digit integer, find the four-digit integer that is equal to the square of the sum of the last two digits and the first two digits” which was on the admission test for Gunma University in 1978 . A student expressed the problem as

$$(a + b)^2 = 100a + b$$

, inputted “solve the

$$(a + b)^2 = 100a + b$$

” to the AI (He expressed that a is the first two digits, b is the last two digits). The AI transformed to

$$a = -b + 50 \pm \sqrt{2500 - 99b}$$

and focused the values of a and b when

$$\sqrt{(2500 - 99b)}$$

is a square number. As a result, the AI outputted “The answer of this problem is 2025, 3025”. He interpreted the results output from generative AI, and discovered mistakes in the mathematical result. Finally, He concluded “The correct answer of this problem is 2025, 3025, 9801”. The research question is as follow: How do the computer results output by generative AI affect mathematical models of modeller? To address this problem, we will conduct a new modelling workshop using generative AI and another ICT. In the previous modelling workshop, the students used generative AI as CAS (Computer Algebra System). In a new modelling workshop, we will focus on exploring uses of mixing ICT packages (e.g. generative AI and scientific calculator).

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[83.] Metanoticing in Mathematical Modeling Instruction: A New Lens on Teacher Noticing

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Successfully incorporating mathematical modeling into school curricula not only presents challenges but also offers valuable opportunities, as it enables teachers to foster students' mathematical proficiency, critical thinking, and metacognitive awareness by guiding them through complex, real-world problem-solving processes (Hidayat et al., 2023; Stillman, 2011; Wilson & Clarke, 2004; Yang et al., 2025). However, studies show that because of the complexity of the modeling cycle and the cognitive demands it takes on both teachers and students, teachers frequently find it difficult to facilitate modeling activities (Blum & Ferri, 2009). Mathematical modeling requires teachers' situation-specific skills ("as the process through which knowledge and beliefs become relevant in practice" (Santagata & Yeh, 2016, p.163)) to notice student thinking, as students engage in real-world problem formulation, validation, and refinement of solutions (Alwast & Vorhölter, 2021; Borromeo Ferri & Blum, 2010; Cai et al., 2022). This study explores the role of teachers' metacognitive noticing in mathematical modeling instruction, introducing the concept of "metanoticing"—the ability to regulate and refine one's own noticing in the teaching process. This study contributes to ongoing discussions on developing teachers' strategies for effective mathematical modeling instruction. In this study, three math teachers' experiences with metanoticing during a mathematical modeling activity are investigated using a phenomenological research methodology. The data collection tool of this study involved the video records of the implementation of the modeling task, video records of stimulated recall sessions, and semi-structured interviews, allowing researchers to capture teachers' instructional decisions and reflective processes in depth. The modeling activity, carefully selected to align with students' prior mathematical knowledge, required teachers to assess students' reasoning, and make real-time instructional adjustments. During video-stimulated recall sessions, teachers reviewed and analyzed recordings of their own lessons, providing insights into their cognitive and pedagogical reasoning. Thematic analysis was conducted following van Manen's (2023) hermeneutic phenomenological approach, identifying key patterns in teachers' metacognitive and noticing competencies in the context of modeling instruction.

According to the data analysis obtained from the modeling interviews, it was determined that one teacher's modeling knowledge was more qualified than the other two teachers. Findings demonstrated that the teacher with good modeling knowledge generally paid more attention to the instructional dimension and benefited all dimensions of metacognition. This teacher's interpretation of the situation she paid attention to was found to be reflective. The other two teachers didn't know much about modeling instruction, paid more attention to task dimension, and didn't generally use metacognitive strategies. These teachers' interpretation of the situation they paid attention to was found to be descriptive. In conclusion, teachers' interpretation of

dimensions, which they notice, is related to their metacognitive strategies. By systematically examining teachers' metanoticing processes, this study provides empirical evidence in mathematical modeling instruction, offering practical implications for teacher education and professional development programs.

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[86.] **Education for Sustainable Development through Mathematical Modelling Activities in Primary School: implemented tasks, suitable teaching methods and learners' solutions**

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Education for Sustainable Development (ESD) is a cross-cutting task that is required as part of the transformation towards global sustainability to achieve the 17 Sustainable Development Goals. Target 4.7, ESD, states that education in schools must also include education *about* sustainability (UN, 2015). Current research has shown that ESD can be promoted through modelling activities (e.g. Vorhölter & Siller, 2023; Borromeo Ferri, 2025) in mathematics teacher education and with secondary and high school learners (e.g. Bulut & Borromeo Ferri, 2025; Siller et al., 2025). Accordingly, we know little about the teaching of ESD through mathematical modelling (MM) in primary school or about the potential of ESD-MM tasks for young learners. An ESD-MM task is defined, if it combines and features the concept of ESD and the criteria for modelling tasks (Wiegand & Borromeo Ferri, 2023). The ESD-MM tasks developed according to the design criteria (Wiegand, 2024) were conducted with secondary learners (Borromeo Ferri, 2025). Even so, complexity, topics and mathematical models set different standards for primary school learners. In Germany, ESD is one of the 'overarching educational areas' in primary schools (KMK, 2015), which can be integrated into many subjects. However, ESD is primarily anchored in the 'Sachunterricht' (general studies), due to its tradition as a central subject for environmental education as well as for other areas of responsibility (e.g. geographical education) (Hauenschild, 2023). Ideas for teaching ESD in primary schools are offered (e.g. Karaarslan-Semiz, 2022), but the mathematical perspective, in particular modelling, is neglected.

The qualitative study presented here is intended to fill the research gap. We will present, how ESD-MM tasks look like and which topics (SDGs), mathematical content and associated teaching methods are possible in primary school. The basis for the data (10 developed and implemented ESD-MM tasks, associated lesson plans, written reflections) built an ESD-MM seminar over a 5-month semester with 20 pre-service primary teachers (PSTs). They developed a task in pairs according to design criteria. The PSTs were free to choose the topic of the task and the year group (in Germany grades 1-4,; age: 6-10 years). The following research questions were in the focus, aiming to promote mathematics as a core discipline in ESD besides 'general studies':

1. Which SDGs and mathematical content (models) are focused and promoted on in the ESD-MM tasks by the PSMTs?
2. Which teaching methods and techniques were used to integrate the tasks into the taught lesson to deepen understanding for ESD through modelling?
3. How does solutions of the young learners look like in the ESD-MM context?

The data are analysed with the method of thematic analysis (Clarke & Braun, 2017). Although data analysing process is not complete yet, but will be for the conference, first results are very interesting. Since the tasks were challenging in their complexity, also younger kids at the age of

8 to 9 years were able to model. In particular, the topics of CO₂ emissions and water consumption played a role, albeit with different issues. The ESD-MM tasks were prepared based on concrete materials experimentally. This deepened learners' understanding during the modelling process and later of the real results. For the kids, this was an important basis for formulating conclusions for sustainable action based on mathematical facts. This also highlighted the importance of mathematics in the context of ESD.

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[87.] Problem solving at the interface of maths and chemistry: Modelling processes, cognitive resources and epistemic framing

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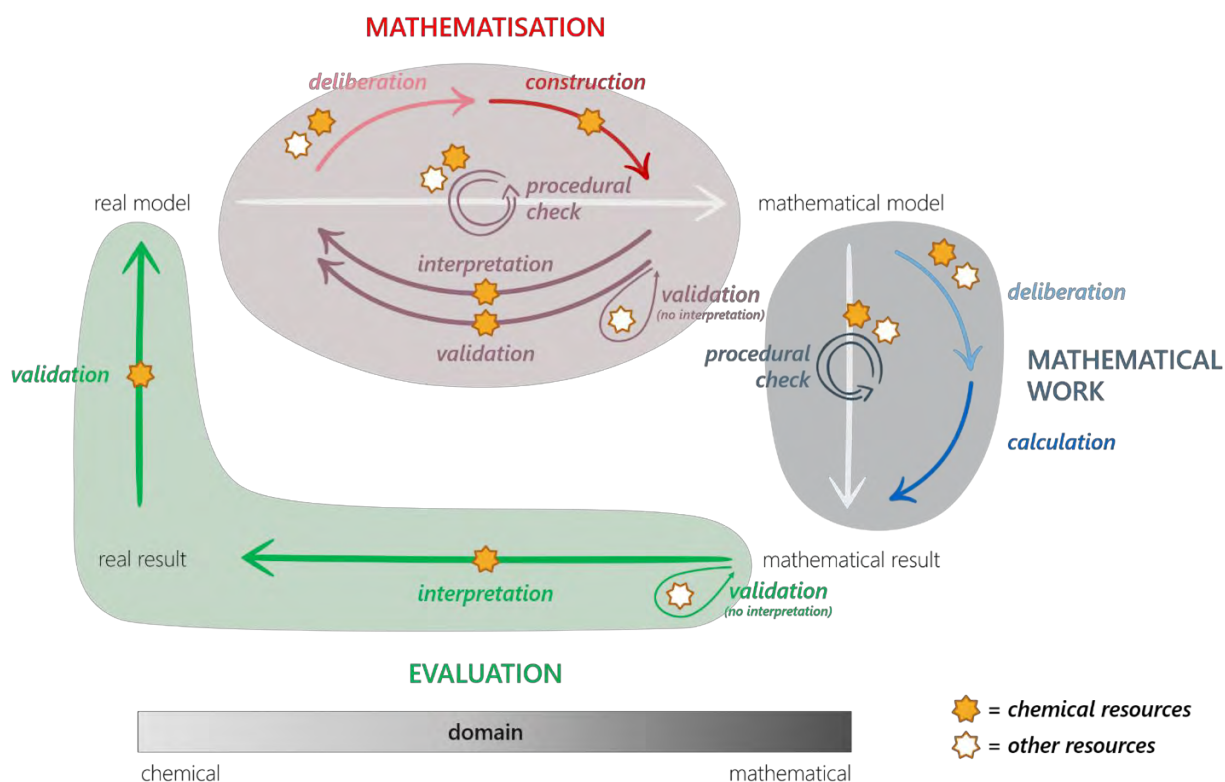
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Problem solving at the interface of mathematics and other scientific disciplines presents significant challenges for students across a range of fields (Biza et al., 2016), including chemistry. The procedure involves more than algorithmic mathematical manipulations; meaningful application and integration of chemical and mathematical knowledge at a conceptual level is essential for constructing models and theories that explain and predict chemical behaviour. Mathematics is integral to the knowledge structure of chemistry, enabling the advancement of understanding as well as the generation of new insights. Research has shown that integrating mathematics into other disciplines involves a range of processes and that difficulties extend well beyond technical mathematical manipulations (Uhden et al., 2012; Bain & Towns, 2016; Ye et al., 2024, 2025). Clearly, the issue is not ‘just maths’—the picture is far more complex. Understanding *what makes problem solving at the interface of chemistry and mathematics so difficult* is the central focus of our research.

In this communication, we will present findings from two studies where we investigated how second-year chemistry students solved tasks that required varying degrees of integration between chemical and mathematical knowledge (Ye et al., 2024, 2025). Building on the mathematical modelling cycle (MMC; Borromeo Ferri, 2006) and the resources framework (Hammer et al., 2005), we investigated both the *modelling processes* students engaged in and the *cognitive resources* they activated. This led to the development of the *extended MMC* (Figure 1), which offers a more fine-grained account of the subprocesses and resources involved in each modelling step. In particular, our analysis revealed that students engage in evaluative activities (e.g., procedural checks, interpretation, and validation) and rely on input from extra-mathematical resources throughout all stages of the MMC. Furthermore, while extra-mathematical input often primarily refers to non-mathematical *disciplinary knowledge*, our findings suggest that it also includes resources grounded in *situational knowledge*—knowledge about what is typical or appropriate in a given problem context. For instance, students sometimes referred to an algebraic expression as ‘too long’ or ‘too complex’. Such judgements are not rooted in explicitly articulated chemical or mathematical rules, but rather reflect students’ expectations based on their prior experiences and intuitive ideas about the nature of a reasonable solution.



We will also share insights from our ongoing research exploring the role of students' *epistemic framing* during mathematization of chemical phenomena. In this study, students were given a task requiring them to construct a mathematical model from chemical (spectroscopic) data. The task was designed to target the domain-bridging steps of the MMC: it was based on familiar chemical concepts and basic arithmetic operations, though combined in a form unfamiliar to the students. Preliminary analyses reveal that students struggle to make decisions where their expectations about what counts as legitimate come into conflict across disciplinary boundaries—for example, when determining whether certain mathematically feasible approaches are also chemically acceptable. These findings highlight the additional cognitive demands placed on students during mathematical modelling contextualised in another discipline, such as chemistry.

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[88.] Empirical analysis of authentic aspects in reality-based mathematical test items

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Enabling students to use mathematics in a variety of real-world contexts is one of the central goals of mathematics education (Niss & Blum, 2020). For assessing this goal, reality-based tasks (mathematical tasks that address an extra-mathematical real-world context) are crucial. Since the references to reality in dealing with such tasks should actually be real and not artificially invented, authenticity is usually demanded as a key criterion for these tasks (Geiger, 2017). This requirement for authenticity applies not only to learning tasks but also to test items in large-scale tests (OECD, 2021).

Although authenticity has become a prominent ‘domain issue’ in the mathematical modelling community (Galbraith, 2013), there is little research on the construct of authenticity in test items and there are many unanswered questions. To this regard, the main goal of the study is to gain a deeper insight into the theoretical construct and the empirical conceptualisation of authenticity in reality-based mathematical test items (in large-scale tests). For this purpose, existing mathematics test items from the German standardised large-scale test VERA-8 (exemplary object of investigation) are analysed with regard to the following research questions:

RQ1: To what extent do reality-based test items (of VERA 8) contain authentic aspects?

RQ2: How do several aspects of authenticity correlate with general task criteria (in VERA 8)?

RQ3: Do different ‘clusters of items’ (based on aspects of authenticity) exist and how can those potential clusters be described by general task criteria in terms of cognitive activation (in VERA 8)?

The study analysed $n=109$ test items from VERA-8 that are publicly available (from the years 2014 to 2019) and that were assigned to the competence of mathematical modelling. In response to the research questions, two constructs were assessed: On the one hand, the construct of authenticity was assessed in the test items using the Model for Authenticity in Modelling Test Items (MAMTI, see Schlüter & Besser, 2024). The model combines different theoretical ideas on authenticity from mathematics education (Palm, 2009; Vos, 2018) and assesses the authenticity of eight test item aspects in binary form. On the other hand, the construct of cognitive activation is used to assess general criteria of high-quality tasks. For this, the established COACTIV classification scheme (overview in English language see Neubrand et al., 2013) was used. The coding was performed independently by two raters (significant agreement at a substantial level: for the MAMTI Cohen's kappa = 0.69, for the adapted COACTIV scheme Cohen's kappa = 0.68), any disagreements were subsequently resolved by consensus.

The results show that different aspects occur authentically with different frequencies (RQ1), that certain relationships exist with general item characteristics (RQ2), and that different ‘clusters of items’ exist based on aspects of authenticity (RQ3). In our presentation we will present these

results in detail and discuss them in the light of implications for authenticity in reality-based tasks in general as well as for the development of standardised test items in particular.

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[89.] “How big is the panther?” - mathematical modelling in kindergarten

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Young children possess informal knowledge of mathematics that is amazingly broad, complex, and sophisticated (Clements & Samara, 2009). Based on this observation, we assumed that mathematical modelling activities in kindergarten should be feasible. We conducted an exploratory study guided by the research questions: (1) *how can mathematical modelling be implemented in kindergarten?* and (2) *what type of ‘early mathematics learning’ can emerge from mathematical modelling activities in kindergarten?*

The empirical basis comes from the project *How big is the panther?* carried out at five kindergartens in South Norway. Central was a large plush black panther. At the start of the activity, the children were informed about a stuffed animal in the next-door room. They were asked to draw the animal. Three rules were given: (1) the children should take turns in going in pairs to the other room to explore the animal, (2) the drawing should be just as big as the ‘real’ animal, and (3) the children were only to collect the animal to their own room after having finished the drawing.

The study used a theoretical framework to define mathematical modelling competences (Niss & Blum, 2020; Kaiser & Brand, 2015), and to assess whether the activity could be perceived as a modelling activity. The second framework was used to characterize aspects of early mathematics learning (Björklund et al, 2020; Clements & Samara, 2009; Flottorp, 2018). The data collection consisted of classroom observations, photographs, feedback from the kindergarten teachers, and recall reports.



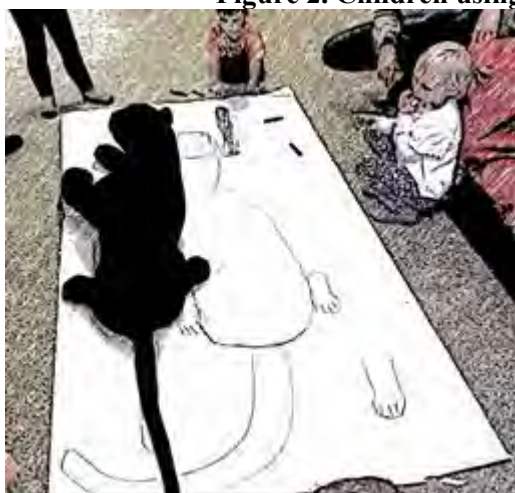
We consider the activity as mathematical modelling, since the children had to create a 2-dimensional model of a 3-dimensional physical object. The 1:1-scale imposed mathematical activities related to size comparison, placement, orientation, visualization and measurement. Since most children could not yet count or use metric measurement. A particular challenge was that the measurement had to be ‘transported’ from one room to another. In all groups, the children used their body parts as a unit of measurement (see Figure 1).

Figure 1. A child measures the panther's tail using the forearm.



In all groups, the idea arose of carrying objects into the other room and place them in a row next to the panther. Some groups took their shoes, other used books (see Figure 2), and one group used pencils. For this strategy, there was no need to count, because the set of objects could physically be moved to-and-fro. The children clearly understood that the set of shoes or pencils maintained their length when moving.

Figure 2. Children using a row of books to measure the panther.



After around half an hour, the children's drawing was finished and the panther was collected. It was a great experience for everyone to compare the panther with the drawing. All the groups drew the animal quite precisely (see Figure 3).

Figure 3. The panther has been picked up to check the dimensions of the drawing.

The paper will end with conclusions, discussions and recommendations for mathematical modelling teaching in kindergartens and the research thereof.

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[90.] Pre-service mathematics teachers' investigations of a genuine campus problem through statistical data modeling

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Statistical modeling is one of the powerful and natural approaches as it enables a holistic approach to learning statistics (Kazak & Pratt, 2021). To effectively cultivate essential statistical skills and mindsets in their students, the training of teachers in statistics should encompass a thorough examination of the complete statistical investigation cycle (Burrill, 2023; da Ponte & Noll, 2018; International Data Science in Schools Project [IDSSP], 2019; Suh et al., 2020). Pre-service mathematics teachers (PSTs) should first engage in statistical investigations as learners to gain experiences that will inform and enhance their future teaching of statistics (Makar & Fielding-Wells, 2011). Our understanding of the development of in-service and pre-service teachers in statistical investigation cycles is still limited (Biehler et al., 2017; 2018; Leavy & Frischmeier, 2022; MacGillivray & Preira-Mendoza, 2011; Zapata-Cordana & Martínez-Castro, 2021). This study explores the development of statistical investigation projects carried out by PSTs following the stages of a statistical investigative cycle known as PPDAC (Wild & Pfannkuch, 1999).

In this study, we conducted an interventional statistical investigation project with PSTs using a design-based research method (Cobb et al., 2003). To support PSTs in developing statistical modeling skills, we examined how they carry out a statistical investigative project related to a genuine issue (dining hall crowd) they encounter daily on campus. For the implementation, we utilized the PPDAC framework, which includes the stages of problem, plan, data, analysis, and conclusion as proposed by Wild and Pfannkuch (1999). Hence, the research question in this study was as follows:

How do PSTs develop statistical investigation projects based on the PPDAC stages?

26 PSTs participated in the implementation that lasted six weeks. PSTs worked on a genuine problem they encountered on campus daily with groups of 3-4 members. The written artifacts, audio, and whole-class video records of the groups were the data sources for the study. We employed formative assessment, provided timely feedback specific to each group, and revised the next week's class work following each group's needs and development. We used content analysis to analyze groups' audio and classroom video recordings.

The study findings showed that the PSTs encountered several difficulties, including reaching a meaningful statistical investigative question in the early phase, securing the best data collection strategy, and deciding on a statistical research method appropriate for their statistical investigative question. After posing an appropriate investigative question, the groups proceeded with their statistical investigations according to the PPDAC framework. They faced some challenges in data, analysis, and conclusion steps, such as organizing data and not reaching the planned sample size. These results highlighted the importance of careful and comprehensive planning. All groups conducted their statistical investigations using only categorical variables.

The researchers provided feedback that helped them conduct the investigation according to PPDAC to the PSTs during the investigation process. Furthermore, the PSTs tried to present compatible statistical models such as graphs, tables, and analysis methods and report the story behind the data in their conclusion reports. Lastly, we discuss the study's implications for the statistical education of teachers and future research.

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[91.] Programming as a support for mathematical modelling – a teacher perspective

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With the new curriculum in Norway in 2020, mathematical modelling (MM) and programming have become part of the mathematics teaching in grades 1-10 (Ministry of Education and Research, 2019). It is therefore a task of the mathematics teachers to implement both in their teaching, and it is relevant to explore the connections they see between the two. In our study, we focus on exploring how mathematics teachers view programming as supporting MM. The first author collaborated with two mathematics teachers to formulate and implement a modelling session integrating computer programming in two grade 10 classes. The topic chosen was personal economy, and the students had to write a Python code to plan their savings for buying their first car. Both teachers teach programming as electives, and their students are thus known with programming. The teachers' implementation of the session, in addition to a final interview with them after the teaching session, were audio-recorded, and then transcribed to be used as data for this study.

Previous research has found advantages for using digital tools in modelling, such as supporting the understanding of MM or possibilities for exploring different approaches in solving MM problems (Cevikbas et al., 2023). Few studies are focusing on programming and MM. In here, programming is defined as «the process of developing and implementing various sets of instructions to enable a computer to perform a certain task, solve problems, and provide human interactivity» (Balanskat & Engelhardt, 2015, p. 7). Ang (2021) emphasizes that programming is introduced in schools to help students develop computational thinking (CT). Further, “CT describes the thought processes entailed in formulating a problem so as to admit a computational solution” (Bocconi et al., 2016, p. 18) and it involves a set of skills that Ang (2021) means are relevant for MM. The latter is also supported by Villa-Ochoa et al. (2022) who identified computational thinking skills such as decomposition, abstraction, algorithms, representation and generalization embedded in a MM project. And Villareal et al. (2018) found that programming can support MM phases, such as finding mathematical solutions or the validation process. In our study, we focused on the recorded teaching episodes and teacher interview to identify the elements of CT, that were previously mentioned in literature, in students' work with the MM problem.

Preliminary results from our study show that the two teachers had several moments in which they saw programming to support MM. From the data analysis, we have identified four elements of computational thinking that support MM in different ways, and they fit with the Villa-Ochoa et al. (2022) findings. Decomposition helps in understanding, abstraction is a support for simplified models, making algorithms is connected to working mathematically and generalization is a support to making dynamic models. While generalization and making algorithms are elements that the teachers mentioned specifically in the interview, decomposition and abstraction were identified in the teaching episodes based on the theoretical descriptions. Our study adds to the research in MM by further specifying how programming can support MM

and is also relevant for the practice field by offering examples of how practitioners can combine the two.

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[92.] Integrating mathematical modelling into mathematics classrooms: An analysis of Korean secondary teachers' documentation work

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This study investigates how Korean secondary mathematics teachers design lesson plans for mathematical modeling classrooms. Their documentation work reveals how they select, adapt, and synthesize various instructional resources. Drawing on the documentational approach to didactics (Trouche et al., 2020), this study analyzes the teachers' documentation work not merely as static lesson plans but as dynamic processes shaped by professional knowledge, beliefs, and interactions with teaching materials. By examining these lesson plans as artifacts of documentation activity, the study explores how teachers mobilize resources to integrate mathematical modelling into their mathematics classrooms. It sheds light on how teachers develop and refine their professional knowledge and instructional practices through engagement with resources.

This study was conducted within a graduate-level course at a university in Korea designed to develop competencies for teaching and learning mathematical modelling. As mathematical modelling has become an increasingly emphasized component of the Korean mathematics curriculum over time, the ability of Korean teachers to design lessons for mathematical modelling has gained importance. Participants were 21 (prospective) secondary mathematics teachers enrolled in the course. They were asked to design a lesson plan for implementing mathematical modelling in Korean secondary mathematics classrooms. These lesson plans reveal the participants' instructional intentions, the modelling tasks they designed, and the sequences of activities as reflections of their documentational work. The analysis focused on identifying the types of resources mobilized and the ways in which modelling activities were structured within the lesson plans.

The analysis of the lesson plans revealed two aspects of the participants' documentation work. First, while participants focused on constructing modelling tasks based on contexts that students could realistically experience, rather than on artificial scenarios, there were differences in the types of resources they mobilized. Participants drew on a range of resources, including curriculum documents, textbook problems, and experience-based examples, to create realistic situations for modelling tasks. Second, participants adopted varied approaches in structuring the modelling process. Some participants designed lesson plans that focused on specific stages of the modelling cycle, while others aimed to enable students to experience the entire cycle of mathematical modelling. These findings illustrate the diverse ways in which teachers select, adapt, and structure resources when designing mathematical modelling lesson plans.

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[93.] Effects of collaborative modelling lesson series and debriefing on students' ability to engage in mathematical modelling, assumption making and on students' self-efficacy

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A number of studies have demonstrated that each phase of the modelling cycle can pose cognitive barriers to students (Galbraith & Stillman, 2006; Goos, 2002; Klock & Siller, 2020; Stillman, 2011). Galbraith & Stillman (2001) highlighted assumption making as an important part of the modelling process. Seino (2005) argued that assumptions are 'the bridge' that connects the real world and the mathematical world. For students who have not been previously engaged with mathematical modelling, the teacher's primary function is not to provide assumptions, but rather to clarify what assumptions are and to inform students of their importance.

Small group discussions have been shown to promote the development of modelling competencies (Galbraith & Clatworthy, 1990; Maaß, 2006). Wester (2021) investigated the opportunities what can be learnt when small-group discussions are used as an integrated part of whole-class teaching. The findings indicated that the absence of subsequent whole-class discussions following small-group discussions have less effect on student learning. However, there remains a lack of knowledge about what effect collaborative modelling and debriefing have on students' ability to engage in mathematical modelling and also have on students' self-efficacy. Additionally, it is unclear how to provide support for making assumptions when using collaborative modelling tasks and what the effects are. The objective of the present study is to investigate the effect of a lesson series with collaborative modelling tasks, whole-class discussions and the opportunity to improve assumptions have on students' ability to engage in mathematical modelling and on students' self-efficacy.

The study involved four teachers (two males and two females, with a mean experience of 12.5 years) and 199 grade eight students (aged 13-14) from four schools in the Netherlands. A quasi-experimental research design was employed, with a pre-test, post-test and with two conditions. One of the intervention conditions was collaborative modelling with focus on assumptions in debriefing (CMAD). In the debriefing the teacher devoted additional attention to the assumptions that students had made and stimulated them to reflect on how they could improve their model. The other intervention condition was collaborative modelling and debriefing (CMD), the teacher did not undertake an additional step in the whole-class discussion and limited their discussion to the findings of the groups in class. To examine the effect of the interventions we used mathematical modelling tasks, recordings of student interactions in the collaborative learning groups, student questionnaires and interviews with participating teachers.

The results indicated that the students improved significantly in their mathematical modelling ability and in their self-efficacy in both conditions. The main effect of intervention was not

significant, suggesting no overall difference in test scores and self-efficacy between conditions. However, it was evident from their work that CMAD demonstrated a greater concern with making assumptions. The findings of the students' interactions indicated that the CMAD students demonstrated a higher level of proficiency in collaborative modelling. It is important to note that students employed the use of drawings and gestures to simplify the modelling problems. Furthermore, a marked discrepancy in assumptions was observed between schools. Following the interventions, semi-structured interviews were conducted with the participating teachers. The teachers would like to teach mathematical modelling in the future. However, they indicated to find it challenging. One notable challenge was to intervene and resist the temptation to offer solutions.

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[94.] Exploring the link between tertiary students' prior mathematical knowledge and their performance in a modelling unit

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This paper reports on connections between tertiary students' prior mathematical knowledge (at the start of their calculus course) and their performance in the pre- and post-test for a five-lesson mathematical modelling unit developed in the project Comparative Studies into Teaching Approaches for Mathematical Modelling (CoSTAMM). Since 2019 four studies have been implemented as part of the project with the main aim to advance students' mathematical modelling competency through different teaching styles (a 'method-integrative' and a 'teacher-directed' style; for details see Durandt, Blum & Lindl, 2022). In all studies so far, irrespective of the educational context, the modelling unit proved to be effective, and more effective for students that followed the method-integrative style (compare Lindl, Durandt & Blum, 2025). One aspect that often receives attention in research is the level of university students' prior knowledge and the effects that this knowledge has on students' performance (Di Martino, Gregorio & Iannone, 2023). Thus, we explored in detail the connection between the students' prior mathematical knowledge and their performance in three of our studies which have been conducted at the tertiary level.

For this purpose, these three studies, all implemented at a public university in South Africa (in 2019 and 2022), were analysed simultaneously. The altogether 465 participants were exposed to a diagnostic test before the start of the modelling intervention (for details regarding this test see Durandt, Blum & Lindl, 2021) and a pre- and post-test before and after the intervention. In our analysis, presented here, we searched for correlations between the results for the different sections (content areas) of the diagnostic test, on the one hand, and the different sections of the pre/post-test, on the other hand. The results show some expected correlations between certain content areas of the diagnostic test (algebra, analytical geometry, functions, modelling) and certain parts of the pre- and post-test (intra-mathematical and modelling sections), but also that the effects of the modelling unit (students' competence gains) are largely independent of students' prior knowledge.

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[95.] Exploring Pre-service Mathematical Literacy teachers' mathematical modelling competency through the solving of simultaneous equations

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The main purpose of this research project is to gain insight into the competence of pre-service mathematical literacy teachers in solving real-world problems using simultaneous equations. The inclusion of real-world problems and applications is an essential component of the mathematical literacy curriculum (Department of Basic Education [DBE], 2011). Durandt et al. (2021, p. 3) posits that mathematical modelling competency is often defined as an individual's ability not only to create but also to apply mathematical models for addressing real-world problems. This study was conducted to prepare pre-service mathematical literacy teachers for these challenges.

This research paper focuses on the performances of three groups of pre-service mathematical literacy teachers in their attempts to solve four different-worded real-world problems through simultaneous equations. The participants were third-year pre-service mathematical literacy teachers enrolled in an undergraduate education program at a South African university.

A mixed-method approach to this study was employed within an interpretative paradigm to ascertain the levels of mathematical modelling competencies of the pre-service mathematical literacy students. Task-based activities and observations were used as a means to collect data, as well as semi-structured interviews to measure participation.

The results of this research study indicate that the pre-service mathematical literacy teachers have a rudimentary understanding of analysing and making sense of the real-world problems they are faced with. This degree of incompetence serves as a stumbling block, preventing the pre-service mathematical literacy teachers from easily navigating from simple problem solving to advance problem solving in the mathematical modelling process. The participants struggled with constructing models and experienced difficulties interpreting the solutions. A link exists between comprehending what the real-world problem constitutes and solving the real-world problem.

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[96.] Empowering Mathematical Modelling through AI and Coding: An Exploratory Study in Indonesian Secondary Schools

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This article explores the integration of Artificial Intelligence (AI) and coding within mathematical modelling instruction, conducted in five secondary schools across Semarang City, Indonesia. Drawing upon Indonesia's national education reform agenda (Kemdikdasmen RI, 2025a; Kemdikdasmen RI, 2025b) and UNESCO's AI Competency Framework (UNESCO, 2024), the study aims to strengthen students' mathematical reasoning, digital literacy, and engagement through modelling activities grounded in real-world contexts and supported by emerging technologies.

A combination of project-based learning and simulations was implemented, with digital coding environments helping students to construct, analyse, and refine mathematical representations of everyday phenomena. The initiative also incorporated AI-enhanced feedback systems that responded dynamically to learners' inputs, allowing for customised support and increased independence in problem-solving (Yi et al., 2025; Li, 2024). Participants showed improved capacity in abstracting problems, applying mathematical structures, and evaluating results critically—skills aligned with key AI competencies, such as ethical design, iterative improvement, and learner-centered approaches (UNESCO, 2024).

Teachers observed higher levels of enthusiasm and improved modelling performance among students. At the same time, they noted the importance of sustained teacher professional development and responsive curricular design. These reflections echo findings from earlier studies that stress the role of instructional scaffolding when embedding AI and coding tools into classroom environments (Walter, 2024; Ateş & Gündüzalp, 2025). Practical exercises involving block-based coding also contributed to deeper generalisation and flexible problem-solving strategies (Çakıroğlu & Muştu, 2025).

The study draws upon related innovations that emphasise creativity in STEAM contexts—for example, motion capture tools supported by AI for choreographed “mathematical dance” (Cahyono et al., 2025), and local cultural expressions such as batik pattern design through 3D modelling, which enhanced creativity and computational awareness (Yunianto et al., 2025). In addition, the RoboSTEM initiative demonstrated success in preparing future mathematics educators to integrate robotics in line with 21st-century competencies (Cahyono et al., 2021).

Recent discourse highlights the need to equip teachers to critically engage with AI-powered tools like ChatGPT in mathematics problem solving (Noster et al., 2024). In parallel, global research on mathematical modelling in times of disruption stresses the importance of equipping students with critical awareness to navigate complex societal issues (Siller, Geiger, & Kaiser, 2024).

The findings contribute to ongoing efforts in emerging contexts to implement technology-driven modelling practices. By aligning curricular innovation with national priorities and global AI literacy frameworks, the study underscores the urgency of equipping learners with adaptive,

reflective, and technologically informed mathematical competencies for an AI-augmented future.

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[97.] Student conceptions of the role of data in AI models

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Most modern AI systems are data-driven and rely heavily on mathematical modelling, particularly statistical and probabilistic methods (Friedrich et al., 2022). As society becomes more digitised with AI, it is essential that citizens are able to critically assess AI-related information (Ding et al., 2024) and, by extension, evaluate the mathematical models that underpin it. Mathematical modelling competency includes the ability to evaluate mathematical models that represent real-world situations (Niss & Højgaard, 2019). This evaluation can involve validating the model's outcomes, examining its structural properties, analysing its quantitative features, and comparing it with alternative models (Niss & Blum, 2020). However, AI systems operate in complex, messy, and data-rich environments and are often complex and opaque. This makes some of these evaluation methods challenging, or even impossible, even for experts. Nevertheless, students can still develop a basic understanding that AI models are data-driven—that the training data play a crucial role in shaping what the models can and cannot do.

Both Kim et al., (2023) and Whyte et al. (2024) argue that students often hold naïve conceptions of AI, such as seeing AI as a universal solution or underestimating the importance of data quality. Our study investigates students' conceptions of the connection between training data and AI models, and how training data defines both their possibilities and limitations. Specifically, we investigate the following research question: *What conceptions about the role of training data in AI models do Norwegian 10th-graders hold?*

This study investigates a teaching session where 29 students explored questions about ChatGPT and autonomous vehicles, alongside a data-driven mathematical modelling task within an AI context. Data sources include written responses and follow-up interviews. We identified three conceptions of how AI uses data: (1) AI performs mathematical modelling based on data, (2) AI functions like a search engine, retrieving information from an available dataset, and (3) AI is preprogrammed and not based on data. When evaluating AI outputs, or explaining incorrect ones, students who held conception 1 often discussed whether the training data lacked the representativeness needed to produce accurate or complete answers. Students with conception 2 typically described AI output as the top result that appears when searching for an answer. Some students did not recognize the limitations of the data and attributed errors to the AI's lack of intelligence or to human mistakes.

Our findings show that students who view AI as performing mathematical modelling based on data are better able to critically evaluate AI outputs. They recognize how data quality and representativeness affect model performance.

In contrast, students who see AI as a search engine or preprogrammed often overlook the role of data and misattribute errors to AI's "intelligence" or human mistakes.

This highlights the need to develop mathematical modelling competency to foster critical and AI-literate students.

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[98.] **Meaning in mathematical modelling activities mediated by technology: from philosophical to didactic implications**

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Much attention has been devoted to the concept of meaning in mathematics education, with both philosophical and non-philosophical perspectives addressing how the world can be understood by human beings—for example, Wittgenstein’s perspective on language (Wittgenstein, 1958), as well as studies conducted by Kaiser (2008), Almeida and Silva (2015), and Sousa and Almeida (2019). As a topic of debate in the teaching and learning of mathematical modelling, various factors can influence the construction of meaning, including the participants involved in the process, the mathematical and non-mathematical content that emerges during the activity, the means and modes of expressing this content, the ways of knowing, and the modes of interaction (as detailed by Kilpatrick, Hoyles, and Skovsmose, 2005; Skovsmose, 2016). Elucidating the relationships between meaning and modelling processes remains a challenge for both teaching and educational research.

This study presents the results of research investigating meaning in mathematical modelling activities mediated by technology, with a particular focus on the role of digital videos in these activities. The research is also theoretically grounded in studies that explore the intersection of mathematical modelling and digital technologies in mathematics education. For instance, Greefrath and Vos (2021) provide a framework for classifying ICT-based modelling tasks, highlighting the pedagogical potential of video-based word problems and modelling projects in enhancing students’ modelling competencies. Similarly, Molina-Toro, Rendón-Mesa, and Villa-Ochoa (2019) analyze emerging trends in the integration of digital technologies with mathematical modelling, emphasizing their role in fostering more dynamic and interactive learning environments. These contributions support the present study, which is guided by the following research question: *What meanings can be identified in the development of mathematical modelling activities supported by digital videos?*

This question is explored using a qualitative research approach. Data were collected from two mathematical modelling activities developed by prospective mathematics teachers who participated in mathematical modelling courses. The analysis of students’ documents, videos, and materials led to the identification of actions and relationships that offered a detailed understanding of the meanings constructed during the development of the activities. Data analysis was conducted based on a theoretical framework on meaning in mathematics education, considering two dimensions of meaning in modelling activities: philosophical and didactic. To analyze the data, we employed a method for capturing patterns across qualitative datasets, based on Braun et al. (2019). A Thematic Analysis was conducted to analyze the data, describing the different activities and seeking both personal and objective meaning of mathematical modelling and teaching.

The findings reflect on the one hand, objective meanings emerged in the students' activities and in the interactions enabled by technology, such as an understanding of the modelling process, the construction of algorithms and apps, mathematical reasoning and the solution of a problem; on the other hand, personal meanings were also constructed of mathematics as a useful tool, of modelling as a challenge to solve students' own contextual problems, and the connection with technology as a way to position oneself critically in the face of the demands of the problem and context. This study also highlights students' understanding of the phenomena investigated through mathematical modelling, the use of mathematical concepts, and the role of digital technologies in the modelling process.

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[99.] Student Voice while Modelling Socially Pressing Issues with Digital Technology in a Japanese High School Mathematics Lesson

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Introduction

Digital technologies (DTs) have become integral to mathematics education, serving diverse purposes. A recent review identified five main goals for DT use: enhancing learning, understanding learning processes, designing for learning, promoting equitable access, and transforming curricula, teaching, and assessment (Drivers & Sinclair, 2024). The authors suggest a future direction: exploring whether DTs help students engage with social issues (ibid, p. 240). Aligned with this suggestion, collaborative lesson designs were created to connect mathematics with real-world and socially pressing issues, such as the spread of infectious diseases (Ohtani, 2024). This study adopts a modified Japanese Lesson Study approach (Fujii, 2015), emphasizing collaborative lesson design and reflection. We further adapted it to foreground students' perspectives (Kaur et al., 2013). By engaging students in inquiry-based lessons using DTs, we examined how these tools support understanding of real-world social issues.

Lesson Study Process

Topic Selection: COVID-19 as a Social Issue

The High School reported in this paper has implemented Lesson Studies for over 20 years, drawing on Realistic Mathematics Education (van den Heuvel-Panhuizen, 2020). The topic of infectious disease was chosen using the ICT-based task *Epidemics* from the Dutch National Exam for Mathematics A (Cito, 2003). The COVID-19 pandemic and the Diamond Princess case made the topic timely and relevant.

Lesson Design

Designed for 12th-grade students in the “Inquiry” unit of Mathematics III, the lessons focused on recurrence models involving limits—models not solvable analytically. DT-based simulations supported exploration of:

1. Mathematical models of infectious diseases
2. Simulation of disease spread
3. Simulation of various infection scenarios
 - Lesson 1: Students analyzed current COVID-19 data and explored the Reed-Frost model (S_n, I_n, R_n = susceptible, infected, recovered).
 - Lesson 2 (Research Lesson): A more realistic model introduced recovery rates, reflecting cases where not all infected recover in one week.
 - Lesson 3: Students explored models without immunity, allowing reinfection.

Research Lesson was implemented on February 22, 2020, the research lesson involved 44 students (aged 16–17) and 27 observers. In groups of four, students used laptops with Excel:

- Task 1: Simulate infection with $N=10,000$; $k=0.99979$; $I_0=10$. Adjust vaccination levels to keep weekly infections below 1,000.
- Task 2: Introduce recovery ($g=0.5$ and 1). Assess effects on infection duration, peaks, and vaccination needs.

- Task 3: Modify parameters (I_0 , R_0 , k , g , N). Reflect on understanding and confusion.

Capturing Student Voice

Lesson observers documented student interactions and inquiry processes, which were later analyzed during the post-lesson reflection. Traditionally, these discussions center on the teacher's perspective. To incorporate the student voice, three students participated in the reflection, offering direct insights into their experiences and thought processes.

Emerging Themes from Student Reflections

Overall Impressions

- “It felt realistic. I tried to connect the equations with real graphs from the media.”
- “The equations were confusing at first, but the simulation helped me engage. Still, I felt the model was too simple.”

Task 3 Approaches

- “I focused on how the graphs changed as I adjusted the values.”
- “We increased n and saw everyone got infected—which isn't realistic. It made us question the model.”

Future Simulations

- “I'd like to simulate real scenarios, like a school setting. k feels a bit abstract.”
- “Real diseases are more complex. I'd like to explore different models.”

Discussion and Implications

Including student voices in the reflection process allowed teachers to gain deeper insights into how students engaged with complex, multi-variable recurrence models. While DTs stimulated curiosity and exploration, students also identified the limitations of simplified models. Many expressed a desire to improve the realism of their mathematical modeling.

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[100.] Evaluating a South African Model-Eliciting Activity Using Rubric-Based Assessment: A Case Study on Measurement Concepts

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Mathematical modelling – the process of translating real-world situations into mathematical terms – is recognised as a critical component of mathematics education (Arseven, 2015). However, in South Africa, where mathematics classrooms have limited resources, its integration remains limited due to a lack of structured modelling tasks such as Model-Eliciting Activities (MEAs) (Jakobsen & Mhakure, 2024). MEAs present a promising approach by engaging students to construct mathematical models based on real-life problems (Dede et al., 2017). To ensure their effectiveness and contextual relevance, MEAs should align with established design principles that support the development of high-quality, thought-revealing activities (Lesh et al., 2012).

Grounded in these principles, the study evaluated a measurement-focused, context-specific MEA designed for implementation in a South African higher education mathematics classroom. The study presents the MEA's alignment with Lesh et al.'s (2012) six design principles: reality, model construction, self-assessment, construct shareability, construct documentation, and effective prototype. The central research question was: *To what extent does a context-specific MEA align with Lesh et al.'s (2012) design principles?* The MEA that was evaluated in this study involved estimating the remaining water in a farm's open-cast dam and predicting its sustainability based on consumption rates, providing a culturally relevant and context-rich scenario for engaging with measurement concepts.

A qualitative case study approach (Yazan, 2015) guided the evaluation of the MEA. The MEA, its solution, and a reflective guide from the MEA's designer was collected. The MEA was assessed by three expert reviewers using a rubric adopted from Dede et al. (2017), rating each principle as completely appropriate, partially appropriate, or inappropriate.

Preliminary findings indicated a strong alignment with the principles of reality and model construction. The study suggests that rubric-based expert review is a cost-effective and scalable method for evaluating modelling tasks, particularly in resource-constrained settings. For teachers, this MEA offers a practical and contextually meaningful resource for use in South African higher education mathematics classrooms. Future research could explore the implementation of this MEA with students in real classroom settings to evaluate its impact on students' modelling competencies.

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[102.] Model Construction and Selection in Predictive Modelling by 8th-Grade Students: Focusing on the Determinism of Models

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In predictive modelling based on bivariate data displayed in a scatterplot, students are expected not merely to fit a function to the data, but to construct a model by taking into account its relevance to the real-world context (Stillman & Brown, 2021). Whether the constructed model is deterministic or non-deterministic depends on whether individual data points are interpreted as signal or noise. (Konold & Pollatsek, 2002). This study addresses the following research question: *Are students who are unfamiliar with non-deterministic models able to informally construct and select such models?*

The participants were fifteen Japanese 8th-grade students with little prior experience in constructing non-deterministic models. During two class periods, students engaged in a predictive modelling task in which they were asked to estimate the minimum extent of Arctic sea ice in the year 2050, based on a scatterplot showing the annual minimum sea ice extent from 1982 over a 40-year period. To support their quantitative analysis—such as deriving equations from graphs—students were allowed to use GeoGebra for dynamic manipulation. In the actual classroom implementation, seven students constructed deterministic and qualitative models, four constructed deterministic and quantitative models, three constructed non-deterministic and quantitative models, and one student constructed a non-deterministic and qualitative model. Subsequently, four students presented their different types of models to the class. For instance, one of the three students who constructed a non-deterministic and quantitative model dynamically drew a line through the central cluster of points — perceived as the signals — and then added two parallel lines above and below to include the noises, visually demarcating the "volume zone" of the data. Following these presentations, the number of students who selected a non-deterministic and quantitative model for their written report increased from three to five. These students recognized the model's strength in terms of accuracy and likelihood.

These findings suggest that even students with limited prior experience — albeit a small number — can, through the dynamic use of technology, construct non-deterministic and quantitative models to make predictions for unknown values, and can appreciate the merits of such models. Furthermore, students who understood and engaged with non-deterministic and quantitative models shared by others were able to appreciate their merits and select them. However, for many students, two class periods were insufficient, indicating a need for medium- to long-term instructional support.

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[103.] Integrating Practical Problem-Solving Activities into the Mathematics Curriculum: A Multi-Perspective Analysis of Three Seventh-Grade Textbook Series in China

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Interdisciplinary education has received growing attention in recent years, as it emphasizes the cultivation of students' comprehensive abilities to address complex, real-world problems. Among the most representative approaches is STEM education. In the United States, the CCSS-M (Common Core State Standards for Mathematics) encourages students to engage with cognitively demanding STEM tasks (Walker & Sherman, 2017), while *The Australian Curriculum: Mathematics* (ACARA, 2011) highlights the importance of cross-disciplinary learning to enrich student experiences (Curriculum, 2012).

In China, the *Comprehensive and Practical* domain, one of the four core content areas in the lower secondary mathematics curriculum, likewise underscores interdisciplinary learning and the development of students' abilities to solve real-world problems. However, it has historically been underemphasized in both textbook development and classroom instruction. Since its initial introduction in the 2001 curriculum standards, the domain remained ambiguously defined. Although the 2011 revision provided further elaboration, it still lacked clearly articulated curriculum goals, stage-specific objectives, and concrete implementation or assessment guidelines—leading to limited effectiveness in actual teaching practice. The 2022 revision of the Compulsory Curriculum Standards for Mathematics (MCSCE 2022) offers a comprehensive and systematic refinement of this domain.

Following the release of MCSCE 2022, lower secondary mathematics textbooks across various publishing presses in China began incorporating *Comprehensive and Practical* Activities (CPAs) starting in the fall semester of 2024. As this represents a newly introduced module, the study focuses on the following research questions:

1. How are CPAs organized across different textbook series? Are there any identifiable patterns in their organization?
2. What theoretical perspectives on the teaching of CPA, if any, are presented in the textbooks?

To address these questions, this study analyzes three textbook series developed in accordance with MCSCE 2022, published by the People's Education Press (PEP), Beijing Normal University Press (BNUP), and Phoenix Science Press (PSP). PEP and BNUP represent the most widely adopted series in China's lower secondary mathematics education (Liu et al., 2022), while PSP textbooks—primarily used in Jiangsu Province, China's second-highest GDP province—offer a valuable perspective from a highly developed educational context.

Using thematic text analysis (Kuckartz, 2014), the study divided CPAs into three main sections: Pre-activity Preparation Section, Activity Tasks Section, and Post-activity Reflection and Extension Section, in order to examine how different textbook series organize content within

each section. The results reveal that the three textbook series adopt different approaches to structuring the three sections, particularly the Activity Tasks Section, which reflects distinct models of inquiry-based learning. Furthermore, the CPAs embody two major instructional perspectives: (a) an epistemological or theoretical perspective and (b) a realistic or applied perspective (Kaiser, 2017). These results provide meaningful insights for educators aiming to implement CPAs effectively in mathematics classrooms.

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[104.] **Minecraft Education: didactic benefits and challenges from the perspective of mathematical modelling**

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Minecraft Education is a popular video game, increasingly being introduced into classrooms (Kipnis, 2018). In this virtual environment, players build using a variety of cubic building blocks. This is a “sandbox” type of video game (Kim & Park, 2018), meaning that the game designer has not set any precise objectives or strict guidelines. This encourages creativity and imagination, as players can realize the scenarios of their choice and build their own worlds using building blocks.

While its pedagogical potential has been discussed in various mathematics education studies (e.g. Jensen & Hanghøj, 2020; Kim & Park, 2018; Koroglu et Yildiz, 2021; Moore, 2018), empirical research on its role in mathematical learning remains scarce. As digital tools and serious games become increasingly prevalent in classrooms, it is crucial to take a step back to examine both the benefits and challenges of using Minecraft Education for mathematical learning. Mathematical modelling plays a crucial role in helping students make sense of real-world phenomena, yet the integration of technology in modelling tasks raises important didactical questions. We therefore targeted this research question: What are the didactic benefits and challenges of using the Minecraft Education virtual environment for teaching and learning mathematics in primary and secondary schools?

While Minecraft Education offers a dynamic environment for mathematical exploration, creation and application, little empirical research has explored how it supports or constrains mathematical modelling in classroom settings. Based on sub-competencies involved in modelling (Vorhölter et al., 2019), we consider mathematical modelling as the process of using mathematics to construct, simplify, mathematize, interpret, validate and/or expose real-world situations with models. Since mathematical modelling education offers opportunities to reflect, to use creativity and to find new pathways for the future of mathematical learning, this study underscores the need to critically assess how Minecraft Education shapes the development of modelling sub-competencies.

In this collaborative study, our research team worked with eight teachers and six pedagogical consultants to co-develop mathematical tasks using Minecraft Education. Some of these tasks have a clear potential to support mathematical modelling. Data collection (observations, student productions, teacher interviews, and group discussions with students) was conducted in 10 classrooms with 220 students aged 9 to 14 years (grades 4 to 8) in Quebec (Canada). Thematic analysis (Paillé & Mucchielli, 2016) of this data highlights didactic benefits such as the use of virtual manipulatives to support model-based reasoning and the development of student comprehension. Our analysis also reveals didactic challenges, including constraints related to block size and shape in model construction. For this presentation, as a way to exemplify those

didactic benefits and challenges, we focus on a complex task (at primary level) with mathematical constraints. For this task with a creative intent, we present students' productions, to show how Minecraft Education can be used to introduce mathematical modelling at primary level.

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[105.] AI and mathematical modelling education – challenges and opportunities

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Artificial Intelligence (AI) is now an indispensable technology for our daily lives and society, from smartphone image recognition and online shopping recommendations to credit card fraud detection, self-driving cars, infectious disease and environmental predictions, and more. AI combines machine learning algorithms with large amounts of data to predict the likelihood of future events and generate new text, images, video and audio. Building on their broad applications, AI tools such as ChatGPT have been increasingly used in mathematics and statistics education. The topic of AI systems such as machine learning algorithms has also attracted attention as an important learning object in computer science and data science education.

It is an unavoidable challenge for us, the ICTMA community, to explore how AI tools and the topic of AI systems can be incorporated into mathematical modelling education, including AI's impact on teaching pedagogy and student learning. We still know little about how teachers can leverage AI tools to teach modelling, and how students can use them to advance the modelling process. In addition, AI systems rely heavily on the use of mathematical models and modelling processes. These systems typically operate as “black boxes”, i.e. the relevant mathematical models and operating modelling processes are not obvious, nor are the underlying data and algorithms. This may imply a shift in modellers' agency from building models themselves to focussing on critiquing, interpreting and validating models produced with the help of AI. However, students still need hands-on experience of creating and exploring models in order to critique, verify, understand and further develop models and AI systems. Teachers, lecturers and students also need to understand how they can utilise AI within the modelling process. An understanding of the modelling process will be key for these learning experiences, and the teaching and learning of mathematical modelling at all levels of education is expected to play a role.

In this dual plenary talk, we will each draw from our own areas of expertise and present an overview of the current state of AI and mathematical modelling education, explore potential uses of AI in the teaching and learning of mathematical modelling, and give some tips for considering AIs introduction into modelling education practice within primary, secondary and tertiary education. Finally, we aim to conclude with presenting practical tips to consider for incorporating AI into pedagogical practices and developing students understanding of AI systems. It is our hope that open questions for further research into utilising AI as an innovative tool within mathematical modelling education are generated from our plenary talk.

[106.] Analysing non-optimal implementation of modelling tasks – What do mathematics teachers notice in classroom cartoon vignettes?

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Building on approaches to teachers' noticing and knowledge-based reasoning about aspects of classroom situations, this study focuses on teachers' analysis of mathematical modelling classroom situations. To build up these key competences (OECD, 2022) in modelling, learning opportunities embedded in classroom situations are necessary, which support students in learning and actively carrying out modelling activities (Blum et al., 2007). Such situations bring challenges for teachers: In addition to the complexity of students' modelling processes (e.g., Borromeo-Ferri, 2006), and given multiple possible modelling approaches, the students' individual modelling ideas and the students' potential difficulties require the teachers to be able to notice a variety of potentially relevant situation aspects and to analyse them in the sense of knowledge-based reasoning (Sherin et al., 2011). While teachers' noticing is being considered as a key aspect of profession-related expertise (Sherin et al., 2011; Amador et al., 2021; cf. Kersting et al., 2012), there is a growing body of research about noticing regarding specific criteria (e.g., Choy, 2014; Dreher & Kuntze, 2015). However, relatively little is known about teachers' noticing related to criteria relevant for mathematical modelling in classroom situations. The modelling cycle (e.g., Blomhøj & Jensen, 2003) can contribute to structuring key analysis requirements: Teachers should be able in particular to analyse how a task with modelling requirements is introduced (cf. Kuntze et al., 2018) and framed by the teacher, how students' modelling attempts in the sense of mathematising are being dealt with, how learning opportunities related to validating are being used, and whether and how the modelling process is reflected upon and evaluated in a post-hoc, retrospective way.

Teachers' analysis of classroom situations can be elicited particularly well by classroom cartoon vignettes (Buchbinder & Kuntze, 2018), on which teachers can be asked to report their observations or suggestions. Analysis criteria such as those introduced above can be implemented in vignettes so as to create analysis requirements for mathematics teachers in which they can detect non-optimal aspects of the classroom situation through corresponding knowledge-based reasoning (cf. Buchbinder & Kuntze, 2018; Skilling & Stylianides, 2019). It is particularly primordial to explore pre-service teachers' analysis of classroom situations, as findings can directly inform the design of learning opportunities in pre-service teacher education.

In line with these research needs, the mainly explorative research question is: *Do pre-service successfully analyse aspects of a non-optimal use of learning opportunities related to mathematical modelling in classroom situations represented in cartoon vignettes?*

This study uses vignette-based methods: A set of specific cartoon vignettes were designed, the theory-based vignette design enabled a normative perspective according to which an expert

teacher can be expected to notice non-optimal teacher reactions, allowing also for a top-down coding procedure. The sample consists of almost 200 pre-service teachers.

The findings reveal rather low frequencies of successful noticing related to the modelling-specific analysis criteria. The evidence hence suggests a need of fostering the pre-service teachers' criteria-based noticing, e.g., through an increased focus on modelling-related practices in classroom situations.

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[107.] Analysing teachers' examples of modelling activities

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Due to varying objectives of the teaching of mathematical modelling, classroom activities may appear diverse. There are various frameworks in the current literature that can be useful for identifying aspects of modelling activities, e.g. Maaß' (2010) classification scheme including the different steps of a modelling cycle, type of data, nature of relationship to reality, type of situation, model and representation. Other perspectives in modelling activities emphasise characteristics of the context (Goos et al., 2017), the radius of action (Jensen, 2007), or elements from the Spirit of mathematical modelling (Frejd & Vos, 2024). Different aspects of mathematical modelling are emphasised in these frameworks. For teachers it can be challenging to find or develop appropriate classroom activities, and a lack of resources including examples of modelling activities is previously reported as a constraint (Schmidt, 2011). In this study we will explore the characteristics of activities teachers have already conducted in their classrooms. The analytic framework is developed from earlier research, and includes mathematical content area, context, and openness. The research question is formulated as:

What are the characteristics of modelling activities that teachers use when teaching mathematical modelling?

We interviewed ten mathematics teachers (grade 1-10) regarding various aspects of their teaching of mathematical modelling. They were asked about their understanding of mathematical modelling, their approaches to teaching it, and opportunities and challenges. Additionally, they were inquired about the specific modelling activities they had implemented in their classrooms, and about what they acknowledge as criteria for a good modelling task. These two themes from the interviews are analysed in the present study. The teachers had diverse levels of experience in teaching mathematics, ranging from 1 to 42 years. During the interviews they mentioned 40 activities in total. Some activities were mentioned briefly, whereas others were described in greater detail. The preliminary results indicate that most teachers emphasised the importance of relevance for students in the context of a well-designed modelling task, highlighting its role in fostering student engagement. Regarding the examples of activities provided by the teachers, only a few included authentic problems presented by an enterprise or for providing arguments regarding relevant questions in society. The majority of activities were constructed around the context of working with

mathematical content, such as variables and functions. The openness of these activities was limited in terms of providing students with opportunities to make assumptions and simplify. As spelled out by one of the teachers "*Sometimes I want a more open tasks [...] other times I want a more closed task where I want the students to do a certain thing. It depends*". This reflects the varying objectives of the teaching of mathematical modelling.

We argue that creating a list of criteria for a modelling problem offers limited value; however, identifying characteristics of the modelling activities provided can yield insights into the current teaching practices of mathematical modelling. This can help identify types of modelling

activities that teachers may be lacking in their teaching materials, and further address this constraint highlighted by Schmidt (2011) by developing relevant teaching materials.

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[108.] **Scaling and Disseminating a Modelling-First Curriculum for Life Sciences Majors: Drivers of Success**

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Mathematical modelling supports conceptual development in mathematics and fosters students' appreciation of mathematics through helping them understand real-world situations from a variety of disciplines, including physical sciences, life sciences, and civic life. Yet, as with many research-based instructional practices, implementing modelling-first curricula and pedagogies in tertiary mathematics classrooms trails behind the state-of-the-field in STEM education (Shadle et al., 2017). While it's relatively common for an individual faculty member to learn about how students benefit from learning mathematics through modelling and become duly inspired, transforming that inspiration into a fully-fledged course is an undertaking that many keen faculty are unable to realize. Still, for those who do develop a course, research shows that sustaining and maintaining it, especially beyond the initial faculty's efforts, is rare. Scaling up these initiatives and rolling them out at other institutions is nearly unheard of.

In this talk, we share stories of success and optimism – we present case studies of large research-intensive universities that have transformed their approach to teaching introductory mathematics to life science majors, through modelling. The focal curriculum is the *Life Sciences 30* course from UCLA, which was designed for first year biology and pre-medicine majors. It uses a modelling-first approach to teach advanced mathematical concepts, focusing on dynamical systems and computational strategies, without a calculus pre-requisite. The curriculum has been adopted by at least 7 other universities.

Our study addresses the question: *Why were these sites successful at implementing and sustaining this course?* Using a blend of institutional ethnography (DeVault & McCoy, 2006), narrative inquiry (Martinie et al., 2016; Mueller, 2019; Polkinghorn, 1995), and qualitative case comparison analysis (Ragin, 2004), we identified the drivers and barriers (Shadle et al., 2017) experienced by various stakeholders at each of the sites and how the barriers were overcome. We will share the successful origin story at one university and provide an overview of how the other sites are building and sustaining their transformed course. We conclude with (1) recommendations for overcoming specific barriers and (2) specific modifications to the original curriculum design that worked at other sites.

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[109.] Conditions for modelling in mathematics curriculum reforms in engineering programmes in Sweden and Spain

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Despite the progress in integrating modelling in university mathematics, research highlights persistent institutional constraints rooted in a reduced understanding of modelling as an ‘application’ of mathematical knowledge given in advance (Barquero et al., 2013). In this contribution, we aim to analyse the conditions and constraints that emerge from how lecturers conceive what mathematical modelling is and how it should be integrated into mathematics courses, to characterise how these conceptions affect modelling implementation. In particular, we address these issues in the context of engineering degrees and the particular challenges that engineers initial training opens (Gainsburg, 2006).

This paper approaches this problem within the framework of the anthropological theory of the didactic and analyses the decisions made when engineering programmes are worked to be reformed. The context of our research is framed in the special case of the curriculum reforms for engineering programmes at the University of Gävle in Sweden and the Escola Universitària Salesiana de Sarrià (EUSS) in Spain.

Our initial hypothesis is that one of the main components of this prevailing epistemology is characterised by a specific way of interpreting and conceptualizing the relationship between mathematics and engineering based on an ‘applicationist’ approach. Under its influence, modelling activity tends to be identified as an ‘application’ of previously constructed mathematical knowledge or, in the extreme, as a simple ‘exemplification’ of mathematical tools to some extra-mathematical contexts, often artificially built to fit these tools.

We start by adapting the indicators of ‘applicationism’, initially defined to analyse the role of modelling in mathematics courses for natural sciences degrees, into the case of engineering education. We integrate previous research developed about modelling in the particular case of engineering training and profession (i.e., Gainsburg, 2006; Moore et al., 2013). They will be used to guide our empirical study to contrast the prevalence of ‘applicationism’ in the context of mathematics teaching within engineering university programmes and their reform. In the first step, this study is based on analysing the proposed changes in mathematics subjects and syllabus in the previously mentioned universities, based on these indicators. The second step consists of analysing the discourses of mathematics university teachers when asked about the changes introduced for modelling in mathematics courses and the criteria used. Five lecturers have been selected in each university context, all of them involved in teaching mathematics courses for several years, as well as being part of the team in charge of the curriculum reforms of these subjects.

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[110.] Mathematical Modelling as a method for civic understanding, interpretation and evaluation of media claims

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Our society is confronted with various, sometimes contradictory claims in the media daily. Further, mis- and disinformation are increasing and influence public opinion (Lewandowsky et al., 2017). Recognising and dealing with these is therefore necessary in today's society. However, current studies show existing challenges: For example 50 % of participants from 27 EU countries stated that they only (very) rarely recognise disinformation (Unzicker, 2023). These challenges require an understanding and comprehension-orientated interpretation and evaluation of media claims as basis to participate maturely in social discourse. In addition, when dealing with media reports, mathematical capabilities and data literacy are necessary, because media reports increasingly contain maths and data, for example as part of an argumentation (Gal & Geiger, 2022; Schüller, 2020). Thus, the mathematical content in media reports must be identified, interpreted and understood, as well as evaluated using mathematical methods and external data. This represents an additional challenge for the society.

In summary, understanding, interpreting and evaluating media claims is a social challenge in the currently changing world. This can be addressed with mathematical modelling as a method (Siller & Vorhölter, 2025), because mathematical modelling and “an understanding of mathematical models [...] are key to responding to the challenges of an increasingly changing world” (Siller et al., 2024, p. 318). Thus modelling makes it possible to implement and promote the capabilities “to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen” (OECD, 2004, p. 15), which are also required and important in relation to media reports.

The aim of this contribution is to provide a theoretical characterisation of modelling as a method for civic understanding, interpretation and evaluation of media claims. For this purpose, a holistic perspective on modelling is focused on, as study results demonstrate that this approach is more motivating and authentic (Blomhøj & Jensen, 2003). The characterization is done with the help of various existing general and modelling-specific theories: In addition to characterisations of mathematical modelling (e.g. Pollak, 1979), Lompscher’s activity theory (Lompscher & Irrlitz, 1985) is used and modelling is thus characterised as a learning action with three different orientation bases (trial orientation, pattern orientation and field orientation; e.g. Bruder & Schmitt, 2016).

Through the theoretical characterization of modelling in relation to the understanding, interpretation and evaluation of media claims, initial insights are gained, and a basis is created to illustrate the beneficial role of modelling for society. In future, building on this it will be possible to implement the topic in the classroom.

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[111.] Boundary Artefacts for Mathematical Modelling in Vocational Education

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The International Community of Teachers of Mathematical Modelling and Applications (ICTMA) has developed a rich body of research over recent years (Geiger & Frejd, 2015). We want to contribute to it by highlighting the role of mathematical models in vocational education. We present findings from a research project conducted over the past ten years, aiming to reduce school dropout rates in vocational education in Italy. We conducted several design-based studies that aimed at offering students with high risk to drop out access to *meaningful* mathematics (Wake, 2014) through innovative teaching practices.

The specific aim of the research is to explore how co-disciplinary teaching paths co-designed and co-taught by mathematics and vocational subject teachers can reveal and strengthen links between vocational education and mathematics. Within this framework, we focus on artefacts drawn from vocational practice that serve as shared reference points between disciplines. An example of such an artefact is the paper pattern used in fashion lessons, see Figure 1. The artefact can be perceived as a 2-dimensional model of 3-dimensional real-life trousers.

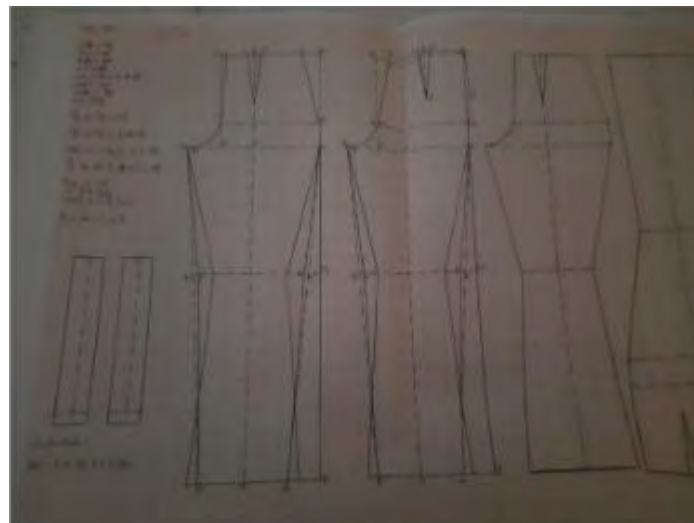


Figure 1. A paper pattern for making trousers

Inspired by the notion of Boundary Objects (Akkerman & Bakker, 2011; Star, 1995), we propose a refined concept: the Boundary Artefact (BA). While a Boundary Object enables communication across different social worlds, a BA is conceived as a didactic tool that retains its meaning in two different domains, while allowing students to engage in co-disciplinary processes that are contextually grounded. When the mathematical and vocational domains are combined, the BA potentially can make the vocation more mathematical and the mathematics more vocational. The dual interpretability positions BAs as mediating tools in the sense of the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008), enriched by their vocational

relevance. Through carefully designed tasks and triggering questions, a BA can become a source for mathematical modelling activities embedded in a vocational domain. An example of such a task is to find out how the given paper pattern in Figure 1 can be used for smaller or larger trousers.

Introducing and implementing BAs requires a co-learning and inquiry-based teaching approach. According to Blanchard-Laville (2000), co-disciplinarity involves generative cooperation among educators working across disciplinary boundaries. In our study, co-disciplinarity becomes visible through shared problem-solving situations, where mathematical modelling competences emerge from the solving of concrete vocational tasks mediated by BAs. The use of BAs and the deployment of their semiotic potential through student-student- and teacher-student-discussions leads students to produce personal mathematical and vocational knowledge (Bartolini Bussi, 1998). These are orchestrated by both the mathematics teacher and the vocational subject teacher and gradually transform into shared knowledge.

Our approach aligns with existing research in mathematics education that emphasizes contextualized modelling (Frejd & Muhrman, 2022; Wake, 2014) and extends it by introducing BA as a means to deepen conceptual integration between mathematics and vocational training. In this paper we present the analyses of the semiotic potentials of BAs and some preliminary results relating to their use in two interventions involving Mathematics and Fashion and Mathematics and Chemistry.

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[112.] What kinds of models did modelers make while solving the problem in the real world?: The case of an arranged ice cream problem

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The first author take as an example of ‘The similarity of ice creams problem’ in the grade 9 mathematics textbook in Japan, and focus on the gap between the mathematical world and the real world occurred while mathematizing (Suzuki, 2024). ‘The similarity of ice creams problem’ is ‘Two ice creams containers are cylinder and similar. We decide which is less expensive based on volume.’ Mathematical models of the ice creams problems in the textbook are volume models, and have already mathematized. We used ice cream containers sold in a real shop, and arranged the problem. The arranged ice cream problem is as follow: There are three ice creams sold in the real shop. How would you like to buy ice creams for 900 yen?

We plan and conduct a workshop to solve an arranged ice cream problem for 17 undergraduate Japanese students. The workshop focus on how they lead mathematical results and real results based on modelling cycle (Blum & Leiß, 2007). What kinds of models did modelers make to get real results? The research question is as follows: What kinds of models did modelers make while solving the arranged ice cream problem? At that time are models made by modelers the same in the textbook or not?

In the workshop the students decide how they buy ice creams. They made some kinds of models in addition to a volume model. The models can be classified six types as follows: (i) ice cream types models (5 students), (ii) number models (4 students), (iii) ingredient models (4 students), (iv) price models (2 students), (v) scene model (1 student), (vi) volume model (1 student). Five types models made by 16 students were different from the model in the textbook. Only one student made the same model , type (vi) volume models, as the textbook.

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[113.] In search of synergy in students' conceptual understanding and the teaching of calculus and chemistry

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The relationship between mathematics and other disciplines in science and engineering, as well as the application of mathematics in these fields, is a topic of research that has attracted increasing attention in the mathematics education research community in recent decades (Goos, Carreira & Namukasa, 2023). This research includes, for example, the conception of instructors and students about the role of mathematics in other disciplines, ranging from a mere tool for computation, to being an integral part of the structure of the discipline (Uhden et al, 2012; Goos, Carreira & Kamukasa, 2023; Koerfer et al., 2025). There is also much interest in processes involved in and factors influencing mathematical modelling (Stillman & Brown, 2019), including our own recent studies in the context of chemistry (Ye et al., 2024, 2025). Furthermore, pedagogical affordances and challenges associated with the use of calculus in teaching science and engineering have been examined in the literature, including the issue of the application of calculus being in practice relegated to an instrumental role, leverage little of the fundamental insights and understanding afforded by a more conceptual treatment of the contextualised mathematics (Faulkner et al., 2020; González-Martin, 2021).

In this reflective presentation, possible ways to create synergy across disciplinary boundaries will be explored, through the lens of a number of contexts and cases involving single and multivariable calculus in the teaching and learning of physical chemistry at both the lower- and upper-division university level. As a discipline heavily reliant on mathematics as part of its knowledge structure and language, physical chemistry poses significant challenges to students and teachers alike (Bain et al., 2014; Bain et al., 2016), but also offers many rich and unexplored contexts for interdisciplinary efforts in mathematics and chemistry education research. Drawing from chemical thermodynamics, statistical mechanics and kinetics, possible inroads for enhancing conceptual understanding of both chemistry and calculus will be outlined and discussed. Specific examples include state and path functions in thermodynamics and their connection to partial derivatives, interdisciplinary treatment of multivariable calculus in the multidimensional parameter space in thermodynamics, as well as the connection between the discrete at the microscopic level and the continuous at the macroscopic level in statistical mechanics, and its potential to further student understanding of the integral. Questions concerning the associated demands placed on the pedagogical content knowledge of instructors in their teaching practice will also be raised for consideration and discussion.

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[114.] Influence of creativity on mathematical modelling activities

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Creativity is seen as an important characteristic of modern life and has been refocused as part of the learning and innovation skills of the 21st Century Skills. Due to its complexity, empirical research on it, especially in relation to mathematics education, was relatively rare in the twentieth century, but has now increased significantly (Leikin & Sriraman, 2022). In our everyday life, where many processes are taken over by machines or artificial intelligence, the human ability to think and act creatively will remain necessary to enable self-determined and self-regulated ways of working in the future. This need is also reflected in the focus of the latest PISA study, which for the first time measured creative thinking among young people worldwide (OECD, 2023). Creativity arises from the need to understand anomalies in the world (Schank, 2004). Therefore, this ability should be particularly promoted in mathematics classrooms to help students understand our world through creativity as it is considered to be an important cognitive skill that students should develop (Pellegrino & Hilton, 2012).

In the quest to understand the world, the creation of models becomes a central process – especially in mathematics education, where real-world phenomena are often explored through mathematical modelling. Modelling activities serve as a bridge between everyday reality and abstract mathematical concepts, so these translation processes are at the heart of mathematical modelling (Niss et al., 2007). Recent international discourse highlights the growing importance of creativity in modelling activities (Wessels, 2014). Consequently, studying the role of creativity in students' transitions from real-world contexts to mathematical representations is crucial for understanding and improving modelling activities in education.

In addition to the creativity component of modelling activities (e.g. Lu & Kaiser, 2022), we investigate the extent to which learners need mathematical creativity to make the transition from the real world to the world of mathematics. In a survey of 41 Year 11 students, participants were asked in two tasks - in the sense of problem posing (Siller et al., 2024) - to develop mathematical questions about a non-mathematical context (sports and games). For these items, we first have a look at how large the actual proportion of mathematical questions is. We then want to explain this proportion of mathematical questions with the help of the participants' creative dispositions. It is particularly noticeable here that there is a high correlation between the number of questions asked and the flexibility of the learner's thinking.

Our study provides empirical indication that mathematical modelling activities involve creativity, especially in the transition from the real world to the mathematical one. It can be concluded that a creativity perspective on modelling activities can contribute to a more comprehensive understanding of modelling processes in mathematics classrooms and thus facilitate the promotion of mathematical modelling as well as mathematical creativity. Modelling tasks can therefore be used to easily and naturally integrate creativity into mathematics education.

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[115.] Constructive Resources in Mathematical Modeling

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Within the cognitive perspective of modelling, the ICTMA community has made significant progress in developing accounts of student thinking as students construct mathematical models of real-world scenarios. Specifically, the ICTMA community has produced various types of modelling cycles (e.g., Blum & Leiß, 2007) and used those modelling cycles as analytical tools to describe students' modelling routes (Borromeo-Ferri, 2007), decision making strategies (e.g., Wickstrom & Jung, 2024), typologies of validation (Czocher, 2018) and to diagnose student challenges in modelling (e.g., Jankvist & Niss, 2020). While modelling cycles and the research on student thinking grounded in them serve as powerful descriptive schemes, they sacrifice explanatory power that attends to modelers' fine-grained thinking (Czocher, in press). In her plenary talk at ICTMA-21, Czocher (in press) emphasized the importance of accurately diagnosing students' representational conventions through an anti-deficit lens (Adiredja, 2019), framing what students know as resources students' draw upon as they construct models.

Consistent with this call, we found that networking theories (Radford, 2008) from quantitative reasoning (Thomson, 2011) and resources (Hammer, 2000) have been beneficial in describing beyond *what* students did, to *why* they did so (e.g., Roan et al., 2025). Following Sherin (2000), we propose a constructive resource for mathematical modeling (CRMM) framework with potential to develop second-order theories of students' modeling activities (Steffe & Thompson, 2000). We operationalize CRMM as the resources—consisting of *quantification processes* and *cognitive resources*—that contribute to students' construction of mathematical models (Lee, n.d.). *Quantification processes* are the mental operations involved in conceiving a real-world situation consisting of quantities and relationships among quantities (Thompson, 2011). According to Thompson (2011), a quantity is a mental construct consisting of an object, attribute, and a conceived measurement process. Relationships between quantities can be conceived through operating on quantities or reasoning about variations among quantities. *Cognitive resources* are fine grained knowledge elements that are networked together to make sense of a given situation (Hammer 2000). Literature has identified three types of sense-making resources students draw upon when making decisions as they work on scientific tasks (diSessa, 1993; Hammer, 2000; Swanson, 2025). These include conceptual, epistemological, and intuitive resources. Conceptual resources are knowledge elements that learners have acquired regarding mathematical and scientific concepts. Epistemological resources can be characterized as learners' knowledge elements for making sense of the provenance of acquired knowledge (Hammer & Elby, 2002). Finally, by intuitive resources, we refer to the knowledge elements acquired through experiences that may contribute to the activation of certain quantification processes.

In this talk, we will elaborate on the CRMM theoretical framework using illustrative vignettes from two pairs of pre-service teachers' (PSTs) work on the Laundry and Basket tasks. Specifically, we address the question: *what constructive resources do PSTs draw upon as they*

construct models for real-world situations? Finally, we will make conjectures about the CRMM framework's capacity to prescribe pedagogical intervention that responds to student thinking.

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[116.] **How students' knowledge of benchmarks influences their process of solving Fermi problems**

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Fermi problems are characterized by the absence of data, whereby they require making assumptions and estimating quantities in the real world (Ärlebäck, 2009; Ärlebäck & Albarracín, 2019b). Because of this real-world connection and because Fermi problems require various steps of the modelling process, they represent a special type of mathematical modelling problem (Greefrath & Frenken, 2021). At the same time, Fermi problems are closely related to estimation problems (Albarracín & Gorgorió, 2014). When solving estimation problems, students have been found to use benchmarks to estimate quantities (e.g., Holland et al., 2023; Huang, 2014), and previous research has found that using benchmarks can influence how students process estimation problems. For example, when students compare the to-be-estimated objects with benchmarks, they get more accurate estimates (Holland et al., 2023), and when students make errors in estimating lengths, this often arises from unsuitable benchmarks (Holland & Hoth, 2024). Since estimating quantities is also a necessary part of solving Fermi problems, benchmarks are also likely used when students process Fermi problems. Indeed, a qualitative study has shown that early secondary school students use benchmarks for lengths and benchmarks for weights when solving respective Fermi problems about estimating lengths and weights (Meyer & Greefrath, 2025). In that study, students used benchmarks to make implicit estimates to build a mathematical model, to estimate missing quantities or to validate their estimates when solving Fermi problems (Meyer & Greefrath, 2025).

Given this context, students' use of benchmarks and their knowledge of benchmarks may influence their process of solving Fermi problems, especially the steps that involve estimating quantities and validating their estimates. To conduct a more detailed analysis of students' use of benchmarks in the process of solving Fermi problems, we are conducting a quantitative study to investigate how students' knowledge of benchmarks for lengths and those for weights influences their Fermi problem solving process. Our quantitative study also investigates whether students' knowledge of benchmarks influences the realisticness of the estimates they make to solve Fermi problems.

Since Fermi problems are suitable for initial modelling activities in school (e.g., Ärlebäck & Albarracín, 2019a), our study involves early secondary school students. We examine how students' knowledge of benchmarks affects the process of solving Fermi problems with a test instrument on benchmark knowledge and Fermi problems. This paper-and-pencil test was used in several sixth-grade classes towards the end of the school year and in several seventh-grade classes at the beginning of the school year. The test consists of two parts: The first part includes tasks related to benchmark knowledge of lengths and benchmark knowledge of weights; the second part includes Fermi problems that require estimating lengths or estimating weights. At the ICTMA conference, we will present our initial results to answer the following research question: *How does students' knowledge of benchmarks influence the Fermi problem solving processes?*

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Exploring the Interplay Between Chinese Secondary Mathematics Teachers' Professional Knowledge and Attitudes Toward Mathematical Modelling: The Mediating Role of Self-Efficacy

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Since the beginning of this century, mathematical modelling has been a compulsory element in school curricula and a critical mathematical competence for students in several countries worldwide (Blum, 2015; Niss & Højgaard, 2019; Schukajlow et al., 2018). However, there are key differences in the historical trajectory of mathematical modelling's integration and the

objectives of mathematical modelling in school curricula, diverging between Western contexts, such as Germany, and Eastern contexts, such as China and Japan, owing to discrepancies in pedagogical traditions and curriculum structures (Brown & Ikeda, 2019). In contrast to the Western context, which has a long tradition of emphasis mathematical modelling in school curricula, mathematical modelling has only recently been emphasized in secondary school mathematics curricula in Eastern Asian contexts, such as China (Lu & Huang, 2021).

This delayed integration, compounded by China's entrenched tradition of prioritizing foundational knowledge and procedural skills in mathematics education, has resulted in a widespread lack of teacher readiness in both professional knowledge and attitudes toward mathematical modelling (Xu et al., 2022). Consequently, the effective classroom implementation of mathematical modelling remains a critical challenge for Chinese mathematics educators.

Against this backdrop, this proposed study investigates the relationship between secondary mathematics teachers' professional knowledge and their attitudes toward mathematical modelling, with a focus on the mediating role of self-efficacy in mathematical modelling and its instruction. Drawing upon validated instruments from prior research within the Chinese context (Asempapa & Brooks, 2022; Wess et al., 2021), the study surveyed 197 in-service Chinese secondary school mathematics teachers. Descriptive, correlation and path analysis results revealed that, overall, participants demonstrated limited professional knowledge of mathematical modelling and held predominantly negative attitudes toward its integration into teaching. Furthermore, while teachers' mathematical modelling knowledge exhibited a direct effect on their attitudes, self-efficacy only partially mediated this relationship. The study also examines the broader socio-cultural factors influencing these dynamics, offering insights into the unique challenges faced by educators in China's evolving mathematics education landscape.

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[117.] Effectiveness of a Teacher Professional Development program on Mathematical Modeling and Cognitive Demand for elementary school teachers

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Mathematical Modeling (MM) is present in several school curricula, implying a challenge for teachers, since they must understand it before bringing it to the classroom. However, they have difficulties in selecting, designing and implementing mathematical tasks (MT) with a high Cognitive Demand (CD) (Estella et al., 2020; Tekkumru-Kisa and Stein, 2015), in particular, those of MM because they consider them complex (Borromeo-Ferri et al., 2021), or because their characteristics overwhelm them (Cai et al., 2022). This reveals the need for Teacher Professional Development (TPD) programs that prepare teachers for their teaching. A TPD program was carried out to strengthen teachers' knowledge of MM and CD. This work shows its effectiveness in achieving the proposed training objective.

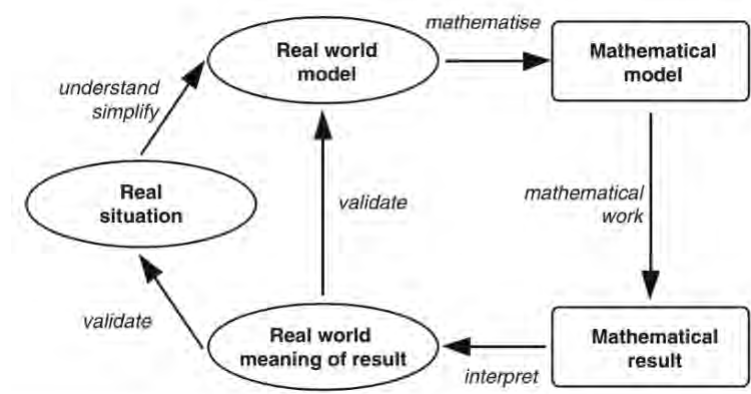
An effective TPD is "structured professional learning that results in changes in teacher practices and improvements in student learning outcomes" (Darling-Hammond, et al., 2017). Ramos-Rodriguez et al. (2021) define eight principles of effectiveness for TPD programs in mathematics education.

For the design of a TPD program on MM, we rely on the theoretical-practical model of Borromeo-Ferri (2018) that seeks to deepen the knowledge about this competence.

MM is a competence that is progressively developed at educational levels; it involves identifying a problem in a real situation, translating it into mathematical language, interpreting and validating the solution obtained (Greefrath et al., 2022), viewing it as a cycle (Figure 1).

Figure 1

Mathematical Modeling Cycle



Note. Kaiser and Stender (2013, p. 279).

A MT of MM satisfies certain characteristics (Borromeo-Ferri, 2018; Maaß, 2010; Niss and Blum, 2020), summarized in Bustos-Osorio et al. (2025) (Figure 2).

Figure 2

Characteristics of a mathematical modeling task

Open:	it grants students the possibility to ask diverse and new questions and does not provide all the information that allows them to identify the mathematical model.
Complex:	cognitively demanding for students, because they must understand the context and search for relevant data
Realistic:	based on students' experiences in consideration of their age, and give way to authenticity
Meaningful:	must make sense to students
Clear:	wording appropriate for the level of education and language
Process:	It involves all phases of a mathematical modeling cycle.

Note. Bustos-Osorio et al., 2025.

CD is the level of demand faced by students when performing an MT and the Task Analysis Guide (Smith and Stein, 1998) can be used to analyze the CD of a MT.

The TPD was designed based on the principles of effectiveness for mathematics education programs (Ramos-Rodríguez et al., 2021), and the competency model for teaching MM (Borromeo-Ferri, 2018). It was implemented in two schools in Chile during 16 sessions, involving eight elementary school teachers. The definition of MM (Greefrath et al., 2022), the MM cycle (Figure 2) and the Task Analysis Guide (Smith and Stein, 1998) were used.

The results are obtained from the analysis of the teachers' responses to a questionnaire, validated by experts, applied *before (CI)* and *after (CF)* the TPD. They modified their initial conception of MM, going from considering in the IC that it is a teaching strategy to show students the steps to solve a MT, to understanding it, in the CF, as a competence that is progressively developed by going through the different phases of a cycle when solving a real situation using mathematics. In addition, they characterize a MT of MM, mainly, as open-ended and not providing all the information (Bustos-Osorio et al., 2025).

In addition, out of a set of four MTs, where MM is the one with the highest CD, only three out of eight teachers recognize MM as such in the IC, while six do so in the FC, showing progress in the identification of the CD of a MT.

The findings evidence a positive impact on the teacher's knowledge of MM and CD, considering that the TPD was effective in these terms.

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[118.] **Developing Pre-service Teachers' Real-world Problem-posing Abilities: A Three-year Study in Thailand**

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Developing pre-service teachers' ability to create problems connected to real-world contexts is critical for mathematics teacher education in Thailand and internationally. Studies show that even where mathematical modelling is included in national curricula, textbooks contain few authentic modelling problems, and teachers rarely design their own (Hansen & Nerida F, 2015; Paolucci & Wessels, 2017). As a result, teachers need to develop the ability to design and create mathematical modelling tasks suitable for their classrooms. While researchers such as Ferrando et al. (2025) have examined the effects of different training strategies on pre-service teachers' ability to pose modelling problems, a significant challenge remains in understanding how pre-service teachers, especially those without personal experience with modelling—develop this competence. Recent studies by Unver et al. (2018) emphasize the need for further research on specific pre-service teacher training strategies and their impact on teachers' abilities to pose modelling problems. This study aimed to: (1) analyze the redesign of learning activities promoting real-world problem-posing abilities over three years (2022–2024) and (2) examine the characteristics of problems created by pre-service teachers during this period. The study involved 76 Thai pre-service teachers with no previous modelling experience in a mathematics teaching program. Employing content analysis, we evaluated course materials, instructors' reflection, and pre-service teachers' real-world problems, using thematic coding based on five criteria for modelling tasks proposed by Siller & Greefrath (2020): Reality relation, Relevance, Authenticity, Openness, and Promotion of modelling sub-competencies. To ensure methodological rigor, we implemented researcher triangulation and member checking with research assistants to validate our interpretations. Findings revealed that (1) learning activities improved through implementation of Fermi problem-solving and problem-posing activities that bridged general problem-solving with mathematical modelling; utilization of the "what-if-not" technique; practice of atomic modelling competencies; and reflection on problem authenticity—approaches that effectively addressed the challenge of developing authentic modelling competencies in pre-service teachers with no prior experience. (2) Analysis of pre-service teachers' work showed a clear progression: in 2022, students primarily created word problems like the *Ferris Wheel Problem*, incorporating real contexts but lacking meaningful connections to real-world objectives. By 2023, they produced problems with more diverse contexts (e.g., *Macaroon Packaging Problem*, *Robot Vacuum Cleaner Problem*), though still limited in openness and authenticity. In 2024, significant improvements emerged as pre-service teachers posed problems with greater authenticity and alignment with modelling processes, exemplified by the *Recycling Bins Problem* addressing environmental sustainability. Despite some inconsistencies, the overall trend indicated a shift from problems focusing merely on calculations with real-world connections toward comprehensive modelling problems, allowing learners to engage in meaningful modelling processes connected to significant real-world issues. While limited to a single institution, results suggest that structured, progressive exposure to problem-

posing activities effectively develop modelling task designing competencies even among pre-service teachers with no prior experience. Future research should investigate the long-term impact on classroom practices post-graduation.

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[119.] Students' metacognitive behaviour identified on the construction of mathematical modelling cycles

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The relevance of metacognition on goal-oriented activities, like modelling problems, has already been addressed in the literature (e.g Almeida & Castro, 2024; Castro & Almeida; 2022; Vorhölter, 2024; 2025). On the one hand, knowledge of cognition itself, as well as its regulation, has been offering contributions regarding the recognition that working on mathematical modelling is challenging for students (Frenken, 2021; Krüger, et al. 2020; Vorhölter & Krüger, 2021). On the other hand, facing challenges in relation to knowledge about the modelling process requires an instruction that includes information about the modelling process, such as that which can be presented through modelling cycles (e.g Ferri, 2018; Geiger et al. 2022). A modelling cycle is a schematic diagram showing mathematical modelling as a cyclic process and it is not only a theoretical model which characterizes the modelling process, but also a multi-purpose learning instrument for students and could even be a diagnostic instrument for teachers. So, a modelling cycle can act as both a conceptual and an instructional tool (Almeida & Silva, 2021; Ferri, 2018; Vos & Frejd, 2022). It is important, therefore, to pay attention to whether a modelling cycle allows students to think about their modelling process retrospectively on a metacognitive level. In this way we can investigate how students indicate metacognitive behaviour through the adoption of metacognitive strategies (Almeida & Castro, 2023). These strategies, in turn, may reveal metacognitive behaviour that includes planning, monitoring and evaluating the task processing when students construct modelling cycles, both before performing a modelling activity and after completing it. Focusing on this theoretical framework, the research question is: How can mathematical modelling cycles serve as a tool looking for students' metacognitive behaviour? The methodological approach aligns with qualitative research. The data were collected from modelling activities developed by mathematics preservice teachers who participated on modelling courses in two Brazilian universities. The analysis of students' reports, scenes from the videos of students work as well as the modelling cycles produced by the students enabled the identification of relationships between metacognitive behaviour and modelling cycles. Data analysis was conducted considering: theoretical background, the sorting of the data generating theoretical approximations; the description of modelling activities and the relationships established by students in the mathematical modelling cycles used before and after acting on modelling processes; the interpretation of how students expressed their metacognitive behaviour in the modelling cycles. This analysis has made it clear that to produce a modelling cycle, whether with conceptual or procedural characteristics, activated students' metacognitive strategies, allowing the identification of metacognitive behaviour that guided students' actions during the modelling activity.

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[120.] When should students start making assumptions: a thinking aloud study

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Students who are new to mathematical modeling tasks face a range of challenges — from reading and understanding the problem, making assumptions, mathematizing, and validating their solutions (Kaiser & Brand, 2015; Galbraith & Stillman, 2006; Stillman et al., 2013; Maaß, 2005; Schaap et al., 2011). It is well known that the cognitive processes involved in solving modeling tasks differ significantly from following the modeling cycle step by step (Borromeo Ferri, 2007; Blum & Leiß, 2005). Language proficiency also plays an important role in how students perform modeling tasks (Leiß et al., 2019).

This study explores the question of what activities students, aged 11 – 16, engage in when solving mathematical modeling tasks for the first time, and what difficulties they encounter.

We conducted a think-aloud analysis with 14 students, each solving two modeling tasks. We developed a coding scheme based on both the modeling cycle and observed student competencies and difficulties. Subsequently, we analyzed the resulting time-series data in R. Our findings reveal that students spent the most time on reading and the least time on validation. A common strategy was to reread the task in an effort to understand what was expected. Making assumptions emerged as the main source of confusion — and students typically postponed making assumptions until late in the process. Interestingly, students with stronger reading comprehension skills spent significantly more time making assumptions and experienced more confusion than their peers with weaker reading skills.

This study aims to shed new light on the complex role of assumption-making in the modeling process — and raises questions about how and when students are to be supported in this critical phase.

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[121.] A Misconception about Number and Quantity

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A proper understanding of the concepts of quantity and number is crucial in mathematical modeling processes. Mathematical modeling processes are recognized as cyclical processes (i.e. Haines and Crouch, 2007) that include converting processes of going back and forth between the real and mathematical worlds. At a deeper level of the conversion processes between the two worlds, we find that a conversion exists between quantities in the real world and numbers in the mathematical world. For example, the distance y km traveled by a car moving at 40 km/h in x h can be expressed as $y=40x$. In the example, distance, speed, and time are quantities. However, x , y , and 40 in the function can be regarded as variables and a constant that represent numbers because the total cost of x products purchased at \$40 each can also be expressed as $y=40x$, where the total cost of x products purchased at \$40 each is \$ y . Therefore, in the mathematical world, the function $y=40x$ is regarded as an abstract model of the relationship. However, it is unclear whether learners perceive this approach.

This study aimed to investigate learners' perceptions of the difference between a number and quantity. The target population comprised university students enrolled in teacher-training programs in Japan. Participants were asked to describe the difference between numbers and quantities. As a result, the typical response was that numbers are discrete and applied to countable objects, such as apples or people, and each is distinct, such as one or two. Meanwhile, quantities are continuous and applied to uncountable elements, such as weights and lengths. In fact, in a survey of undergraduate students enrolled in teacher-training courses, 94% of the students responded with an inappropriate understanding of the difference. However, the answers make it difficult to explain that rational and real numbers are also numbers, which may lead to misconceptions. In fact, the problem could be unique to the Japanese mathematics curriculum because in Japan, students learn the idea of a quantity in elementary school, but quantities are rarely dealt with after junior high school mathematics, when the concepts of natural and irrational numbers are learned. Of course, science always deals with quantities, but it is separate from mathematics, with few opportunities to deal with it in an integrated manner. Finally, we conclude that the curricula of more advanced mathematics, such as middle- and high-school mathematics, should be improved to enrich and better relate the perception of numbers and quantities.

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[122.] The importance of mathematics in artificial intelligence

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In the age of generative artificial intelligence (AI), early cultivation of mathematical literacy is vital to understanding AI systems (Zhang et al., 2025). The rapid advancement of generative AI, exemplified by GPT models, underscores the need to foster AI literacy beginning in high school and the first years of university (Relmasira, Lai & Donaldson, 2023). Foundational math concepts and modelling skills provide a natural gateway to AI algorithms and data-driven models (Lozano & Dominguez, 2025). Research also indicates that teachers' AI literacy and beliefs mediate the adoption of AI-based tools in math classrooms. Thus, developing AI literacy can yield benefits for both teachers and students (Wijaya et al., 2024).

One promising approach is modelling-based teaching, a form of contextual modelling (Kaiser & Sriraman, 2006) that embeds simple AI modelling tasks into mathematics learning. For example, guiding students to fit a straight line by adjusting slope and intercept simulates training a single-layer adaptive linear neuron. Learners observe how weights affect outputs and how error reduction improves models (Lozano & Dominguez, 2025). Such tasks make abstract AI mechanisms tangible and empower students to engage with algorithmic reasoning, drawing parallels to advanced systems such as how GPT predicts words based on learned patterns (Kalantzis & Cope, 2025). Activities may even incorporate generative AI tools at a “meta” level—for example, by prompting students to ask ChatGPT to explain an ADALINE and reinforce neural-network fundamentals. Each activity embeds a core mathematical concept within a simple AI task to simulate training an adaptive linear neuron (ADALINE). Moreover, this sequence of modelling-based teaching activities can be adapted from middle school to first-year university students.

Integrating AI through mathematical modelling yields multifaceted benefits: students strengthen both math and AI literacies by understanding how models are trained, how they generate predictions, and where their limitations lie. This comprehension empowers critical engagement with AI tools rather than treating them as black boxes. However, carefully crafted instructional materials are needed to guide educators in leveraging AI effectively as a pedagogical tool (Kalantzis & Cope, 2025). Since increased AI literacy and trust can also correlate with higher dependence on AI and potential declines in creative problem-solving (Wijaya et al., 2024; Zhang et al., 2025).

Despite the existence of such promising approaches, systematic searches across SCOPUS, IEEE Xplore, and Dimensions found only a handful of studies on mathematical literacy related to core AI concepts, highlighting a critical research gap. Specifically, Dimensions returned 208 results,

IEEE Xplore returned 8, and Google Scholar returned 6 for key terms in math education and AI literacy between 2023 and 2025. This study proposed to conduct a systematic literature review on this topic.

In conclusion, early incorporation of modelling-based AI activities into mathematics curricula equips learners with analytical skills and ethical awareness necessary for participating fully in the AI era. Through straight-line fitting exercises, neuron analogies, and generative AI partnerships, educators can demystify weight optimization, error minimization, and predictive modelling while cultivating resilient critical AI literacies. Grounded in empirical research, such instructional materials position mathematics education at the forefront of 21st-century learning, preparing students to engage thoughtfully and responsibly with generative AI technologies.

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[123.] Entering the next DIM²ENSION: How mathematical modelling in the context of sustainability can be supported by a digital device

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Mathematical modelling requires students to apply their mathematical knowledge in real-world situations (Blum, 2015; Niss & Højgaard, 2019). Particularly, it offers rich potential to connect STEM subjects from a mathematical perspective and to address sustainability issues in mathematics education (cf. Hallström & Schönborn, 2019; Maass et al., 2019; Siller et al., 2024).

In the presentation, we introduce the Erasmus+ Cooperation Partnership DIM²ENSION (2024–2027) focusing on mathematical modelling in the context of sustainability. At the centre of the project is (1) the provision of ready-to-use material in the form of tasks. To bring this idea into mathematics classrooms, (2) a digital device, technically based on the ASYMPTOTE system (Barlovits et al., 2021), has been developed and researched to facilitate the use of these tasks. Both aspects, the material-based and the technical development, will be addressed in the presentation.

Material-Based Development: During the project, an open-accessible database of peer-reviewed modelling tasks has been created. This database includes tasks for lower and upper secondary level, ranging from Fermi tasks to STE(A)M modelling tasks, as well as from short modelling activities to multi-day modelling projects. In order to support students' modelling processes, scaffolded support is provided for all designed tasks.

Technical Development: Recent literature ascribes many potentials of digital technology to support mathematical modelling (cf. Cevikbas et al., 2023; Siller et al., 2023). Given these potentials, we aim to support the preparation, implementation, and follow-up of modelling activities through the development of a digital device called DIM²ENSION. To do so, we provide a twofold system, consisting of a web portal for teachers and a corresponding mobile application for students. In the web portal, teachers can select tasks from the open database, monitor students' current work progress and receive advanced learning analytics on the modelling activities. The mobile application presents the task formulation to the students and provides needed resources. Moreover, it allows two different work modes, i.e., a free working mode for more experienced learners and a scaffolded working mode for learning how to model. Whichever

mode is used, the application encourages students to validate their results and reflect on sustainability issues.

The presentation and the paper will focus on the technical and material developments of the DIM²ENSION project. In particular, the first prototype of the DIM²ENSION device including task examples will be presented.

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[124.] Why model through machine learning in upper secondary mathematics? A Danish case study

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In recent years an emphasis has been put on what it means to educate students for critical and responsible citizenship (Geiger, 2024), and if mathematical modelling in schools can be said to relate to modern modelling practices in scholarly contexts (Frejd & Bergsten, 2016). Data science has gained attention at all educational levels, but it is still an open question, if it assures develop students' concepts, attitudes and knowledge regarding machine learning and algorithms, in particular for primary and secondary education (Fleischer, Podworny & Biehler, 2024). Boaler and colleagues (2024) argues that upper secondary mathematics courses should reflect the dominant role of data science in societal decision making. This is supported by other researchers arguing that more specifically machine learning has such a dominant role in the data-driven world we live in, that it should be taught in school (Martins & von Wagenheim, 2022; Martins et al., 2024). Still, mathematicians are raising concerns how these new trends align with current need for mathematical content knowledge, preparing students for further education in science and technology (Conrad & Mazzeo, 2022). This presentation pursues the following research question: how can machine learning be taught and learned in upper secondary mathematics and explicitly linked to mathematical modelling?

We revisit a study implementing machine learning in ordinary upper second mathematics education in Denmark. The study strived to create explicit linkages to curriculum requirements, students' prerequisites and drew on examples of modelling from the real world (Tarp & Frantzen, 2023). The original study aimed to teach machine learning through active learning and inquiry (Artigue and Blomhøj, 2013), instead of the traditional "use-modify-create" approaches from informatics (Martins & von Wagenheim, 2022). The theory of didactic situations (Brousseau, 1997) was originally the theoretical background for developing and analysing the teaching activity. Revisiting the study to analyse the potential of machine learning as a way to teach mathematical modelling, we draw on the Anthropological Theory of the Didactic (ATD, Chevallard, 2019, Jessen & Kjeldsen, 2023).

We analyse a situation where students are to determine gender of a person based on prior data and the methodology of kNN (k nearest neighbour) conducted in the program Orange Data Mining (<https://orangedatamining.com/>). We analyse the situation through the Herbartian schema as well as question answer dynamics, to discuss the potentials and limitations of doing mathematical modelling through machine learning. The explicit need for scaffolding when engaging non-computing students in light programming is evidently a challenge. Still, the students' engagement with the designed milieu provided to them with autonomy and they were able to carry out productive inquiries, correct own mistakes and gain knowledge about the modelled system. They explicitly linked their analyses with prior learning and made sense of notions like residuals from their initial work with linear regression (Tarp & Frantzen, 2023). We conclude that there are rich potentials of teaching students critical and responsible citizenship

(Geiger, 2024) drawing on elements of machine learning and light programming within existing constraints and conditions of upper secondary mathematics in Denmark.

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[125.] Relationship between pre-service teachers' flexibility and performance in solving Fermi problems: does the setting make a difference?

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Fermi problems (FPs) pose a real situation, without data, requiring an estimate that needs the development of a mathematical model (Ärleback, 2009). They are suitable for introducing mathematical modelling in primary school classrooms, as they require prior knowledge – measurement of magnitudes, proportional reasoning – accessible to students at this level (Segura et al., 2025). Despite this, FPs are hardly introduced in primary education. Previous research indicates that some pre-service teachers can improve their performance: they have shortcomings in their prior knowledge (Segura et al., 2025) and make errors when solving FPs (Segura & Ferrando, 2023).

FPs are open-ended and can be approached using multiple strategies (Ferrando et al., 2021; Montejo-Gómez et al., 2024). Segura and Ferrando (2023) define flexibility across a sequence of FPs as the ability to switch strategies from one problem to another depending on certain contextual features. Segura and Ferrando (2023) found that pre-service teachers with greater flexibility made fewer errors and therefore had higher performance. Subsequent studies (Segura et al., 2025) have shown that the relationship between flexibility and performance is robust when FPs are solved in the classroom.

Nevertheless, FPs can also be solved outside the classroom, and several studies have found that the problem-solving setting affects performance. Hence, Buchholtz (2017) and Jablonski (2023) have shown that working outdoors enhances the development of mathematical models compared to evoking the problem indoors. Segura et al. (2023) found that experimenting outdoors promotes the pre-service teachers' success in completing their strategy for solving FPs and facilitates their perception of contextual features. However, beyond the outdoor setting, another problem-solving setting with huge potential has emerged, Artificial Intelligence (Noster et al., 2024). Despite this, little is known about how an AI setting would foster competence in solving modelling problems (Spreitzer et al., 2024). In that sense, Gallart et al. (2025) found that pre-service teachers assisted by ChatGPT tend to delegate part of the solution process to AI, which could affect their flexibility and/or performance.

Consequently, considering these three settings (evoking indoors, experimenting outdoors, or querying ChatGPT), a research question not yet addressed is the following:

(RQ) Does the setting influence the relationship between flexibility and performance of pre-service teachers solving a sequence of FPs?

To answer this question, we conducted a study with 137 pre-service teachers from the Teacher Training Faculty of the Universitat de València (Spain), distributed among the three settings: evoking indoors (n = 55), experimenting outdoors (n = 41) and querying ChatGPT (n = 41). Participants individually solved a sequence of four FPs set in a context close to their faculty and

familiar to all of them. An analysis and coding of participants' flexibility and performance allowed us to correlate these variables, differentiating by setting. The results show that, although flexibility and performance are significantly lower evoking indoors, the relationship between the two is significant. The relationship between flexibility and performance also holds, even more strongly, when experimenting outdoors. However, the relationship between flexibility and performance is weak and not significant when querying ChatGPT. These results have implications for the design of training to ensure that teachers develop competence (flexibility, performance, etc.) in solving modelling problems and can implement them successfully in primary education.

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[126.] What cue lead students to begin mathematizing in a human-centered way? — An exploration through the example of gifting oranges to relatives —

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The PISA2022 has shown that students need to be able to make judgments and decisions based on evidence. For this reason, I think it is also important to use teaching tasks that involve student's decision making in mathematical modelling. However, there are several studies that have revealed the difficulties of using tasks that involve decision-making in mathematical modeling (e.g. Jung and Lee, 2021; Frejd, 2017; Ng, 2011). From these previous studies, it was found that the difficulty of being aware of both the mathematical world and the real world when making decisions in mathematical modeling remains a problem. This challenge highlights the importance of promoting a human-centered approach to mathematization, where individual values and perspectives influence the mathematical modeling process, encouraging students to recognize that they can shape how they interpret and apply mathematical concepts in real-world contexts. Therefore, the aim of this research is to investigate and implement lessons that not only help students make decisions using mathematical reasoning while taking into account real-world considerations but also emphasize how students' personal values and perspectives can influence their mathematization process. The goal is to clarify the results and issues involved in a case study, with a focus on fostering a deeper, more personalized approach to mathematical modeling.

The task deals with a familiar situation of gifting case of oranges to relatives. Assuming that they buy them online, students make decisions based on their own reasoning from among the five sites presented in response to the question “Which site do you think is appropriate to make everyone HAPPY?”. In addition, they are given a budget and the number of relatives as preconditions.

Before the main task, we conducted a 15-minutes survey with 12 seventh graders (ages 12-13) to explore the criteria they use for decision-making. The survey followed this structure: (1) the teacher explained the context, (2) the teacher explained his concerns and asked the students to think about them, and (3) the students independently worked out the answers on their own. Students independently solved the task without teacher intervention, with the teacher only answering questions.

The results showed that students focused on the number and price of oranges but ignored the preferences, personalities, or positions of their relatives. By considering these factors, we believe a more diverse approach to mathematization could emerge. Ikeda (2007) discusses how taking into account family members' preferences can lead to various ways of mathematizing, such as when dividing a cake. This study emphasizes the importance of incorporating relatives'

personalities and preferences into problem-solving. The research question is: How can teachers encourage students to consider these factors when solving mathematical problems?

To address this research question, we make two assumptions and will conduct a class based on them. First, we aim to intentionally provoke differences of opinion. The teacher will ask, "If you were in the position of receiving the oranges, what kind of oranges would you like to receive?" This question is designed to bring out a variety of responses, such as "I want to eat lots of oranges" or "One orange per person is enough, isn't it?" These differing viewpoints can help students recognize how people's personalities influence mathematizing.

The second assumption focuses on the context and presentation of the task. The task in this study requires students to collaborate with the teacher in solving a problem, and it is believed that the extent to which students engage in problem-solving depends on how much they perceive the teacher's concerns to be genuine. Therefore, the context and presentation are key factors in helping students understand the importance of considering human factors when solving mathematical problems.

As the teaching practice is scheduled to be held in June, the actual class and discussion will be reported at the time of the presentation on the day.

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[128.] A Case Study Emphasizing the Importance of Idealization in Mathematization of Real-World Phenomena for Pre-Service Teachers: Focusing on Application of Fermi problems

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This study focuses on the concept of idealization, which is important in mathematical modelling (Kaiser, 2014). The research question of this study is whether cognitive and emotional improvements can be achieved, and what challenges arise. This is examined through a case study involving pre-service teachers (hereafter "students") who do not necessarily specialize in mathematics. Based on the practices of natural science experts and the understanding of mathematical models, we have identified the following three areas where improvement is expected: the first point is that idealization in mathematical modelling involves the intentional use of "deliberate falsification". This relates to the fact that the primary motivation for experts to use idealization in mathematical modelling is to make it mathematically manageable (Pincock, 2006; Batterman, 2009). The second point is that the elements to be idealized are selectively chosen. Mäki (2020) mentions that most models exclude various elements of the real world and focus on the relationships between specific elements. This suggests that the degree and nature of idealization produce different models depending on the creativity and perspective of the problem solver. The third point is that, according to "de-idealization thesis", a model is considered more explanatory if it passes through a process of de-idealization (Jebeile & Kennedy, 2015).

This case study was conducted as part of the "Introduction to Arithmetic" lesson, with one of the authors serving as the instructor for 60 minutes. To help students understand the characteristics of idealization from the above three points, we have chosen Fermi problems as the topic for our case study. The reason is that Fermi problems emphasizes the need to make various assumptions in the process of organizing a complex reality and narrowing down the necessary conditions. We thought this approach would effectively emphasize the significance of idealization. Fermi problems focuses on the need to make various assumptions in the process of organizing complex realities and narrowing down the necessary conditions (Ärlebäck, 2009; Albarracin et al, 2022). By eliminating complex elements of reality, students are expected to recognize the existence of idealization and focus on the characteristics of the act of idealization among the multiple models presented.

In the class, students were made aware of the importance of setting conditions for Fermi problems and were asked to consider what combination of conditions is necessary to approach real-world situations. Furthermore, at the end of the class, students were encouraged to review textbook problems and identify which parts had already been idealized. This process corresponds to de-idealization and is intended to provide an opportunity to understand how mathematical problems simplify reality.

After the class, students are encouraged to reflect on their learning and write down their thoughts. The students' responses are analyzed using content analysis, categorizing them into three

perspectives: (1) how the three characteristics of idealization are described, (2) the type and appropriateness of examples required for problems involving idealization, and (3) emotional improvements in students through activities. Following the initial content analysis, thematic analysis is conducted to identify new themes beyond the predetermined categories and provide characteristics of the descriptions.

In summary, this case study confirmed that participants had diverse interpretations of the intentional use of deliberate falsification and the selective setting of idealized elements. It also suggested that experiencing the process of de-idealization may provide an opportunity to deeply understand the significance of idealization.

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[129.] Authenticity in vocational mathematics exam tasks in Norway.

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In vocational education mathematics often aim to be connected to workplace settings (FitzSimons & Boistrup, 2017). However, a task in a school setting cannot be fully authentic (Vos, 2011). Exams show what is valued and measured in the curriculum (Burkhardt and Schoenfeld, 2018). Bjørnset et al. (2020:01) argued that the Norwegian 10th grade mathematics exams had issues with high reading skills needed, and a high use of low-frequency words. Illustrations were most often not needed to solve the tasks. In vocational programmes, in 11th grade, some students must do a paper-based mathematics exam. The mathematics course is connected to the students' future vocation, and as argued in Sundtjonn (2025), the students authenticated several vocational aspects of tasks in a classroom situation. My aim here is to find if and what kind of authentic aspects that were present in paper-based vocational mathematics exam tasks.

Schlüter and Besser's (2024) Model for Authenticity in Modelling Test Items (MAMTI) framework is made for analysing authentic aspects within paper-based mathematical modelling tasks. MAMTI contains the categories real-world context, event, objects, data, question/assignment, figures, use of mathematics and purpose (Schlüter and Besser, 2024). Here I discuss the results of analysis by using MAMTI of exam tasks from two vocational mathematics courses connected to the vocational programmes Building and Construction (BC), and Restaurant and Food Processing (RF).

I analysed total 12 exam sets, 6 from each vocational programme, from 2022, 2023 and 2024. I analysed the tasks intended to be connected to the students' vocational programmes. One task example is *the drawing [above] shows the front of a building. Nora is told that the scale of the drawing is 1:50. She measures the wall with a measuring tape, which shows that the wall is 8000 mm long. When she measures the drawing with a ruler, the wall is 16 cm long. Do the calculations and check if the scale is correct.* (The Norwegian Directorate for Education and Training, 2023).

My results show that the exam tasks had authentic aspects with reference to categories in the MAMTI framework, *real-world context*, *event*, *objects* and *data*. The *illustrations* were in some tasks authentic, and in some tasks not authentic. The exam tasks from BC and RF differs in the regards of the illustrations. The illustrations in the RF tasks were mostly not needed for the solving of the task. However, in the BC tasks the illustrations were often necessary to solve the tasks, and relevant information were contained in the illustrations.

Regarding the *question/assignment*, would the question be asked this way in the real-world, most of the tasks are coded as not authentic. The aspect *purpose* of the tasks, do the work done fulfil a need in the real world, is though often coded as authentic. The probable *use of mathematics* to solve the tasks is often coded as not authentic. Therefore, an interesting issue is which kind of solution methods and tool use is expected and encouraged in the exam tasks.

I argued in Sundtjønn (2025) that students' expertise from out-of-school communities can be used for authenticating task aspects in a classroom setting. However, here the results that the question/assignment and use of mathematics often seems be not authentic aspects, and I therefor argue the vocational expertise can be seen as a distractor for the students' possibility to solve the exam tasks in ways valued by examiners.

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[130.] **Challenges in Implementing a Technology-Integrated Mathematical Modeling (MM) Course: A Mentorship-Based Collaborative Study**

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Technologies that support computational modeling enhance learning across disciplines by fostering systems thinking and engaging students in authentic modeling practices (Wang et al., 2021). Such tools have been widely applied in modeling-based instruction across science domains. While these technological advancements present promising opportunities to increase student engagement and deepen conceptual understanding, they also introduce distinct instructional challenges—particularly when combined with the inherently complex nature of MM. Navigating these challenges requires both pedagogical flexibility and professional development among mathematics teacher educators (MTEs). One promising approach for fostering such development is collaborative self-study, wherein educators jointly reflect on and analyze their teaching practices. This method supports professional growth through shared experiences, critical dialogue, and mutual feedback (Kastberg et al., 2020; Pocalana and Robutti, 2024).

The present study contributes to this growing body of research by investigating the implementation of a technology-integrated MM course co-developed and taught by a senior and a junior mathematics educator. Specifically, it explores the instructional challenges encountered and examines how mentorship shaped their professional development throughout the course. The aims of this study are twofold: (1) To explore the challenges instructors face when implementing a technology-integrated MM course. (2) To examine how a senior researcher mentors a junior researcher during the instructional process.

This study used a qualitative research approach, specifically a collaborative self-study. Two experienced MTEs, each with over three years of experience and a research focus on MM education, collaboratively designed and implemented the course. Despite teaching independently, the two instructors held weekly Zoom meetings to reflect on course progress, instructional strategies, and emerging challenges. These collaborative discussions formed the foundation for the data collection and analysis process in the study. Each meeting was recorded verbatim and the transcripts were used as the main dataset for descriptive analysis.

Instructors encountered various student-related obstacles, such as insufficient mathematical foundations, unfamiliarity with digital tools like GeoGebra and Excel, hesitancy toward group collaboration, and infrastructural limitations that hindered effective group work. These challenges were amplified by students' limited prior exposure to MM with technology, creating difficulties in engagement and classroom management. Additionally, pedagogical challenges—such as developing appropriate assessment tools, managing instructional time, and gaining confidence in facilitating tech-enhanced lessons—emerged during course implementation. A structured mentoring relationship between a senior and junior educator proved instrumental in addressing these issues. The senior instructor offered strategic guidance on fostering inquiry-

based learning, organizing collaborative activities, and managing tech-rich classrooms. In return, the junior brought in innovative ideas (a new learning management system) for assessing MM processes and student engagement. This bidirectional exchange not only supported the junior's development but also encouraged reflective practice for both educators, emphasizing the value of mentorship in complex instructional settings.

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[131.] Modeling moral change: A case study in developing explanatory and predictive mathematical models

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The use of mathematical models is ubiquitous in the natural sciences, and economists rely on them for diverse applications. However, compelling examples of mathematical models that successfully describe sociological and psychological phenomena remain less common. This lecture will explore one such example, stemming from our work at the Institute for Futures Studies, where my colleagues and I have developed a framework for understanding and predicting changes in public opinion on moral issues.

This model, refined and validated across numerous studies, is notable for its intentional simplicity. It's remarkable performance against empirical data underscores a key principle in modeling: complexity is not always a prerequisite for insight.

In this talk, I will first present the core modeling principles and the considerations that guided the development of our basic model of opinion change. I will illustrate how this foundational model can be extended for more nuanced predictions and discuss the methods we've employed to test its validity. I think this journey of model creation, application, and validation offers rich insights for the teaching of mathematical modeling. It provides a concrete case study on translating complex social dynamics into mathematical terms, the iterative process of model refinement, and the power of parsimonious approaches.

[132.] **Students Conceptual Understanding in Exam Tasks with Models in Danish High Schools**

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In Danish high school mathematics, modelling competence is a goal and a didactic means to enhance students' understanding of mathematical concepts. Particularly, this is the case in written tasks where students apply and analyze mathematical models. Such tasks are common in exams and thus play a crucial role in teaching. This article analyzes students' progression from variables and functions to derivatives, based on 232 exam tasks and 2552 written responses from 11 students in a mathematics B-level course.

The article contributes by developing a theory-based model to uncover students' conceptual understanding. The model is constructed through a synthesis of Sfard's model of concept formation (Sfard, 1991) and the epistemological triangle (Steinbring, 1989, 2005). The model enables a nuanced picture of concept formation by considering the significance of representations.

A methodological framework ensures consistent assessment of tasks and responses, providing a solid basis for comparison (Kvale, 1980; Schoenfeld, 2002; Qvortrup & Ljungdahl, 2019). Morgan & Sfard (2016) provide linguistic parameters for assessing written responses and encapsulate how students express their understanding (Sfard, 2008). Additionally, criteria for the assessment of tasks are based on the three stages in Sfard's model and its relation to central representations. These criteria are inspired by Blomhøj (2016), Knuth (2000) and Sfard (1992). The model, frameworks and results are further elaborated in the article.

With a quantitative treatment, deviations can be identified (Køppe & Helles, 2014; Lemnitzer & Zinsmeister, 2015). One deviation is tasks with a switch in representations, e.g., setting up a model based on a context. The understanding of modeling and the transition from text to mathematical symbols is problematic (Blomhøj, 2006). Also, the meaning of learned algebraic processes in graphical representations is problematic, as seen in Knuth (2000). This insight suggests that teaching should use different representations to assist students in relating their processes to a structural understanding.

It appears that concepts are mainly identified with one representation, typically algebraic, and when the assignment is in another context, the epistemological triangle collapses, leading to the vicious circle in Sfard's model. Reification of a concept presupposes that each representation is an object for the student, according to our extended model of reification.

To summarize:

- Students exhibit a strong process-oriented approach, where communication and dissemination promote structural understanding.
- A conceptual understanding within one representation is not necessarily transferable to another representation.

- It is easier to solve tasks based on existing models with complex functions than to set up simple models based on a realistic context.

Our model is inspired by a thesis that students in their acquisition of concepts may be at different stages in the reification process depending on whether they are working with an algebraic or graphical representation. The analysis confirms this thesis and shows that written tasks with models provide a basis for uncovering students' conceptual understanding. Based on the results, we propose a new thesis; increased focus on students' written work with models with accompanying feedback can be a didactic means to strengthen their conceptual understanding within the domain of differential calculus.

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[133.] Assessment of Mathematical Modelling in Digital Portfolios – What Competencies Can Be Identified?

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The assessment of mathematical modelling competencies has been a central yet challenging issue in mathematics education research and practice (Frejd & Vos, 2022). Traditional written assessments often struggle to capture the process-oriented and creative dimensions of modelling (Lu et al., 2025). Alternative assessment formats such as portfolios, which had already been proposed as a response to these challenges (Frejd, 2013), have since gained increasing attention for their potential to comprehensively and contextually capture students' modelling competencies.

Building on current discussions on the conceptualization and measurement of modelling competencies (Cevikbas et al., 2022; Schukajlow et al., 2023; Frejd & Vos, 2022), this study explores the potential of digital portfolios as an assessment format in lower secondary mathematics education. Specifically, the research question is: what competencies can be identified and assessed through digital portfolios? The study is situated within the AIPMo project (Alternative forms of assessment in mathematical modelling, in German: Alternative Prüfungsformen beim mathematischen Modellieren), which was implemented with 144 ninth graders in five classes. As part of the project, students solved mathematical modelling problems and developed digital portfolios. These portfolios included documentation of students' modelling processes, individual reflections and solutions that were created using the dynamic geometry software *GeoGebra*.

The analysis follows a qualitative content analysis approach (Kuckartz & Rädiker, 2023), combining deductive and inductive coding procedures. The deductive framework is based on established sub-competencies of mathematical modelling, such as simplifying, mathematising, working mathematically, interpreting, and validating (Kaiser & Brand, 2015; Hankeln et al., 2019). Inductive coding complemented this framework to identify additional competencies and portfolio-specific characteristics reflected in the students' work. In the conference presentation, findings will provide insights into which aspects of students' modelling competencies are made visible and to what extent they become evident in digital portfolios. The findings aim to contribute to the ongoing discourse on assessment formats for mathematical modelling and to inform future practice regarding the implementation of digital portfolios in mathematics education.

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[134.] Inside the Modeling Mind: A Dynamic Framework for Cognitive Processes in Mathematical Problem Solving

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Mathematical modelling is a cognitively demanding activity that requires students to integrate multiple processes, such as interpreting contextual data, activating mathematical knowledge, constructing formal representations, and validating results (Ferri, 2018). While existing frameworks, - such as the modelling cycle proposed by Blum and Leiss (2007) - describe the procedural stages of modelling, few attempts have been made to formalise the underlying cognitive dynamics that unfold during the process.

This work presents a novel mathematical model that sheds light on how six key cognitive variables - fatigue, comprehension, modelling quality, problem-solving, validation, and knowledge - evolve over time as students engage with modelling tasks. The model is expressed through a system of nonlinear differential equations, where each cognitive variable evolves over time in response to learner-specific traits - such as abstraction ability, capacity to learn from feedback, and resilience to cognitive load. The model integrates theories from multiple domains: the iterative modelling cycle (Blum & Leiss, 2007); Sweller’s (1988) Cognitive Load Theory, which links overload and fatigue to learning failure; Flavell’s (1979) concept of metacognition, crucial for validation and error recovery; and Van Geert’s (1991) dynamic systems approach to cognitive development. The framework is further enriched by developmental insights - particularly regarding the emergence of abstraction and formal reasoning in adolescence (Piaget & Inhelder, 2013) - and by research on the disruptive effects of math anxiety on cognitive load management (Ashcraft, 2002), contributing to a coherent epistemological foundation.

Simulating different learner profiles reveals complex and nonlinear dynamics among the cognitive variables. For example, fatigue impairs comprehension and suppresses validation, whereas knowledge grows only when cognitive load is effectively managed and reflective processes are active. Procedural learners, despite strong initial knowledge, often fail to consolidate or deepen their understanding due to low abstraction ability and limited metacognitive control - preventing them from transforming technical competence into effective modelling. In contrast, strategic learners tend to progress steadily thanks to a balanced regulation of effort, abstraction, and validation.

By simulating diverse cognitive profiles (e.g., varying sensitivity to load or resilience) the model allows researchers to explore which types of students might benefit from specific strategies, such as structured pauses, metacognitive prompts, or reflective phases, based on their internal dynamics.

In unifying cognitive, metacognitive, and emotional dynamics into a formal, time-sensitive model, this work offers a powerful conceptual and computational tool for research, instructional

design, and the development of adaptive learning systems responsive to students' real-time cognitive states.

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[135.] Using Mediated Writing to Facilitate Communication in Mathematical Modelling Class

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Introduction

Communication is essential for sharing ideas and deepening understanding. It allows for reflection, refinement, discussion, and the public dissemination of ideas (NCTM, 2000). In mathematics, effective communication is crucial—without it, even the most sophisticated computations may remain inaccessible or unusable. Recognising this, educators and policymakers have emphasised the need to integrate communication skills into math instruction, believing it enhances students' learning and comprehension (Khadka, 2024; Serio, 2014).

Preparing students to be communicators of mathematical ideas is often overlooked in K-12 (e.g., Forrester et al., 2008) as mathematics teachers mostly regard teaching communication as a language teacher's responsibility rather than a mathematical one. For example, Linhart (2014) noted that “writing and communication are often the last thing that instructors think about incorporating into a mathematics course.” Mathematical modelling—where math is applied to real-world situations—offers a powerful context for students to develop and articulate their mathematical thinking (Linhart, 2014). In response, we have designed a structured sequence of activities that embed meaningful communication within a modelling curriculum for secondary students. Our talk will highlight the development of a mediated writing strategy, specifically the use of a *Modeling Memo*, and share preliminary findings on its implementation in the classroom. A Modeling Memo is a written document that details the modelling process.

Activity Design & Method

Drawing on the Activity Theory approach (Leontiev, 1978; Vygotsky, 1978), we developed a *Modeling Memo* as a mediated writing learning tool to promote mathematical communication and support students' engagement within the modelling process. In our study, mediated writing is a technique in which students refine their understanding by evaluating and improving upon provided, and yet incomplete, modeling memos, thereby producing more comprehensive documentation of their modelling process. Students then leverage their own criteria for what should be in a Modeling Memo to interpret and critique sample solutions to a modelling problem, and write and revise their own memos as they communicate about modelling approaches and potential solutions with their peers. Our intervention is grounded in constructivist teaching philosophy, which emphasises student autonomy in learning (Akpan & Beard, 2016). Specifically, we align with two key constructivist principles (Howe & Berv, 2000; Zadjia, 2021):

1. Learning is an active process, with experiential learning playing a central role.
2. Learners are self-regulated, constructing and monitoring their own understanding, with metacognition being crucial for meaningful learning.

By critiquing incomplete *Modeling Memos*, students identify aspects of the solution that require improvement. Through this engagement, students develop their own criteria for what constitutes

an effective Modeling Memo, which they then use to guide the drafting and revision of their own memos. This analysis draws them into the modelling process, prompting them to refine a solution and document their strategies as they progress.

Findings and Discussion

Classroom implementation revealed that students leveraged the memo as a central, iterative tool for modelling that supported reflection, revision, and deeper engagement with the task. For example, as students identified additional constraints, they updated their memos to reflect emerging ideas, and the act of writing often led them to generate new modelling approaches to explore.

Students also demonstrated awareness of this iterative process. In interviews, they explained how writing helped them to identify flaws in their models, and that they would revise their memos as they worked. The memo evolved from a reporting tool into a meaningful part of the modelling workflow, supporting both communication and critical thinking.

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[136.] Epistemological aspects of teaching mathematical modeling to biology students: do today's approaches meet tomorrow's needs?

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Mathematics plays an increasingly important role in life sciences (Cohen, 2004), yet biology education does not offer the amount needed (Jungck, 2011). Main challenges in mathematical biology are related to formulation, analysis, and comparison or fitting models to data (Gyllinberg et al., 2023); all these activities are difficult for students (Rogovchenko & Rogovchenko, 2024a, 2024b; Rogovchenko, 2021). In this paper, we discuss two important epistemological aspects that require a critical revision of approaches to teaching of mathematical modelling.

Many biological systems are described by ordinary differential equations. Classical examples brought by mathematicians into mathematics courses include logistic growth and predator-prey models. In a quest for “a small number of universal models which explain as many biological phenomena as possible” (Gyllinberg et al., 2023, p. 4), mathematicians enthusiastically teach them, unaware of biological limitations (Jungck et al., 2020). The first epistemological challenge for mathematicians is to accept that “the nuances of what is a persuasive model to biologists may not be the elegance of the mathematics nor the power of a prediction for a specific case, but may instead be its power for developing new experimental designs, identifying different factors to measure or phenomena to observe” (Jungck, 2011, p. 3). Starting with idealised mathematical models, students conduct experiments, collect data, and create empirically driven visions of biological systems. Since data-driven models “are just representations of the data itself, rather than insights into that data” (Gyllinberg et al., 2023, p. 6), students should link them to concept-driven models. This often requires a deliberate use of false or contradictory assumptions, and several iterations for understanding of what and why does not fit the expected biological description.

When scientists study open-ended complex systems, “there is no uncontested way of telling” what components included in a model are crucial and what are irrelevant (Gyllinberg et al., 2023, p. 2). An increasing need to “process, visualize, and interpret huge quantities of multi-dimensional data” calls for “innovative models that may include nonlinear, multi-causal, multi-scale, or stochastic components” (Jungck, 2011, p. 2). The second epistemological challenge regards the shift from ‘universal models’ capturing most of a system’s dynamics with a minimal number of variables to sets of models, each viewing a complex system through a different lens. The details now are less important since the global picture combines complementary approaches that mutually inform each other, as shown by Enderling et al. (2020) who implemented a “multi-model, multi-scale approach” involving both mathematics disciplines (calculus, linear algebra, discrete mathematics) and biological subjects (genetics, immunology, molecular and cell biology) to address specific questions for different forms of cancer.

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[137.] A practitioner's giving meaning to differentiation in a mathematical modelling class through teaching tactics

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Modelling commonly occurs in classrooms in small groups. Heterogeneous groups are seen as providing support for individual group members in modelling as they offer a diversity of ideas and backgrounds, with opportunities for safe voicing and discussing ideas, concerns and challenges through social within group interaction. Modelling tasks are considered self-differentiating (Borromeo Ferri, 2018) allowing all students to access and pursue different pathways as they bring what mathematical and real-world knowledge, they activate in the moment, to bear on understanding the situation they are modelling in pursuit of a shared modelling goal. When autonomous modelling is the teaching goal, teachers are often advised to intervene minimally; however, situations arise where this intention appears to be overridden by a strongly felt professional obligation to differentiate their practice for particular student groups. Considering the mix of teaching tactics (Stillman & Brown, accepted) and the teachers' purposes in using them in these circumstances is of relevance to modelling related professional development. The research questions addressed in this qualitative case study are:

(1) *What was the teacher's intention in differentiating his practice?* (2) *Which teaching tactics did he use to differentiate his practice for the students he perceived needed this?*

Data were purposively selected from video recorded modelling implementations by a Year 10 teacher who was teaching a problem solving and modelling elective class that ran for one semester. We compare and contrast the practices the teacher used for one student group with what occurred for other groups for tasks conducted early and late in the semester and examine responses to pre and post lesson teacher interviews and post lesson student interviews.

Rather than presenting the teacher's actions as conflicted practice for modelling from trying to appease the competing demands of allowing autonomous resolving of impasses versus facilitating for all students, no matter their capabilities, a successful modelling experience of a challenging real- world situation, we articulate his practice from the perspective of integrating different demands simultaneously. Implications for teacher professional development in modelling practices include the laying bare of teacher's intentions as they wrestle with the interplay of different demands in the classroom unfolding in the moment. The purposes of using a different mix of teaching tactics may help teachers who struggle with the notion of providing minimal help with challenging tasks such as modelling tasks and allowing all students to thrive mathematically by using such tasks.

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[138.] Teachers Connecting Compassion and Empathy to Dilemmas Related to Social and Environmental Justice Decision-Making

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The challenges of the Anthropocene ask of us as educators to find ways that support students in understanding and generating ways to respond to the impacts of disruptive phenomena (Watson & Smith, 2022). Disruptive phenomena, such as global warming and systemic social injustice, lead to consequences that include the displacement of individuals and communities from homes and homelands, poverty, and unemployment. Responding effectively and responsibly to the impacts of disruptive phenomena requires citizens to develop informed judgements and make prudent decisions (Geiger et al., 2023). Decisions about how to respond often rely on models and modelling to gauge the cost and time to recover from a disruptive event. They must also, however, accommodate other factors, for example, the effects on families that must move from where they live to avoid further flooding. These factors often required community leaders and advisors to think beyond financial aspects of a problem alone to find ways to address the greater good for families and communities at large (e.g., Geiger, in press).

Effective decision-making about responses to disruptive events involves the capacity to empathise (understanding others' experiences/perspectives) and feel compassion (concern for others' suffering and desire to alleviate it) (Geiger et al., under review). Beswick (2018) has further argued that the entailment of emotions and emotionally charged beliefs suggests drawing upon students' empathy and compassion has potential to enhance their mathematics learning. There appears to be little advice in the literature, however, related to how teachers can introduce empathy and compassion into mathematics classrooms when engaging with tasks related to disruptive phenomena. Engagement with such tasks must include pedagogical approaches that support and maintain sensitive conversations about the challenges faced by those being disadvantaged, in addition to solutions that take account purely mathematical perspectives. In seeking to generate insight into the dilemmas and tensions inherent within modelling solutions that embraces empathy and compassion, we draw on data from a larger, nationally funded project in Australia on enabling students' Critical Mathematical Thinking (CMT). These data were generated through student engagement with a task related to the current Australian housing crisis. Participants consisted of four teachers and their intact classes of Year 10 students. Tasks were constructed to align with the dimensions of Geiger et al.'s (under review) critical mathematical thinking framework — mathematical capability, critical capability, evaluation capability, reasoning capability and dispositions.

In this presentation we focus on the actions of teachers aimed at promoting empathy and compassion in concert with the mathematical modelling needed to inform decision-making about the influence of short-term rental arrangements, such as AirBnB, on the Australian housing

crisis. We conclude the presentation by speculating on what additional teacher led actions may have further supported this aim, including aspects of task design and approaches to instruction.

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[139.] Using an index to trace the long-term trajectory of a Continuous Professional Development project to enhance achievement in a high-stakes school mathematics examination

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In a similar vein that Ärlebäck and Frejd (2025) deal with rankings as models, this presentation focuses on indexes as models. Drawing on Giordano, F. R., Fox, W. P., & Horton, S. B. (2013) an index is taken “as a mathematical construct designed to study a particular real-world system or phenomenon.” Indexes are widely used for tracing changes in social phenomena such as, house prices and levels of poverty. A well-known index is the Consumer Price Index, described by Flaspohler, Mastrianna & Pulskamp (2012) as “an “index”—a mathematical concept used to measure over time the change of quantities in Economics and other fields.” (p 355) and that “the concept of *index* gives an indication of impact.” (p. 356). Calling an index a mathematical concept is indicative of it falling under the area of teaching and learning of modelling/applications. The focus is on the impact of a continuing professional development (CPD) project to enhance the achievement of school mathematics results in a high-stakes national examination.

The research question pursued is: “What is the impact, in terms of an index, on the implementation in classrooms of the issues dealt in a specific CPD project over an extended period of time?”

The index used is based on the Human Development Index (HDI) (2009), which is a normalised index. The index for this research is:

$$\frac{\text{Achieved Score}}{\text{Maximum Score of the Cohort of Schools for the Specific Year}}$$

It differs the HDI in that for any examination a minimum achievable score is zero. The extended period is from 2015 to 2024.

The CPD project of import is concerned with the enhancement of scores in a high-stakes mathematics examination—an examination that has direct consequences for the examinees. The examination the National Senior Certificate: Mathematics in South Africa. It is written at the end grade 12. The examination is externally set and scored, with practicing teachers having virtually no input in the setting of the examination. Nor do they participate in the administration of the examination in their schools. The examination is written at the culmination of the non-compulsory phase of schooling in November. Teachers participate in the CPD sessions voluntarily and they teach in schools in low socio-economic status (LSES) areas. The learners are also from LSES environments. The data used for constructing the indexes were sourced from publicly available documents released by the South African Department of Basic Education every year. Scores for individual learners are not provided.

It was found that the indexes increased minimally over the extended 10-year period

The findings are interpreted using Bhaskar's (1979) Transformation Model of Social Activity (TMSA). The TMSA purports that for transformation both structure and agency must be addressed and this is not a task for CPD project alone.

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[140.]

[141.]

[142.] Prospective teachers developing modeling projects: difficulties, learning and interest

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In the modeling project approach by Borba and Villarreal (2005), students collaborate to choose a topic of personal interest and formulate a problem that can be addressed using mathematics. This strategy fosters meaningful connections between mathematics and real-life contexts, enhancing students' engagement and appreciation for the subject. In the teacher education, it is crucial to equip teachers with competences to incorporate modelling activities in their lessons (Rendón-Mesa et al., 2024). In this study, the approach was applied to understand the challenges and learning experiences of prospective mathematics teachers during the development of modeling projects.

The study follows a basic qualitative design (Merriam & Tisdell, 2016). Participants were 11 prospective mathematics teachers in their 7st semester of a teacher education program. They formed seven groups based on shared interests, with some working individually while maintaining class-wide and teacher interaction. At least one member of six of the groups wrote his narrative. Each group made a project report, a video describing the project and a narrative of the experience. The projects addressed themes such as football statistics, video game addiction and healthy life.

Narrative analysis revealed three main categories: difficulties, learning outcomes, and the role of interest.

Difficulties included selecting a mathematically viable topic, managing large datasets, formulating relevant problems, and translating real-world situations into mathematical terms.

- **Choosing a topic that could be addressed through mathematics**

G3E1: At first, it was difficult to select a topic that was of personal interest and could also be addressed mathematically. (...) I think that's the biggest challenge when starting a modeling project.

- **Managing and organizing large volumes of data**

G1E2: (...) one of the main challenges was managing and cleaning large data sets. Ensuring that the data was accurate and well-organized for effective analysis was complex but enjoyable.

- **Formulating the problem**

G2E2: The main issue I encountered was in formulating the second problem about gaming hours during vacation, for which there is limited information (...)

- **Mathematizing the situation or problem**

G6E1: During the process, interesting challenges arose such as: "How do I translate this into mathematics?" (...)

Learning outcomes focused on defining meaningful problems, selecting relevant and realistic information, and interpreting data within specific contexts.

- **Proposing a problem**

G3E1: (...) I'd highlight the importance of defining the problem that drives the modeling project, including researching and selecting an appropriate topic (...).

- **Selecting relevant information**

G2E2: Regarding what I learned, the entire process was new to me, but I'd emphasize the importance of selecting relevant information to present to people unfamiliar with the topic and choosing data that reflects reality.

- **Interpreting data**

G1E2: I learned a lot about the importance of data interpretation in a sports context. It wasn't just about calculating numbers, but about understanding what those numbers meant and how they could be used to evaluate player performance over time.

Interest played a crucial role: participants reported higher motivation and confidence when working on topics they personally cared about, which enhanced their involvement in their work.

G1E1: First, I want to highlight that being able to choose our own topic was a huge motivation for developing the modeling project I worked on. Since my project was about something I really enjoy—soccer—it made me much more invested in doing a good job.

In summary, while prospective teachers faced challenges in the modeling process, the ability to choose topics of interest emerged as a key motivational factor, promoting deeper learning and stronger engagement, in the same way as Galleguillos (2024), along important reflections about aspects such as healthy life and video game addiction.

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[143.] Investigating the structure of digital mathematical modelling competence – Introducing the ModiWe project

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Digital tools such as Excel and GeoGebra can be used when working on modelling tasks. Such tools can support solution processes in different ways, for example, by conducting experiments to discover mathematical relationships or by implementing dynamic approaches (Drijvers & Doorman, 1996). Several studies have already observed that digital tools can be used at different stages of the modelling process, for example, when simplifying and mathematising using experimentation or creating digital drawings (e.g., see Geiger 2011; Shahbani, 2025). Modelling competence, understood as the ability to carry out a modelling process, can be differentiated into sub-competences that relate to skills necessary for carrying out certain sub-processes of modelling (Maaß, 2006). However, digital sub-competences go beyond analog sub-competences, as the use of digital tools and the translation processes between mathematics and the digital tool require additional competences (Galbraith et al., 2003; Greefrath et al., 2018). Previous intervention studies have identified only minor differences in the development of overall modelling competence and in the sub-competences after the intervention with and without digital tools. However, although a digital tool was used by the experimental group during the intervention, the tests to determine the competences were carried out in a paper and pencil format not using the digital tool (Hankeln & Greefrath, 2021).

In addition, qualitative studies have shown that there are advantages to using digital tools in modelling (e.g., see Cohen-Nissan & Kohen, 2023). Therefore, a computer-based survey, considering the possible uses of digital tools, seems to be a sensible option. This is being carried out as part of the ModiWe research project to investigate the effects of using digital tools in mathematical modelling on modelling competences.

The project therefore aims to answer the following research questions:

1. How can the structure of a digital modelling competence be described?
2. How does using a digital tool in the modelling process influence the development of (digital or non-digital) modelling competences in comparison to modelling without digital tools?

Regarding the first question, it can be assumed that digital modelling competence, like analog modelling competence, can be described with the help of sub-competences (e.g., see Hankeln et al., 2019). Concerning the second question, based on previous research, it can be assumed that the use of the digital tool promotes both analog and digital competences, while analog modelling only improves analog competences (Hankeln & Greefrath, 2021).

To answer these questions, we developed a teaching unit on mathematical modelling and a test instrument for digital and analog modelling competences. Concerning the teaching unit, two versions with identical content were designed. In the first version, the students are required to solve the modelling tasks without digital tools, and in the second, the tasks are completed with the help of GeoGebra. The next step is to test the developed teaching units with students. In this

contribution, we present and discuss the teaching units, the initial results of the piloting, and the test instrument.

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[144.] Relevance of extra-mathematical knowledge in the modeling processes of future elementary school teachers

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Despite the recognition of mathematical modelling as a key competence in teacher education, several studies show limitations in its implementation in initial teacher education programmes (Wei, et al., 2022). Most prospective teachers have not had authentic opportunities to experience modelling in their training (Andrews & Sayers, 2012; Moreno et al., 2021; Paolucci & Wessels, 2017; Oropesa et al., 2018; Widjaja, 2013). In addition, classroom tasks are often over-simplified, which restricts the development of complex modelling processes. The ambiguity around what is meant by modelling also contributes to its weak implementation in the classroom, manifesting itself in difficulties in identifying an application of a modelling activity (Guerrero & Borromeo, 2022). It is therefore necessary to implement activities in initial teacher training that contribute to the learning of future basic education teachers. In this context, we ask ourselves what knowledge emerges when future elementary education teachers become involved in a packaging project? How is the modelling process that future teachers develop when dealing with an open-ended situation?

Following a qualitative methodology, we analyze the knowledge that participants construct when exploring an open task that considers the Packing problem. To do so, we rely on the work of Castillo & Guerrero (In press) who point out that teachers' knowledge for teaching modeling should be composed of mathematical, extra-mathematical, technological and pedagogical knowledge. Particularly, we focus on the first three types of knowledge, which later in their training will be the basis for the design and implementation of modeling tasks in the classroom. The experimentation was developed during a semester where participants worked on a project that asked them to pose and answer questions focused on the design of packaging for mass consumer products.

It was observed that this activity allowed them to integrate geometric, environmental, economic and technological knowledge. The participants identified and operationalized relevant variables (volume of the package, type of material, carbon footprint and cost) and they constructed different mathematical models based on geometric bodies (prisms, cylinders). As well as estimations, comparative calculations and price/cost and quality analysis were evident at all times. The modeling process revealed different phases: understanding the problem, selection of variables, formulation and analysis of the model, interpretation of results and validation of proposals.

One of the central findings was that, despite their limited previous experience in mathematical modeling, the future teachers were able to go through the modeling process in a coherent and reflective manner, demonstrating the ability to link mathematics with problems of social and environmental interest. The final proposals included more volumetrically efficient and sustainable packaging models, integrating digital technologies such as GeoGebra and spreadsheets to visualize and validate their solutions.

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[145.] How students validate mathematical models: A case-study of Norwegian 9th grade students

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Partly driven by the advancements in artificial intelligence, mathematical models are increasingly shaping decisions that impact our lives and society. Knowing when, and how much, we can trust these models are more important than ever, but also more difficult. This underscores the necessity for proficiency in mathematical modeling, especially the validation step. Unfortunately, however, many students struggle with mathematical modelling, and model validation seems to present a particular challenge (Blum & Borromeo Ferri, 2009; Blum & Leiß, 2007; Czocher, 2018).

According to the Norwegian mathematics curriculum (Ministry of Education and Research, 2019), students should develop competence in mathematical competence, enabling them to “(1) critically evaluate whether the models are valid and what limitations the models have, (2) evaluate the models in view of the original situations, and (3) evaluate whether they can be used in other situations” (p. 3, our numbering). But what types of validation arguments do Norwegian students use? To shed light on this, 10 lower secondary school students in Norway was examined as they engaged in an atomistic modelling task (Niss & Blum, 2020) designed to emphasize model validation:

- a) Firstly, given a dataset comprising five measurements of a horse’s pulse the first few minutes after it had been running, the students were asked to make a mathematical model to predict the horse’s pulse.
- b) Secondly, the students should extrapolate: What is the horse’s pulse after 30 minutes?

Task-based interviews were conducted while the students were working in pairs to solve the task. The interviews were analyzed by an initial exploratory analysis identifying a set of six task-specific categories. Subsequently, these task-specific categories were examined and interpreted in light of the generic requirements of the curriculum.

The analysis showed that all student-pairs succeeded in performing knowledge-based validation (Borromeo Ferri, 2006), at least to some degree. The observed arguments primarily concerned evaluation in view of the original, real-world situations. For example, all pairs realized that the horse’s pulse cannot be below zero (then the horse would die), and modified their models accordingly (see Figure 1 for an example). Limitations of the models were also mentioned. For example, three student pairs argued that the pulse will increase if the horse starts running again.

Except one comment regarding uncertainty due to the dataset consisting of only five datapoints, we observed no attempts to validate models without a direct reference to the practical situation, such as discussing uncertainty in the training data, measurement errors, sensitivity analysis, extrapolation, etc.

Blum and (Leiß, 2007) observed that a clear weakness among students is that they do not validate or reflect on their models. When the students we interviewed were working on task (a), which (only) requested them to make a model of the situation, most of the students finished without even attempting to validate their model. However, when having to extrapolate, in task (b), discussions concerning model validation were initiated. This demonstrates how task design affects students' validation activities. Of course, also teachers can promote model validation. When the teacher asked the students to explain their models and justify their choices, we often observed students revealing flaws in their models, changing their models appropriately.

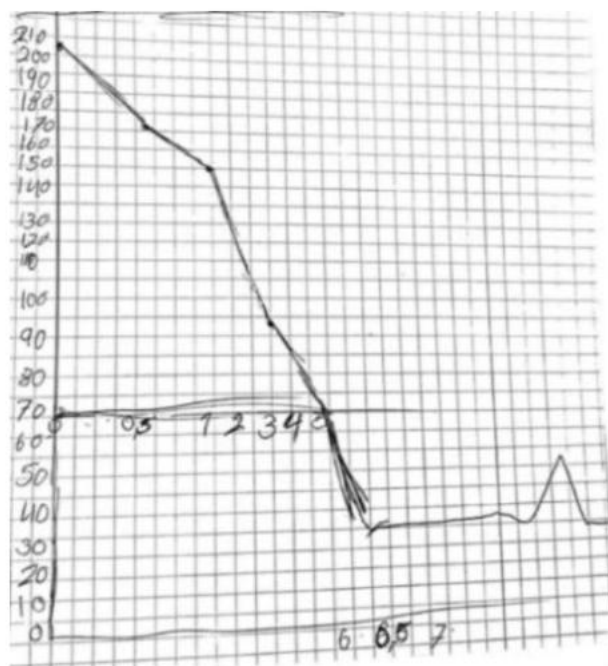


Figure 1: Students introduce a “break-point” in their model to hinder the pulse becoming too low

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[146.] Potentials for fostering the spirit of mathematical modelling in forest education

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Vocational mathematics education should serve as an ideal learning space where instruction is explicitly designed to equip learners for future employment (FitzSimons, 2014). Despite this, students enrolled in vocational programs frequently show limited enthusiasm for mathematics and struggle to perceive its practical value (Muhрман, 2016). According to Dalby and Noyes (2016), incorporating authentic workplace scenarios can enhance students' motivation to engage with mathematics.

The use of mathematical modelling and application within educational contexts offers a way to bridge mathematics with real-world practices beyond the academic discipline (Frejd & Vos, 2024; Frejd et al., 2024), including those found in professional environments. As such, the use of modelling and applications may be particularly effective in vocational settings to help shift students' perceptions and attitudes regarding the usefulness of mathematics in various contexts. However, that require that instructions provided for the students are of relevance for them and that they potentially will support the students to learn about both the mathematics and their future occupation.

This presentation focuses on the Natural Resource Use Programme, which specialises in forestry. It aims to investigate three classroom instructions that encourage for students to develop the Spirit of Mathematical Modelling (SoMM) (Frejd & Vos, 2024). The three instructions refer to: 1. *Buy and own a forest: Responsibilities and profit*, 2. *Marking for cross-cutting*, 3. *Forest clearing*. Based on a content analysis (Frejd, accepted), the research question 'Which aspects of SoMM can be identified in the three different classroom instructions?' will be explored. The results of the analysis, which will show which aspects of SoMM the instructions will potentially foster among the forestry students, will be presented and discussed during the presentation, as will the type of mathematics the students will encounter in their learning activities.

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[147.] Learning possibilities created by the fusion of steam education and origamics

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To respond to the rapidly changing society, there is an increasing demand for the creation of new knowledge and values, raising the importance of STEM/STEAM education. In STEM/STEAM education, engineering involves activities through the process of “Design → Build → Test,” where science (thinking to explore theory/law), technology (thinking to optimize/systematize), and mathematics (thinking with numbers/symbols) are integrated (Yata et al., 2020). In this process, mathematics plays a key role in promoting logical and efficient activities. Furthermore, by incorporating art, mathematical thinking based on learners’ ideas can offer new perspectives and foster further development. Developing educational materials that enable such learning is crucial.

Since 2015, the authors have been developing teaching materials based on “Origamics,” the scientific analysis of origami, a traditional Japanese culture, and have implemented lessons for elementary, middle, and high school students. In particular, the paper container teaching material, which can be designed and created from a single sheet of paper, is environmentally friendly as it can be folded compactly for disposal. In the design and creation process from an engineering perspective, students can apply mathematical knowledge and thinking related to numbers and shapes while pursuing various forms through the lens of art. Additionally, this teaching material includes scientific elements such as experiments and observations performed by folding paper, as well as technical aspects utilizing origami and ICT devices. However, the educational effects of this material have not been sufficiently verified. Therefore, the purpose of this study is to examine how learners apply mathematics to solve problems and the impact on their learning attitudes through lessons using this material.

The study involved 25 students from the third year of X Junior High School in Japan. The students were divided into seven groups of three to four and participated in a 150-minute lesson. First, they created prototype paper containers and observed their shapes and structures. Then, they measured the dimensions of the prototypes and calculated their volumes. Following that, they worked on the task: “Using an A4 sheet of paper (210mm × 297mm), create a paper container in the shape of a square pyramid frustum. Design and create the container so that its volume is as large as possible.” After the lesson, a survey using a 4-point scale was administered (“Strongly agree” = 4, “Agree” = 3, “Disagree” = 2, “Strongly disagree” = 1).

The analysis of the worksheets collected after the lesson revealed that six groups applied mathematics in designing the paper container and formulated equations for calculations. Among these, three groups achieved a volume of approximately 790 cm³ (the optimal solution for the task was around 800 cm³). Additionally, the valid number of responses from the survey was 22, and the average scores (with standard deviations) for each item were as follows: “I found mathematics interesting” = 3.18 (0.59), “My interest in learning mathematics increased” = 3.27

(0.63), “My understanding of mathematics deepened” = 3.23 (0.61), and “Mathematics is useful in daily life” = 3.27 (0.70).

These results suggest that the lesson utilizing this material promotes the application of mathematics in numbers and shapes and has the potential to positively influence students’ learning attitudes.

Acknowledgement

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[148.]

[149.]

[150.]

[151.] The "Data Processing and Organization" A comfortable habitat for teaching mathematical modeling in primary Moroccan school

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In this article, we test the hypothesis that the field of "Data Processing and Organization" represents a comfortable niche for teaching and learning mathematical modeling at the primary level in the Moroccan education system. We start from the following question: **What is the ecology of "mathematical modeling" knowledge in the new Moroccan curriculum at the primary level?**

We seek to characterize the institutional relationship of the knowledge "mathematical modeling" in the new curriculum of July 2021. To do this, we rely on an analysis of the Pedagogical Guidelines (OP) and also the textbook of the 6st-year primary Moroccan students (Age 11 to 12) called " MON LABO EN MATHEMATIQUES». Our analysis is based on the Anthropological Theory of Didactics (ATD) developed by Chevallard (1992) [1] and the praxeological approach [2] which models mathematical activity. This theory posits that every activity falls under a type of task

$(T)(T)$

, which can be accomplished by a certain way of doing things, called a technique

$(\tau)(\tau)$

, which is made intelligible and justified by a technology

$(\theta)(\theta)$

, which is itself justified and made understandable by a theory

$(\Theta)(\Theta)$

. The technological-theoretical block

$[\theta, \theta]\theta, \Theta$

, composed of a technology and a theory, is usually identified as knowledge, while the practical-technical block

$[T, \tau]T, \tau$

, composed of a type of task and a technique, constitutes know-how.

The result shows the existence of an institutional relationship between the knowledge "Mathematical Modeling" and the institution "the new primary curriculum". We develop an analysis grid to examine some activities in the field of "data processing and organization" from " MON LABO EN MATHEMATIQUES» textbook and we found different places that constitute the habitats of Mathematical Modeling as a comfortable niche for teaching it at the primary level. This study leads to perspectives for the teaching and learning of mathematical modeling in other school levels such as middle school or high school or even higher education and also in other fields such as measurement, geometry, numbers and calculation.

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[152.] Modelling for equity: Mathematical modelling and social justice from a sociocritical perspective

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Positioning mathematics as a tool that students can use to understand and act on social inequalities, while at the same time connecting it to reality, is a dual challenge for mathematics education (Gutiérrez et al., 2024). In this sense, solving modelling tasks in the classroom whose contexts focus on sociopolitical phenomena means empowering reflective knowledge - that is, the ability to understand the implications of mathematical results for social practice (Skovsmose, 2007). One possible approach concerns the relationship between social justice and mathematical modelling. From a sociocritical perspective, this connection can support students in developing the ability to interpret social reality, while fostering critical thinking, encouraging dialogic action through reflective discussion, and promoting autonomy (Barbosa, 2009; Skovsmose, 2021). This relationship can be reflected in the middle-school mathematics curriculum through the learning of functions. It involves critical reading of data (Carreira & Blum, 2021; Rubel et al., 2021); making fact-based decisions using mathematical models (Siller et al., 2024; Skovsmose, 2021); and modelling real-life situations using functions (Greefrath & Carreira, 2024).

Under such rationale, this study aims to understand the impact of mathematical modelling involving social justice issues in the learning of functions by middle school students. To achieve this purpose, we formulated the following research question: How do students engage in and value reflective knowledge when working on modelling tasks framed by social justice situations, in the learning of functions? Based on a didactical intervention, a qualitative and interpretative methodology was adopted, with a case study design, focused on two classes – 8th and 9th grade, from different public schools. In addition to students, both the mathematics teachers took part in the research study. Data collection included classroom observations, interviews at different times during the intervention period, audio and video recordings of the observed lessons, students' productions, and the researcher's field notes. The collected data was subjected to content analysis (Bardin, 2016). The modelling tasks were based on real-life situations that were close to the students' lives, related to the themes of water scarcity and labor market.

The results show that critical reading of data emerged as one of the activities in which the students were most involved. The students demonstrated their ability to 'write the world' with mathematics - that is, to develop sociopolitical awareness. Also, the students engaged in group work and communication, developing autonomy and participating in lively discussions. When mentioning the effects of their work on the modelling tasks, the students highlighted the prominence of the mathematics content in the real world, the increasing of their autonomy and the importance of the dialogical action in which they participated. This outcome was influenced, in a symbiotic way, by the development of their sociopolitical awareness and by the students' sociocultural mobilizing experiences. As a result of connecting functions with real world

situations and promoting verbal interactions, the displayed narratives show a reconfiguration of students' perception on the utility of functions for critically understanding the social world.

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[153.]

[154.] Error-diagnostic competence in the context of Math Trails – an intervention study

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There is a growing interest in out-of-school learning approaches in mathematics education (Kuş, 2024), as students need both in-school and out-of-school mathematics experiences to meet the challenges of today's society (Masingila et al., 1996). One such out-of-school learning format are Math Trails, where specific modelling tasks are solved on real objects in the environment (Buchholtz et al., 2020; Shoaf et al., 2004). Despite its high real-world relevance, this learning format can be challenging for learners. While traditional modelling is already demanding for students and can lead to a variety of errors (Blum, 2011; Galbraith & Stillman, 2006), out-of-school learning of mathematics can present additional hurdles in the process of completing modelling tasks that teachers need to respond to with support. This is where teachers' error-diagnostic competence comes into play, which we conceptualize in this study as consisting of the three processes of error perception, cause diagnosis and error handling (Heinrichs & Kaiser, 2018).

While error-diagnostic competence has been well researched for classroom situations, the out-of-school learning context has received little attention in research on this facet of competence. Therefore Ende and Buchholtz (submitted) and Buchholtz (2024) investigated error-diagnostic competence of pre-service teachers (PSTs) in the context of outdoor learning on Math Trails using video vignettes of students' solution processes on a Math Trail and open-ended questions regarding the three processes of error-diagnostic competence. The responses surprisingly showed that a large number of PSTs did not accept the solution strategies chosen by the learners (e.g. using steps to measure the perimeter of a fountain), either because the strategies were seen as errors or because the learners were discouraged from using these strategies in the description of error handling. So, the focus was on the strategies themselves and not on the underlying errors in the strategies. These responses may be due to a lack of knowledge about appropriate strategies in this context, as the PSTs examined had no experience of Math Trails.

Based on these results, the short oral presentation will introduce an ongoing intervention study that examines the development of error-diagnostic competence of PSTs when participating in a seminar on Math Trails. The study follows a pre-post-design where PSTs are asked to analyze videos of error situations on Math Trails in an open-ended questionnaire before and after the intervention. The intervention deals with out-of-school learning, Math Trails, their characteristics and typical strategies as well as error analysis and error handling. In the short oral communication, the design of the study and first results will be presented.

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[155.] “Stochastik digital” (Stochastics Digital): Enhancing Competence through a Flipped Classroom Format with a Focus on Application

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The project “*Stochastics Digital*” involves the redesign of a teacher education course in stochastics into a blended learning format, centred on a modified flipped classroom model (Förster, 2022). The redesign aims to promote students’ autonomous learning and, in particular, strengthen their modelling and application competences.

Didactic Redesign: Blended Learning and Flipped Classroom

The previous format was perceived by students as interesting, but not as professionally relevant – despite numerous interdisciplinary application possibilities. In the new teaching concept, students learn key content independently using interactive explanatory and experimental videos. These are further explored in open attendance events and in exercises. A key challenge is that many students enter the course with limited prior exposure to stochastics, and practical applications were often underrepresented in traditional formats. The integration of everyday phenomena in the videos therefore aims to link theoretical concepts with students’ experiences and prior knowledge.

Structure and Pedagogical Design of the Teaching Format

The modified flipped classroom aims to make optimal use of students’ learning time by maximizing opportunities for mathematical communication – in line with communicative constructivism (e.g., Krummheuer, 1992): with lecturers, tutors and fellow students on site at the university, in video rooms and chats, in order to promote long-term skills development and enable flexible, individualized learning. Formative feedback is provided continuously, scaffolding both conceptual understanding and procedural skills. It goes without saying that stochastics is about making data-based learning tangible – analyzing real data sets, creating interactive simulations of random processes or experiencing randomness visually.

Empirical Evaluation

A questionnaire study was conducted to analyse the effectiveness of the new concept (Förster & Geffers, 2024). The cohort, which took the course prior to the introduction of the new format, served as the control group, while the experimental groups experienced the flipped classroom design. Both groups completed pre- and post-semester questionnaires. The items used are based on instruments from Grigutsch et al. (1998) and Schiefele & Wild (1994), among others, and cover, for example, facets of the stochastics image, learning strategies as well as self-concepts and the willingness to exert effort. Data analysis was conducted using a mixed ANOVA with repeated measures.

Key Findings

The results reveal significant interaction effects in students’ learning behaviour, clearly

favouring the flipped classroom format. According to Cohen (1988), the effect sizes ranged from small to large. The stochastics image also changed: students in the new format rated the content's application relevance and its relevance for later school practice significantly more positively at the end of the semester. In the control group, on the other hand, this assessment decreased. In addition, changes with regard to procedural and formal aspects of stochastics were also analysed. These results suggest that a well-designed flipped classroom can strengthen prospective teachers' modelling competence and their readiness to apply stochastics in school. This highlights the potential of digital, application-oriented formats to improve mathematics teacher education. Future research will examine long-term effects on teaching and student attitudes.

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[156.] Designing for change: Challenges and opportunities in implementing mathematical modelling in Faroese schools

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In response to the significant challenges identified in the mathematical literacy of Faroese students through successive cycles of the PISA assessment, this ongoing study investigates how international large-scale assessment results can be meaningfully implemented into local educational practice. In particular, the project addresses deficiencies in students' mathematical modelling competency and their limited experience with digital tools, both of which were prominently highlighted in the PISA 2022 results (Olsen et al., 2023; OECD, 2023a, 2023b). The study explores the design, implementation, and outcomes of an intervention carried out in three Faroese schools.

Grounded in the PISA mathematics framework and informed by the KOM competency model (Niss & Højgaard, 2019) and the modelling cycle (Blomhøj & Jensen, 2003; Niss, 2010), the intervention applies a design-based research methodology (Bakker, 2018). The dual aim of the intervention is: (1) to develop and iteratively test teaching sequences that foster students' mathematical modelling skills and digital tool usage, and (2) to establish sustainable Communities of teacher Practice (Wenger, 1998), that support the professional development and implementation process. Each teaching sequence focuses on real-world problem contexts and the use of digital tools, such as spreadsheets, as means to support students in formulating, employing, and interpreting mathematics in a real-world context.

PISA data analysis reveals that Faroese students perform notably below both OECD and Nordic peers in modelling processes, with particular difficulty in mathematically formulating real-world problems. At the same time, they report infrequent use of digital resources in mathematics lessons, a finding that further correlates with poor performance on tasks requiring data analysis or technology use (Olsen et al., 2023). These findings serve as both a diagnostic and a design foundation for the intervention.

The intervention is structured over a five-year period (2024–2029) and comprises six teaching sequences distributed across grades 7–9 (age 13–15) in three schools. Each sequence spans approximately 15–20 instructional hours and is accompanied by professional development and collaborative teacher workshops held before, during, and after implementation. These workshops are designed to cultivate a shared professional language and foster ownership among teachers, ensuring sustained use and refinement of the materials. The small scale of the Faroese education system provides a unique opportunity for close tracking of implementation effects, and since all students in the Faroe Islands participate in PISA, the impact of the intervention can be rigorously assessed using population-level data from the 2029 PISA cycle.

[157.] Confirming and Supplementing of a Mathematician's Modelling Competencies

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As previous studies I establish a framework of modelling competencies (Matsuzaki, 2014). In the framework I indicate required handling of variables to progress each modelling process with referring modelling cycle. In that meaning, it can be called a prescriptive framework to grasp modelling competencies. Additionally, I introduce *Gen-Bamen* that comprises images based on modellers' experiences. For describing facts of modelling I use applied response analysis mapping to display processing in modelling progress (Matsuzaki, 2018). In that meaning, it can be called a descriptive framework. Using applied response analysis mapping, it is possible to chase modelling progress, and to confirm modelling competencies on description of prescriptive framework. Variables displayed on maps are mathematical components and non-mathematical components, and components of the modelling based on prior experiences of modeller are identified as follows: components based on real experiences and components based on mathematical experiences (Matsuzaki, 2011).

As previous studies, I applied the above descriptive framework to test persons who was a mathematician (Matsuzaki, 2019, 2023). In this presentation I discuss the mathematician's modelling competencies based on the framework of modelling competencies. As a finding it is possible to supplement a competency for involving with shifting a real situation imaged as a component based on real experiences into a real model imaged as a component based on mathematical experiences.

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[158.] Mathematical modelling competency and textbook design

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Curricula around the world make more and more use of goals trying to capture different kind of processes for the students to master (Højgaard & Sølberg, 2023). This overall development in the approach to learning goals for mathematics education is well covered in the research literature, and many reports focusing on specific countries document this shift (cf. Niss, Bruder, Planas, Turner, & Villa-Ochoa, 2016).

In Denmark, the publication of the KOM report (Niss & Jensen, 2002) initiated a developmental approach, where the mastery-oriented curricular ambitions are described by means of a set of mathematical competencies (Niss & Højgaard, 2019). Since 2009, the set of mathematical competencies proposed in the KOM report has been a fundamental part of the curriculum for compulsory mathematics education (grades K-9) in Denmark. This process of curricular development is thoroughly described and analysed in Højgaard and Sølberg (2023).

One main result from researching reforms focusing on student mastery is that they often do not have the impact hoped for when it comes to actual teaching practices in schools (Cuban, 2013; Hopmann, 2003). Denmark has been no exemption to this trend, and a lot of effort has been put into implementing the formal curricular intentions of competency-oriented mathematics teaching in Danish classrooms.

I have been involved in this developmental process in various ways, of which I will focus on one particular in this presentation: In the years 1998-2020, I was one of the leading persons in the development and writing of a series of mathematics textbooks, *Matematrix*, for the compulsory part of general education in Denmark, grades K-9 (cf., e.g. Gregersen et al., 2020). The three years before the publication of the first book in 2001 were spent on deciding on and didactically designing the fundamental characteristics of the new books to come. That process took place parallel to my involvement in the KOM Project. Hence, one of the main ambitions for the new textbook system became an attempt to systematically facilitate the incorporation of mathematical competencies as a key element in mathematics education for grades K-9 in Denmark.

In this presentation, I will focus on one aspect of this involvement, specified by the following research question:

How can textbooks be designed so they systematically facilitate the students' development of mathematical modelling competency, and what constitutes exemplary types of tasks to meet this endeavour?

In my analysis of this research question, I describe and exemplify the design of the *Matematrix* textbooks and the analysis behind it at three different levels: The structuring of the content for the books in general (Højgaard, 2019), the focal points for each chapter in the various books, and the development of different kind of tasks for the students to work with. When making these descriptions, I focus on and provide concrete examples of how colleagues and I chose to support the teachers in facilitating the students' development of mathematical modelling competency

(Højgaard, 2021). As a pivotal element, this focus of the analysis leads me to a distinction between and exemplification of mathematisation tasks of short duration, constructive modelling tasks of short duration (Ärleback, 2009), and inquiry-based projects (Artigue & Blomhøj, 2013) of longer duration focusing on either the constructive or the receptive facet of mathematical modelling competency (Skovsmose, 1990).

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[159.]

[160.]

[161.] How do students value modeling with and without experiments (differently) and why?

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For students, mathematical modeling is a key competence to learn in school, in preparation for living and participating in modern society (Niss & Blum, 2020; OECD, 2023). Despite its relevance, studies show that students do not value modeling tasks compared to other types of tasks, such as word-problems and intra-mathematical tasks (Krawitz & Schukajlow, 2018). The discrepancy between the importance of mathematical modeling and the low value students ascribe to that topic is the subject of several studies. One approach in research to increase students' value beliefs is to combine modeling tasks with experiments (Beumann, 2016; Ganter, 2013; Geisler & Rach, 2023). Findings indicate that experimenting could improve students' value beliefs regarding modeling tasks. To conceptualize value beliefs, we refer to the expectancy-value-theory, which comprises four components of subjective task value: *intrinsic value* (enjoyment), *attainment value* (importance of doing well on a given task), *utility value* (usefulness for future/daily life), and *costs* (psychological/effort costs) (Eccles & Wigfield, 2024). These components can be characterized as a continuum from value to de-value (Gildehaus et al., 2022; Guthrie et al., 2022; Matusovich et al., 2010). The current study aims to gain differentiated insights into the students' (de-)value and their reasoning regarding modeling tasks with and without experiments, leading to the following research question:

How do students (de-)value modeling tasks with and without experiments differently, and how do they reason their (de-)value?

With a mixed-methods approach, we analyse data from our experimental-control-group study, where 627 students from 27 classes participated. Each class was assigned to one of the following conditions: students worked on modeling tasks (i) with experiment to generate real data, (ii) with given real data, or (iii) with given schoolbook-like smoothed data. After the second of three tasks, all students rated their value components regarding that specific task by answering approved questionnaires. Additionally, 35 students were interviewed about their reasons for (de-)value. The interviews were transcribed and analysed with qualitative content analysis (Mayring, 2015).

To examine differences between the three conditions regarding their task value, we applied an ANOVA to the questionnaire scales. The results show the following significant differences ($N = 627$, $p < .05$): Intrinsic value was highest in condition (i) (experiments) compared to both other conditions (

$$\eta^2=.07 \eta^2=.07$$

). Utility value was higher in condition (ii) (given real data) compared to (i) (experiments) (

$$\eta^2=.02 \eta^2=.02$$

), and costs were highest in condition (ii) (given real data) compared to both other conditions (

$$\eta^2=.05 \eta^2=.05$$

). To explore reasons for (de-)value, we coded the continuum of value components with deductively developed categories (e.g., intrinsic value: positive - "task is enjoyable", not pronounced - "It's not interesting to me", negative - "I'm explicitly bored by the task") and

added inductively categories focussing reasons. Interestingly, different students use the same reasons for explaining different value components: “I could apply mathematics to a real-world example” was a reason for the enjoyment of the task (intrinsic value) and for its usefulness (utility value). The main categories were related to identity, task characteristics, handling results, usefulness, and learning goals. Students of all three conditions reported effort costs but reasoned differently: to improve the fit of a function to the data was difficult for students of all conditions, but only students working with given data reported difficulties in accepting the not perfectly fitting function. Our findings hint that students find working with real data exhausting, but when collecting it themselves through experiments, they perceive the task as more enjoyable and modeling as less demanding.

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[162.] The Art of Sisal Weaving in a Local Community in Brazil: An Ethnomodelling Investigation

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Humans have made use of woven artifacts for centuries either by hand or by machine for centuries. All of which use rich forms of mathematics, many were demonstrated in this research as outlined here. The results of this investigation show that both mathematics teachers and artisans value in this study respect local geometric knowledge related to the art of carpet weaving and in the making of sisal carpets. As they became aware of the relevance of their own experiences, which enabled them to understand the connection between etic (school) and emic (weaving) knowledge in the process of teaching and learning mathematical and geometric content.

As well, this research established relations between the production of sisal carpets in which the development of related activities rely on visuospatial reasoning that depends on the understandings of geometry, visualization, and spatial reasoning of the members of sisal carpet makers. In this regard, artisans and mathematics teachers in this study demonstrated visuospatial reasoning that aligns well with Euclidean geometric content students experience in school (Orey, Rosa, & Rosa Filho, 2024).

The late D'Ambrosio (2016) showed us that this cultural dynamism emphasizes the dialogical communication, which is based on mutual respect and individual differences, as well as Rosa and Orey (2019) highlight the appreciation of the members of this specific cultural group and their knowledge subjectivities. Consequently, the development of mathematical curricular activities proposed for the pedagogical action proposed in this study was in accordance with emic (making sisal carpets) and etic (school mathematical and geometric content) approaches of ethnomodelling.

Since, the main objective of the study was to investigate possibilities found in the local geometric processes related to the making of sisal carpets. This enabled the researchers to both see clearly and understand possibilities for further meaningful, contextualized, and transformative learning experiences, which contrast with the traditional pedagogical model of teaching that is currently used in the school system under study and around the world. Thus, this study promoted respect for, and appreciation of local (emic) geometric knowledge and visualization related to the making of the carpets by artisans in this community through the development of ethnomodels.

The relevance of this pedagogic action became evident when mathematics teachers contributed to the development of the mathematization process of given problems and situations present the daily lives of artisans who make sisal carpets. Thus, this study mathematized ideas, procedures, techniques, and geometric practices typical of everyday artisanal production, which has its natural encounter with activities carried out in the school environment.

The findings have implications for the development of further pedagogical actions that seek to translate mathematical knowledge between distinct knowledge systems that are complementary.

And most importantly, ethnomodelling provided a powerful opportunity to create a unique understanding of curricular geometric activities proposed in this pedagogical action in the context of the local weaving in the making of sisal carpets.

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[163.] Peatland are ugly, or?

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This research focus on a modelling activity on peatland, where 15-year old students estimate carbon storage potential. The activity aimed to enhance students' understanding of modelling complex societal issues involving multiple and conflicting stakeholders. Mathematical modelling is viewed as a cyclic process of translating real-world problems into mathematical representation. Maaß et al. (2019) connects modelling to socio-scientific issues; prompting students to engage in discussions, form opinions, and make decisions that incorporate moral, ethical and social reasoning. The context of such modelling activities is particularly relevant, as it is regarded as “true”, and often controversial.

The author, alongside two ecologists and a secondary school teacher designed an activity that included fieldwork, laboratory work, estimation, podcasts, and poster presentation (Steffensen & Auestad, forthcoming). It was implemented over five weeks in two ninth-grade classes. The data comprises transcripts from group discussions, field notes, 11 group posters, student notes, and teaching materials. Preliminary findings is structured in two themes: (1) estimation and uncertainty and (2) ethical, economic, social and cultural aspects.

Estimation and uncertainty

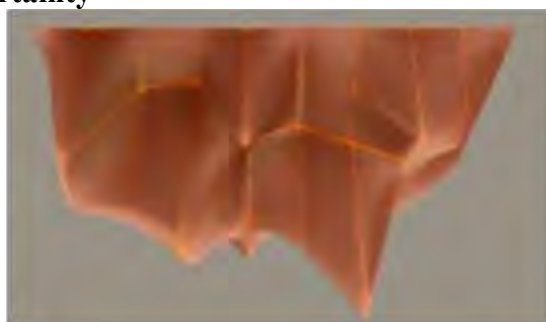


Figure 1 Left side: An online image of the peat. The blue line marked the area of the initial area of the peat, while the red lines marks the lines of the coordinate-system the depth was measured in (with a distance of 5 meters). Right side: A 3D-image of the peat, visualizing the depths.

Online map tools were used to estimate the area of peatland (Figure 2, left side). Clear boundaries between peatland and other natural types are rare. Students had to make assumptions about the extent of the peat, which introduced uncertainty in their measurements. To gauge depth, they measured every 5 meter across the peat, in four lines, allowing them to calculate an average depth. However, transitioning from volume calculations of a cube – where all sides are straight – to estimating the irregular shape of peat proved challenging.

The teacher provided assessment criteria for the poster, one being that students should “Use mathematics correctly and critically evaluate the calculations”. This typically include verifying calculations and algorithms. All posters addressed this criterion, using subtitles such as “Can we be quite certain?” and “Critical evaluations of our calculations”. They explicitly discussed uncertainty, detailing measurement inaccuracies, parameter choices in their formulas, and assumptions made about factors such as the area and volume of all the peatlands in the whole municipality, and the extent of peatlands transformation.

Ethical, economic, social and cultural aspects

Most students initially described peatlands as wet and muddy, and several described it as dangerous, using negative terms like “ugly”, “stinky”, and “unpleasant”. However, in their final presentation, they expressed a newfound engagement in preserving these natural areas, showcasing scenic images of peatlands and vibrant red moss. This suggests a shift in their perceptions, moving from negative to more positive attitudes toward peatlands. All 11 poster urged local politicians to protect peatland by halting development in these areas, including housing, infrastructure, and industrial project like cruise harbour and wind farms. One group’s poster was titled “Save the Peatlands!”, while others emphasised the need to stop destroying peatlands for environmental reasons. Some recommended restoring damaged peatlands, and urged national action, advocating that “at least 30% of all peatlands should be preserved”. They also proposed solutions to balance various interests, such as building fewer detached house and constructing bridges over peatlands.

Using the extended modelling cycle (Maaß et al., 2019) as a basis for preliminary analysis, revealed that students gained practical knowledge about peatlands and the associated controversies. Through hands-on experiences like measuring peat depth and area, along with applying mathematical formulas and scientific concepts in class, they develop competencies to discuss socio-scientific issues, form opinions, and reflect on conflicting societal and political interests regarding land use in their municipality.

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[164.] Pre-service mathematics teachers' self-posed outdoor modelling problems: Analyzing outdoor aspects in the solutions

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Mathematical modelling entails describing, explaining, predicting, and mathematizing the real-life situations (Lesh et al., 2000; Mousoulides et al., 2007). Posing modelling problems engages students and (pre-service) teachers in the modelling process as well as problem-solving process (Hartmann et al., 2021; Sevinc, 2025), particularly due to contextual and open-ended aspects of the modelling problems. In this regard, Borromeo Ferri (2018) highlights the characteristics of real-life context and extra-mathematical information involved in modelling problems, and several researchers investigated different contexts, such as scientific, occupational, or societal, in modelling problems. Nonetheless only a few recently approached outdoor modelling problems (e.g., Barlovits et al., 2025). Therefore, we, in this study, focused on outdoor modelling problems that did not receive much attention yet in the modelling community. Specifically, we aimed to explore Turkish pre-service middle school mathematics teachers' solutions to their self-posed outdoor modelling problems, addressing the following research question: *What are the outdoor-related characteristics observed in pre-service mathematics teachers' solutions to their self-posed outdoor modelling problems?*

To accomplish this goal, we examined pre-service mathematics teachers' self-posed outdoor modelling problems. A total of 66 pre-service teachers were provided with a problem-posing prompt and were asked to produce modelling problems involving outdoor contexts in groups of five to six members. The production yielded 12 modelling problems, accompanied by their respective solutions. Through content analysis, we coded the solutions to identify outdoor-related characteristics utilized in (1) making-sense of the modelling problem, (2) mathematizing the modelling situation, and (3) validating the mathematical model withing the outdoor context. As these three categories denote the stages of the modelling process, open coding of the outdoor-related characteristics, such as physical presence in the museum to understand the phenomenon or physical presence in the café to make relevant estimations, expands our understanding of how outdoor features support the modelling process.

We note that this study is a part of a larger project, and we are currently at the stage of preliminary analysis in two dimensions, one of which is the focus of this presentation. Our initial analysis indicated that not all outdoor modelling problems require problem-solvers to be in the outdoor; but some problems benefit from this aspect in different stages of the modelling process, at least to some extent. As we are distinguishing these two cases of modelling problems, we aim to address the challenges of outdoor modelling problems as well as the opportunities they bring into the modelling process. In this regard, we expect to contribute to the ICTMA22's theme of challenges and opportunities for mathematical modelling education.

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[165.] A model development sequence for understanding calculus concepts: The case of the definite integral

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This paper addresses the difficulties encountered by engineering students in understanding the concept of the definite integral (DI) and explores how their understanding can be improved through modelling activities supported by digital tools. Students often question the relevance of mathematical concepts such as the DI, raising concerns about their applicability in everyday life or in professional settings (service courses). This situation led the authors to review studies on how the DI is taught in some Mexican universities, noting that, although the curriculum typically introduces it with a focus on approximation, sums, and limits, its practical connection to real-world contexts remains limited.

In civil engineering integral calculus courses, the DI is mainly introduced as a means to calculate areas under curves and distances travelled by objects. Although textbooks present other applications, such as calculating centroids and volumes, these are usually addressed at the end of the course, with most of the time devoted to practising techniques and methods of integration. While specialised courses, such as mechanics of materials, do acknowledge the importance and use of the DI, in practice its application is often avoided when solving problems. This results in a disconnection between mathematical theory and professional practice (González-Martín & Hernández-Gomes, 2018). Consequently, students, even when proficient in integration techniques, struggle with problems where the use of the DI is not explicitly stated. This issue may be addressed through the integration of modelling and digital technologies into the teaching and learning process. Technological tools (e.g., *GeoGebra*, *PhET*), may function as mediating instruments that support the construction, reconfiguration, and manipulation of dynamic representations (Arleback et al., 2013; Moreno-Armella, 2013).

Aiming to promote a level of understanding of the DI that enables students to recognise, interpret, and apply it in various contexts, a sequence of modelling activities supported by digital technology was designed. These activities are intended to guide students through a process of exploration and generalisation of the DI across different scenarios. The design of the sequence draws upon the models and modelling perspective proposed by Lesh and Doerr (2003), which involves Model-Eliciting Activities (MEAs), followed by Model-Exploration and Model-Adaptation Activities (MXAs and MAAs), with digital tools serving as mediating instruments to support transitions across various conceptual systems related to the DI. This perspective not only guides the design of the activities but also presents challenges that still need to be addressed through research. As Arleback and Doerr (2020) point out, it is particularly necessary to formulate, test, and evaluate design principles and implementation strategies for model exploration and application activities that enable students to progress from descriptive models to explanatory ones.

Specifically, the proposed sequence includes modelling situations centred around the design of beams that support distributed loads (e.g., trapezoidally) a topic of particular relevance in strength of materials courses. Through a set of activities, students are required to identify and determine the necessary elements for locating the centroid of a distributed load, engaging with concepts such as load, force, moment, and equilibrium. To this end, simulators and *GeoGebra* are used to help visualise and manipulate data, thereby facilitating students' understanding of how moments accumulate and how this affects beam design. Overall,

The sequence of activities not only aims to assess students' mathematical reasoning, but also to foster reflection and the refinement of their ways of thinking, allowing them to generalise and apply these concepts in other contexts.

The use of digital technologies in this process functions as mediating tools that support students' transitions across different conceptual systems, fostering a deeper understanding of the DI, particularly within the context of civil engineering.

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[166.] The students as a decision-making variable: How teachers select and develop modelling tasks

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Tasks are a fundamental element of mathematics lessons, hence their central position in mathematics education (Krainer, 1993; Wess & Greefrath, 2020). They have the power to generate activity, creating opportunities for learning concepts, ideas and strategies. Tasks assume different forms and degrees of difficulty, and may focus on content, working modes, processes and practices (Burkhardt & Swan, 2013). In the teaching and learning of mathematical modelling, tasks play a relevant role and can be chosen from a wide variety of types (Maaß, 2010; Wess & Greefrath, 2020). Maaß (2010) proposed a comprehensive classification scheme that systematizes existing categories, offering an overview of the different types of modelling tasks, which provides guidance in task design and selection processes for predefined objectives and specific target groups. However, from the wide variety of modelling tasks available, teachers may have difficulty to select the appropriate task for a given learning goal and a specific group of learners.

The present study addresses the teachers' criteria for selecting and developing mathematical modelling tasks, aiming to understand which criteria are most valued and how they are substantiated when teachers select a task for their mathematics students. The research took place in the context of a professional development project based on lesson study (LS) with two cycles lasting six months each. The purpose was to plan, teach and reflect on mathematics lessons (for 10th to 12th grades) involving modelling tasks. Three experienced mathematics teachers were involved in each LS. Data collection included audio recordings of work meetings, and documents produced during the lesson planning sessions.

The source material from which the modelling task for the lesson was selected was different in the two cycles of LS: in the first cycle, the teachers selected the task from a set of four proposals presented by the researcher; in the second cycle, the task was selected from a set of tasks proposed by the teachers. The following principles (drawing on Galbraith & Stillman, 2006; Wess & Greefrath, 2020; Geiger, Galbraith, Niss & Delzoppo, 2022) for developing and evaluating modelling tasks were used to analyse the teachers' choices and rationales: reference to reality, relevance and motivation, authenticity, openness, accessibility, feasibility, and didactical flexibility.

In both LS cycles, the reasons given for selecting a modelling task for the lesson emerged from the group discussion. The criteria used in the selection were: the relevance and authenticity of the problems in the environmental and social context of the students, the accessibility and the degree of challenge posed to the students, as well as openness of approaches. After the final selection, the tasks were redesigned, making apparent the group's attention towards didactical flexibility, especially involving the structure of the task. When analysing the tasks, the teachers

always stressed its adequacy to their students, not only weighing the mathematical knowledge required but also seeking to achieve a balance between the challenges posed by the task and the diversity of students' ability. This proved to be a strong decision-making variable. The teachers advocated the inclusion of visual aids (videos and/or photos) in the introduction of the task, to support students in understanding the extra-mathematical knowledge associated with the real-world problem.

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[167.] Mapping the Path: Learning Trajectories for Enhancing Mathematics Teachers' Competence in Mathematical Modelling

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Mathematical modelling has gained increasing recognition as a crucial component of mathematics education worldwide, becoming a key policy goal in many countries including Thailand, aimed at enhancing mathematics teachers' abilities to teach and assess students' mathematical literacy aligned with mathematical modelling processes (Charoensuthipan, 2023; Stacey & Turner, 2014).

Existing literature, including Borromeo Ferri's (2018) course for educating the educators and model for competencies needed in teaching mathematical modelling, has established foundational structures for teacher education in mathematical modelling. However, a significant theoretical gap exists in understanding how to effectively develop teachers' competence in this domain. *Learning trajectories*—conceptualized as coherent sequences of experiences that progressively build teacher competencies, adapted from Simon's (1995) constructivist perspective on mathematics pedagogy—represent a promising yet underexplored approach to teacher development in mathematical modelling. This is especially true when adapting frameworks to educational contexts where mathematical modelling is not yet formally included in curricula and where teachers lack prior experience with modelling processes and without curricular support.

This study is part of a design research project centered on developing and validating learning trajectories as a theoretical foundation for improving teachers' competence in teaching mathematical modelling. The research utilized a professional competence assessment instrument for teaching mathematical modelling (Wess, Klock, Siller, & Greefrath, 2021), adapted for the Thai context. Twelve mathematics teachers enrolled in a master's degree program participated in a 30-hour professional development course consisting of 10 integrated learning units employing innovative teaching strategies. Data collected through pre- and post-tests, open-ended questions, and reflections revealed significant transformations in teachers' understanding and competencies focusing particularly on the theoretical dimension (including modelling cycles, aims and perspectives of modelling, and modelling tasks) and the task dimension (the development of modelling tasks).

Initially, 91.7% of teachers felt uncertain about their prior experience with mathematical modelling, but by the end of the course, they all demonstrated clear understanding of related concepts. Most teachers (75%) initially viewed modelling merely as a mathematical representation tool; however, after the post-course, all understood it as a cyclical process connecting real-world situations with mathematics. In the task dimension, over 90% developed consistent perspectives on effective modelling tasks. Implementation readiness improved significantly, with teachers demonstrating competence through team teaching and detailed lesson plans. The findings contribute to theoretical understanding by empirically validating a specific learning trajectory for developing teacher competence, beginning with conceptual

transformation followed by integrative incremental growth in task design, implementation, and assessment.

Our learning trajectory approach converts the challenge of teacher unfamiliarity with modelling into an opportunity for systematic professional growth, particularly in educational settings where modelling is not yet mainstream. These theoretical contributions suggest the need for further research on learning trajectories in mathematical modelling competence development across diverse educational systems and context, offering a promising framework for designing scalable professional development models that bridge the gap between theory and classroom practice.

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[168.] Chess players' spacial ethnomodel: Emic, etic and dialogic characterization

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Mathematical modelling and ethnomathematics are currently two topics that generate great interest in the mathematic education research area and in the ICTMA community. This is shown in the increasing number of scientific articles that deal with these topics in the last few years, encouraging the incorporation of mathematical modelling in the schools' curriculum and in initial teacher training from the ethnomathematical perspective in Latin America. One target community for the current research is the chess players as a sociocultural group, whose purpose is registering the procedures and mathematical practices developed in this context. To do so, the objective of this research is characterizing emic, etic and dialogic ethnomodels in the chess players sociocultural group from an ethnomodelling perspective.

Cultural groups are the grounding pillars of research articles based on ethnomathematics and ethnomodelling, thus, the diversification of the cultural groups to study are a challenge for the mathematical education researchers' community (Sepúlveda-Herrera & Huincahue, 2024), hence, the multiple characteristics of chess as a game, encourage to explore the chess players community. In order to know how the current study participatively explores this cultural group and the set of symbols and mathematical relationships that represent cultural artifacts that are present in their practice, will be categorized as emic ethnomodel to the representations elaborated by the members of this cultural group from their own reality, and as etic ethnomodel to those that are elaborated according to the vision of external observers, and dialogic ethnomodel which are based on the shared understanding of the emic and etic (Rosa & Orey, 2022).

This research was applied following a qualitative and exploratory-descriptive focus through Grounded Theory (Gasque, 2007), which was developed in 3 stages; 1) theoretical sampling which a ethnographic study was conducted to two chess clubs from the Metropolitan Region of Chile, and semi-structured interviews to 9 chess players belonging to those two clubs, 2) data coding was applied using ATLAS.ti software, with inductive, axial and selective coding criteria, and 3) the drafting of the represented theory in the pedagogical action, that considered the emic, etic and dialogic ethnomodel of the chess players.

One of the main results came from how the space is characterized by the chess players, and the use of this to predict future movements. The space corresponds to an ethnomodel present in his practice, from the emic, corresponds to the number of squares that player controls on the board, from the etic point of view, it is understood as numbering of squares, calculation and sum of the unit area and optimization. To understand the mathematical principles that the chess players follow, the emic, etic and the dialogic approach were fundamental, that is why the ethnomodelling was transcendental to understand in a more holistic way the mathematical knowledge of a cultural group. On the other hand, it also permitted to highlight a cultural group

that has not been studied enough, allowing teachers to democratize the classroom as well, knowing different cultural realities through the incorporation of different cultural groups. Finally, it was discussed about the pedagogical actions of the identified ethnomodels, with implications in the special treatment of mathematics in initial teacher training.

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[169.] Development of Story-Based Instructional Materials: Fantasy and Misdirection Context

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According to the PISA 2022 results, Japanese junior high school students demonstrate high mathematical literacy (OECD, 2023). However, they are less confident in applying mathematics to real-life challenges compared to the OECD average. Additionally, the findings indicate that these students have limited experience linking mathematics to real-life situations. The process of translating between real-world contexts and mathematics in both directions is called mathematical modelling (Blum and Borromeo Ferri, 2009).

The present research aims to enhance students' engagement with mathematics through mathematical modelling. One effective approach to promoting learner engagement is the use of compelling stories in the classroom (Reigeluth et al., 2016). Stories, those that evoke emotions such as conflict and surprise, can captivate learners and deepen engagement (Zazkis and Lijedahl, 2009).

To this end, we developed and evaluated story-based instructional materials designed to improve junior high school students' engagement with mathematics.

First, we summarized the essential requirements for developing story-based instructional materials. Goal-Based Scenario Theory outlines three core elements for creating story-driven learning contexts: role, cover story, and mission (Nemoto and Suzuki, 2014). Regarding story design, Cordova and Lepper (1996) emphasized that integrating learning activities into a fantasy context encourages active student participation. Furthermore, Dietiker et al. (2023) found that the use of misdirection can generate surprise and provoke new questions, sustaining interest in the learning material. Based on these insights, we defined five key requirements for developing story-based instructional materials: role, cover story, mission, fantasy context, and misdirection. We developed story-based instructional materials aligned with these five requirements for second-year junior high school students studying the topic of "similarity ratio." In this scenario, students take on the role of detectives tasked with locating a building where hostages are being held. They must use their knowledge of similarity ratios to solve the problem. The story incorporates all five key elements, including the fantasy context of a rescue mission and the misdirection of lacking information about the height of the building. Students actively engage in mathematical modelling, transitioning between mathematical reasoning and real-world problem-solving.

As a practical application, lessons using these story-based instructional materials were conducted in July 2024 with 121 second-year students across three classes at a private all-girls junior high school in Tokyo, Japan.

The results of pre- and post-lesson questionnaires revealed the following: (1) Lessons using story-based instructional materials improved students' emotional and behavioral engagement.

(2) Regardless of students' initial interest in mathematics, the story-based approach increased emotional and behavioral engagement. (3) The fantasy context positively impacted students' engagement. (4) The story-based materials made students feel excited. (5) Positive emotions such as excitement and surprise were associated with greater emotional and behavioral engagement. (6) Despite meeting the defined development requirements, the story-based instructional materials lacked sufficient narrative development. (7) The learning objectives were generally achieved through lessons using the story-based instructional materials.

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[170.] Productive Failure in Mathematical Modelling

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One of the essential goals of mathematics education is to help students acquire the ability to apply their mathematical knowledge in a variety of contexts (OECD, 2024). This ability is commonly addressed in the study of mathematical modelling. Mathematical modelling refers to the process of representing the key elements of real-world contexts using mathematical concepts and formulating related questions as mathematical problems (Niss & Blum, 2020). The capacity to engage in such processes is known as mathematical modelling competence (Maaß, 2006).

Although it is crucial for students to develop this competence, mathematical modelling is known to be highly challenging. As such, Durandt et al. (2022) argue that targeted instructional approaches are necessary to support students in acquiring mathematical modelling competence. One such instructional approach is “modeling” in the context of cognitive apprenticeship. In this approach, students observe how experts solve tasks, enabling them to form a conceptual understanding of the steps required to complete similar tasks themselves (Collins, 2006). Blum and Schukajilow (2018) compared three lesson designs over ten sessions with secondary school students: teacher-led, student-centered, and integrated. The integrated design included a “Solution Plan,” which outlined the stages of the modelling cycle and provided scaffolding to help students navigate the challenges of mathematical modelling (Schukajilow et al., 2015). This “Solution Plan” served to demonstrate problem-solving processes to students, closely resembling “modeling” in cognitive apprenticeship. Blum and Schukajilow (2018) reported that the integrated approach was the most effective. This result has also been replicated with higher education students by Durandt et al. (2022).

Although explicit instruction in mathematical modelling has proven effective, recent studies suggest that having students first engage in problem solving before instruction—an approach known as productive failure—may yield even greater learning gains (Kapur, 2012). In productive failure, learners attempt to solve complex problems they are likely to fail at initially. This process activates prior knowledge, highlights knowledge gaps, and promotes awareness of key features of the target concepts (Loibl et al., 2017). Additionally, it has been found to enhance student engagement and motivation to learn the concepts (Kapur, 2024). Many studies have demonstrated its effectiveness in fostering the acquisition and transfer of conceptual knowledge (Sinha & Kapur, 2021).

Given these mechanisms, it is plausible that applying productive failure to mathematical modelling may also enhance students’ modelling competence. However, there is currently limited research in this area. Therefore, the aim of this study is to examine whether implementing productive failure in mathematical modelling lesson can effectively improve students’ modelling competence. We plan to deliver this lesson to second-year junior high school students in May 2025 and evaluate its effectiveness. This instructional approach is expected to be more effective in developing mathematical modelling competence than traditional methods.

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[171.] **Adapting the mathematical modeling cycle to work with students with Autism Spectrum Disorder**

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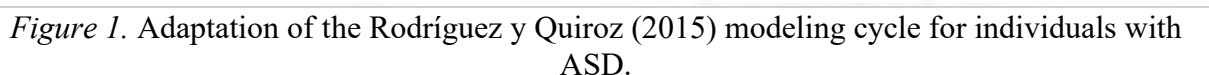
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Inclusive education is a human right that guarantees all individuals can access quality education that enables them to thrive in an ever-changing society (Clavijo and Bautista, 2020). According to the World Health Organization (WHO, 2023), Autism Spectrum Disorder (ASD) encompasses a set of conditions characterized by a high degree of difficulty in social interaction and communication. Barnhart and Dierickx (2021) highlight that people with ASD do not exhibit a medical alteration, but rather a form of human diversity, which is why the concept of “neurodiversity” is often used.

According to López and Pereira (2021), learning mathematical content fosters the development of autonomy and daily life skills, which can be particularly beneficial for people with ASD. As a result, research focused on understanding the mathematics learning process in students with ASD has grown significantly in recent years (Arciniega and Acevedo, 2019; Espinosa Pesántez, 2023; Polo, González and Bruno, 2020; Baixauli, Gómez, Andrés y Berenguer, 2019).

The present research recognizes mathematical modeling as a didactic strategy that bridges everyday real-life contexts with the formulation of mathematical problems. According to Alsina (2007), Aravena and Caamaño (2009) and Bienbengut and Hein (2004), mathematical modeling fosters connections between school mathematics and everyday situations, promotes communication among students, boosts motivation to learn, and encourages the development of strategic thinking.

Therefore, the aim of this research was to reflect on the process of designing mathematical modeling-based situations, making necessary adaptations to support the learning of mathematics for students with ASD. Specifically, adaptations were made following the mathematical modeling cycle proposed by Rodríguez and Quiroz (2015), tailored to the unique characteristics of individuals with ASD (See Figure 1). In order to assess the effectiveness of these adaptations, a modeling situation was designed and implemented with a basic education student diagnosed with ASD level 1 in Mexico.



- **Special interests:** There are diverse topics that individuals with ASD deeply engage with and to which they dedicate a large part of their time. These special interests provide the mathematics teacher with a unique context in which mathematical modeling problems can be introduced.
- **Use of Augmentative and Alternative Communication (AAC) systems:** AAC systems enable the student to better understand the posed problem and facilitate the creation of a pseudo-concrete model.
- **Mathematical diagnosis:** It is essential to assess the student's mathematical knowledge independently of their school grade, as their understanding may surpass or fall short of the expected level for their school grade. Therefore, the mathematical model and results must align with the individual's mathematical diagnosis.

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[172.] Validating an Adapted Mathematical Modeling Knowledge and Attitude Scale (MMKAS) for South African University Students

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Mathematical modelling has gained global recognition as an important component of mathematics teaching that fosters critical thinking, real-world problem solving, and interdisciplinary learning (Hidayat et al., 2022). In South Africa, the integration of mathematical modelling into higher education curricula has been gradually expanding. This reflects broader educational reforms aimed at promoting relevance, contextualization, and applied competencies (Jakobsen & Mhakure, 2024). Despite these advances, validated empirical studies investigating higher education students' knowledge and attitudes toward mathematical modelling—especially within non-mathematics focused disciplines such as social science students in South Africa—remain limited.

To address this gap, this study aimed to validate an adapted version of the Mathematical Modeling Knowledge and Attitude Scale (MMKAS), originally developed by Asempapa and Brooks (2020), for use amongst first-year undergraduate students majoring in Psychology at a South African university. Understanding students' knowledge and attitudes is essential for informing curriculum design, pedagogy, and student support initiatives when mathematical modelling is integrated into the higher education curriculum. However, this study focuses specifically on evaluating the psychometric properties—such as construct validity and internal consistency—of the adapted MMKAS. Therefore, the research question guiding this study is; *how can an adapted version of the MMKAS, for measuring the knowledge and attitudes of first-year Psychology students toward mathematical modelling, be validated in the South African context?*

Using a quantitative, cross-sectional design, the adapted MMKAS was modified to reflect local terminology and contextual examples relevant to South African first-year Psychology university students. The instrument underwent expert review to ensure content validity and cultural appropriateness before being administered to a purposive sample of one hundred and sixty first year undergraduate students enrolled in a Psychology major at a university in South Africa.

Results indicate that the adapted MMKAS demonstrates strong psychometric properties and is well suited for assessing mathematical modelling knowledge and attitudes of South African Psychology students. The validated scale serves as a reliable and contextually appropriate baseline instrument for researchers, educators, and curriculum designers to assess students' initial knowledge and attitudes toward mathematical modelling, enabling the development and evaluation of targeted teaching strategies that foster deeper engagement with modelling practices in higher education. Additionally, the study offers a foundation for further refinement and application of the scale across disciplines and in longitudinal studies.

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[182.] A reflection fika is meant to feed our minds and stomachs. Three senior researchers, **Mogens Niss, Peter Galbraith and Susana Carreira**, will share their impressions and reflections on the day's sessions and put these reflections into different perspectives. The floor will then be open to the audience, who can ask questions, expand on their reflections, dream, criticise or just watch.

[183.] In an era marked by complex societal challenges, mathematics education must evolve to prepare learners not only as problem solvers but as responsible and critical citizens. This panel consisting of **Moderator: Pauline Vos; Panellists: Marcelo C. Borba, Vince Geiger, Britta Eyrich Jessen, Hans-Stefan Siller** explores the intersection of mathematical modelling and education for responsible and critical citizenship, and how mathematical modelling can be a catalyst for civic agency in mathematics classrooms. Through concrete examples, the panel seek to distinguish between responsibility—acting ethically within societal norms—and criticality—questioning those norms and the structures that shape them. A central question in this context is if education for responsible and critical citizenship can thrive without mathematical modelling, or does modelling uniquely empower learners to interrogate data, challenge assumptions, and envision alternatives? The panel also confront and discuss the implications of a mathematics education that sidelines modelling and social context—what is lost when mathematics is taught devoid of ethical and societal relevance?

[P1.] FERMİ Problems for reasoning? – The research project FERMAT

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Fermi problems are a particular type of mathematical modelling tasks that typically contain little to no numerical data (Ärlebäck, 2009). For instance, students might tackle questions such as “How many hours do you spend at school in a year?” or “What is the monthly water consumption of our school?”. To answer these questions, students must determine relevant information and find ways to obtain missing data, such as by drawing on everyday knowledge, estimation, or research (Meyer & Greefrath, 2023). The openness of the problem and solution paths, along with the necessity of handling various possible outcomes, characterize Fermi problems (Ärlebäck, 2009).

This openness not only situates Fermi problems within the context of modelling competencies but also links them to further mathematical competencies, particularly mathematical argumentation. Diverse solution approaches and results could encourage students to discuss their reasoning and the plausibility of their solutions. Questions such as “Which information is important and why?”, “Why did we make these estimations, and are they realistic?”, and “Why is a given result (not) plausible?” could serve as natural prompts for mathematical argumentation among students.

Previous research has already provided insights into the use of Fermi problems. Haberzettl et al. (2018) demonstrated that even young students can engage with Fermi problems and develop their modelling competencies. However, challenges arose in the validation of results. While students found the diversity of approaches interesting, they were often unsettled by the openness of the tasks, leading to initial hesitation in making assumptions (Haberzettl et al., 2018). Similar findings were observed in secondary school students (Surel & Greefrath, 2023). The inherent curiosity, however, could be used to encourage mathematical argumentation.

While theoretically plausible, the empirical validation of Fermi problems as a tool to foster mathematical argumentation remains unexplored. The potential impact of these findings is underscored by research on young students’ mathematical argumentation. Studies suggest that students in grades 3–6, at the transition from primary to secondary education, benefit significantly from structured opportunities for argumentation. Empirical studies have shown that children in this age group generally lack an inherent need to argue mathematically unless explicitly prompted (Schwarzkopf, 2000). However, when provided with targeted opportunities and teacher-initiated scaffolding, they can formulate mathematical arguments with general validity (Jablonski & Ludwig, 2022). The diversity of approaches and solutions in Fermi problems could create cognitive conflicts that serve as genuine occasions for argumentation. Thus, Fermi problems have the potential to enhance mathematics education by integrating modelling and argumentation competencies.

Based on this hypothesis, the FERMAT (*Fermi tasks as an occasion for argumentation activities*) project aims to conduct an empirical, qualitative study on students’ mathematical argumentation activities at the transition between primary and lower secondary education. The

poster gives a deeper insight in the planned study and discusses possible task examples with reference to mathematical argumentation.

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[P2.] Students' perceptions of learning mathematics in Mathematical Modeling and Data Analysis

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This study explores students' perceptions of learning mathematics within the Mathematical Modeling and Data Analysis program at the Faculty of Mathematics, Physics, and Informatics, University of Gdańsk. The curriculum distribution of mathematical and non-mathematical subjects is analyzed, alongside insights from a survey conducted among students.

The survey results reveal varied motivations for choosing the program, with a significant number of students expressing a strong interest in mathematics and its applications. The data indicates that students generally have a positive attitude towards mathematics, appreciating its logical structure and problem-solving aspects. However, challenges such as the theoretical nature of the coursework and the volume of work are highlighted.

Students employ diverse study methods, including independent problem-solving, reading notes, and utilizing online resources. Despite the availability of consultations with instructors, many students do not take advantage of this support due to time constraints or perceived sufficiency of regular classes.

The findings suggest that while students are motivated and engaged, there is room for improvement in the curriculum structure and support mechanisms to enhance learning outcomes. Recommendations include incorporating more practical applications, better structuring of courses, and encouraging the use of consultations.

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- A survey conducted among students.

[P3.] Mathematical Modeling Studies at the University of Gdańsk: Program Structure, Courses, and Student Activities

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This poster presents an overview of the Mathematical Modelling education program at the University of Gdańsk. We outline the curriculum structure, including core and elective courses, and describe the different specialisation paths available to students. We also showcase selected student projects to illustrate how mathematical modelling is applied in practice within our program. In addition, we discuss both the strengths and the challenges encountered in the program's development and implementation, reflecting on successes as well as shortcomings and lessons learned. By sharing insights into the educational approaches and learning experiences of our students, we aim to contribute to the discussion on best practices in mathematical modelling education.

References

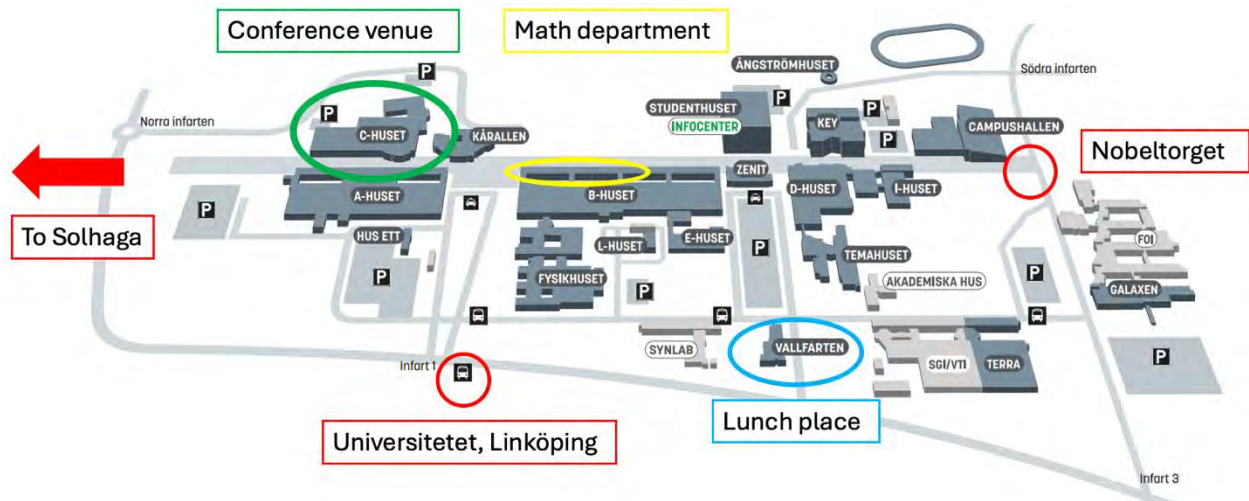
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Conference Venue

The venue of the conference is the campus Valla of Linköping University:

- <https://liu.se/en/article/campus-valla>
- <https://maps.app.goo.gl/5yivFLGhX1wg2Bfx5> (Google Map link:)

In the map below, the **venue for the ICTMA22 conference** is marked in **green** (the C-building), while **bus stops** are highlighted in **red** for your convenience.



Campus Navigation with MazeMap

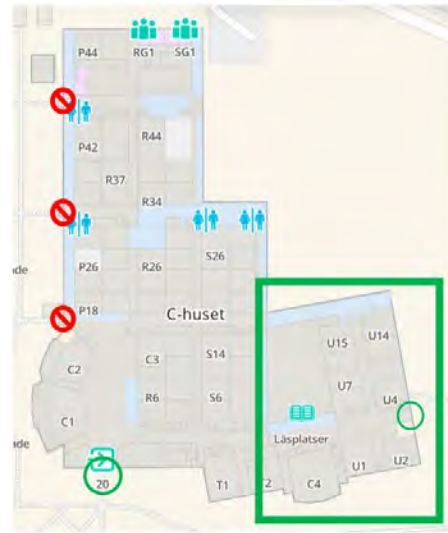
To help you easily find your way around **Campus Valla**, we highly recommend using the **MazeMap app**, which provides detailed and interactive maps of the entire campus.

You can download the app from your preferred app store. Once installed, scan the **QR code on the right** to access a direct link to **Lecture Hall C4**, where the **conference opening**, **keynote sessions**, and **all plenary talks** will be held.



The map to the right highlights the **C-building**, where the **ICTMA22 conference** will be held.

Please note that **some entrances to the C-building are currently closed** due to ongoing reconstruction of the **Corso** area in front of the building. To ensure smooth access, **use the entrances marked with green circles** on the map. These entrances will be **clearly signposted**, and additional signage will guide you from nearby areas.



C-building (Swedish: "C-huset") – the main conference venue (to the right)
The Corso (between the buildings) is under construction work and presently being remodelled.

Transportation

Getting to and from the ICTMA22 Venue Monday 11th – Friday 15th

On the main days of the conference, Monday 11th – Friday 15th, all participants will get a bus ticket valid for travel with Östgötatrafiken. The bus ticket is sent out via email entailing an **activation links**. Please note that

- **Your activation link is personal** and can only be used **once**.
- The ticket can only be activated on **one device**.
- It is **essential** that you open the link in a **browser on your mobile phone** – not on a computer or tablet.
- Activation of the ticket is only possible starting **Monday early morning (just after 00:01)**.

Once activated on your mobile phone, the QR code will appear and serve as your valid bus pass throughout the conference week.

If you encounter any issues or have questions, please don't hesitate to contact us at .

Getting to and from the ICTMA22 Venue and Welcome reception on Saturday 10th

Unfortunately, the bus pass for ICTMA22 will only be valid from Monday to Friday, August 11–15. Participants will need to arrange their own transportation to the venue for the ICTMA22 Early Career Research Day (and the ICTMA22 Welcome reception).

We recommend taking the bus and downloading the “Östgötatrafiken” app (available for both [Android](#) or [Apple](#) devices) to easily check timetables and plan your journey.

In addition to planning your travels, you can also use the app to pay for your ticket by connecting a credit or debit card. Alternatively, you can purchase a ticket onboard by tapping your payment card against the ticket reader. If you're traveling within the city, this will provide you with a ticket valid for the city zone.

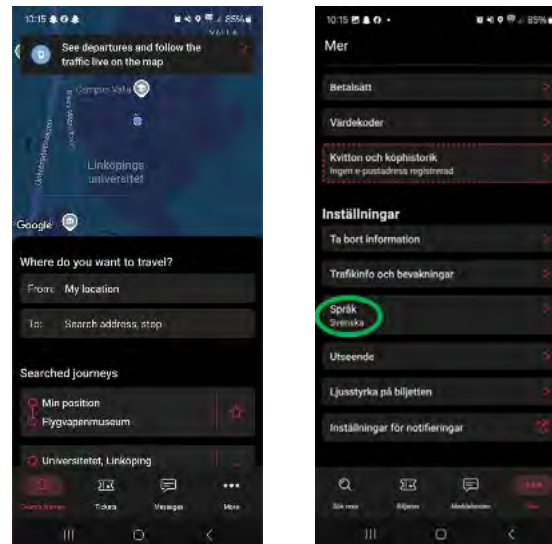


The Östgötatrafiken app and recommended bus lines

The app is relatively intuitive to use, but here is a quick guide to get you started.

The **left image** shows the main screen of the app, where you can search for journeys and plan your routes. You can also view timetables for buses and trains directly from this screen.

If the app opens in Swedish, you can change the language by tapping the **“Mer”** (More) tab in the bottom right corner. Then, under the **“Inställningar”** (Settings) section, select **“Språk”** (Language) to choose your preferred language. (See the **right image** for guidance.)



There are three main bus lines that are convenient for traveling between central Linköping and Campus Valla, where ICTMA22 is held (the names of suggested bus stops are written in *italics*).

- If you are staying at **Scandic Frimurarehotellet**, we recommend taking:
 - **Bus 12** from *Universitetet, Linköping* to *Kungsgatan*, or
 - **Bus 3** from *Solhaga* to *Kungsgatan*.
- If you are staying at **Scandic Linköping City**, you can also use these buses, but note that a short walk is required from the bus stop to the hotel. Alternatively, you may take:
 - **Bus 4** from *Nobeltorget* to *Hamngatan–Nygatan, Linköping*, which offers a slightly shorter walk to the hotel.

The images below show the bus stops **Nobeltorget**, **Universitetet, Linköping**, and **Solhaga**, all located near Campus Valla.



Other modes of transportation in Linköping

Uber is available in Linköping and operates 24/7, making it a flexible choice for spontaneous or scheduled rides. You can request a ride directly through the Uber app.

Traditional taxis remain a popular and reliable mode of transport in Linköping. Several companies offer competitive pricing, fixed rates, and booking options (online bookings are often discounted):

- Linköping 013 Taxi: +46 (0)13 30 30 13; <https://linkopingt看xi.com>
- Gripen Taxi: +46 (0)13 30 30 00; <https://gripentaxi.se/>
- Taxibil i Östergötland: +46 (0)13 14 60 00; <https://taxibil.se>

Social Program, Excursions and “Dining in Linköping”

Welcome reception

Date: Sunday, 10 August 2025

Time: 18:00–20:00

Location: Vallfarten, Linköping University (*see campus map*)

The Welcome Reception for ICTMA22 will be held at **Vallfarten**, a central venue on campus that also serves as the lunch location on Monday, Tuesday, and Thursday. This informal gathering marks the start of the conference and offers a great opportunity to meet fellow participants in a relaxed setting.

Information about suitable bus lines and a timetable for reaching Vallfarten will be sent to all participants via email prior to the event.

Conference Dinner

The venue for the conference dinner is the Swedish Air Force Museum

(<https://flygvapenmuseum.se/en/start/>), where conference delegates will enjoy a dinner and drinks surrounded by aircrafts and dancing. The evening begins with a guided tour and exploration of the museum.

Information about suitable bus lines and a timetable for reaching the Swedish Air Force Museum will be sent to all participants via email prior to the event.



Excursions

Discover Norrköping!



Looking for an exciting getaway? Just a short trip from Linköping lies the charming city of Norrköping, renowned for its rich history and vibrant culture. *Explore the City:* Wander through the picturesque streets and take in the unique blend of old and new architecture. *Sightseeing:* visit some of Norrköping's most iconic sights, including the beautiful Motala Ström river and the historic Industrial Landscape. *Free Museums:* Enjoy free entry to the city's fascinating

museums, where you can delve into Norrköping's past and present.

Experience Linköping like never before: Guided walking tour!



Join us for an unforgettable guided tour of Linköping, where groups of participants will have their own personal guide to lead them through the city's most captivating sights. Enjoy the luxury of having your own guide, ensuring an engaging tour. The tour will discover the rich history of Linköping as you visit landmarks such as the

majestic Linköping Cathedral. You'll learn fascinating stories and facts about the city's past and present from your knowledgeable guide.

A trip to historic Vadstena!



Join us for an enchanting afternoon trip to the historic city of Vadstena, where history comes alive! Sit back and relax as we take care of your journey to and from Vadstena and enjoy a guided tour with a knowledgeable guide who will share fascinating stories and insights. During the afternoon we will visit the city's renowned museums, majestic castles, and ancient monasteries, all steeped in rich history.

Evening cruise on Göta canal and lake Roxen!



Experience an unforgettable evening as you glide gently between sparkling waves and twinkling stars. This picturesque setting is perfect for a successful night out. Enjoy the stunning backdrop as we cruise along Lake Roxen and Göta Canal, passing through a total of 14 locks. Included in the price of the trip is a feast of seafood delicacies, including as many shrimp as you like and it includes two 50 cl beers or two glasses of wine, along with butter, bread, cheese, mayonnaise, and coffee after the meal. We will arrange transfer to the boat that departure at 19:00 from Berg and return back at 23:00 thereafter you will be picked up by a bus, that take you back to Linköping.

Alternative, not centrally organised, options

There are also many other options for shorter (evening) excursions in the city of Linköping and around the Linköping area (please note that none of these are included in the conference fee). Some possibilities are (listed in no particular order):

- Nature experiences: <https://visitlinkoping.se/en/see-and-do/nature-experiences/>
- Norrköping visualization center: <https://visualiseringscenter.se/en>
- Swedish Air Force Museum: <https://flygvapenmuseum.se/en/start/> (*This is presently the venue for the conference dinner*)
- Old Linköping Open Air Museum: <https://visitlinkoping.se/en/do/attractions/museums/gamla-linköping-open-air-museum>
- Linköping Castle and Cathedral Museum: <https://lsdm.se/in-english/>
- Linköping Cathedral: <https://visitlinkoping.se/en/se-och-gora/linkopings-domkyrka/>
- Östergötlands Museum: <https://ostergotlandsmuseum.se> [only Swedish webpage]

“Dining in Linköping”

Going out for a drink in Sweden can be quite expensive, especially when it comes to alcoholic beverages at restaurants and bars. However, during "After Work" hours, many places offer discounted prices on drinks and food, making it a more affordable and social way to unwind after a day at work.



Linköping offers a **diverse culinary scene**, with restaurants to suit every taste and budget. Whether you're in the mood for traditional Swedish cuisine, Mediterranean flavors, Asian fusion, or classic pub food, you'll find plenty of options throughout the city. The area around **Stora Torget**, the main square, is especially lively, with many restaurants and bars offering After Work deals.

Most venues offer After Work specials between **15:00 and 18:00**, while some extend their offers until **19:00** or even later. For example:

- **Brasserie Storan** and **Merwah** offer After Work menus until 18:00.
- **Ciocolata Bistro** extends its After Work until **19:00**.
- **Brasserie Bouquet** serves After Work deals until **18:30**.

A highlight is **Bierhaus**, a German-style pub near Stora Torget, where After Work runs from **15:00 to 18:00**. If you're attending an event and show your ICTMA22 participant badge, you can enjoy After Work prices **until 20:00**.

Explore the flavors of Linköping

Linköping is home to a vibrant mix of restaurants representing cuisines from all over the world. Here's a taste of what you can expect:

- **Swedish & Scandinavian:** Modern takes on traditional dishes at places like *Stångs Magasin* and *Restaurang Jord* [these two are both really expensive].
- **Italian & Mediterranean:** Authentic pizzas and pastas at *Massimo* and *OLYMPIA - DA VINCI*, plus tapas and seafood at *Aioli Tapasbar*.
- **Asian Cuisine:** Sushi at *Sushi Yama* and *Little Tokyo*, and Thai/Vietnamese flavors at *Sol & Ris* and *Miss Búi*.
- **French & European:** Elegant dining at *Restaurant Parisien* and *Brasserie X*, or fusion dishes at *Ciocolata Bistro*.
- **Indian & Middle Eastern:** Rich spices and hearty meals at *Yogi* and *Merwah*.
- **Steakhouses & Grills:** Hearty meat dishes at *Flaming Bull Steakhouse* and *Texas Longhorn*.
- **Cafés & Bakeries:** Cozy spots for coffee and pastries like *Babettes Kafferri* and *Simons Rosteri & Bageri*.
- **Bars & Pubs:** Relaxed atmospheres with good food and drinks at *De Klomp*, *Pitcher's*, and *Bierhaus*.

Whether you're looking for a cozy bistro, a lively beer hall, or a stylish wine bar, Linköping has something for everyone—especially during After Work hours.