

# The Mathematical Colloquia held in 2017 at Linköping University

Organized by **Anders Björn**, **Milagros Izquierdo Barrios**, **Vladimir Kozlov**, and **Hans Lundmark**.

## 2 Onsdag 11 januari 2017, Visa Latvala, University of Eastern Finland, Joensuu, Finland

*Weak Harnack estimates for quasisub- and quasiperminimizers with non-standard growth*

Sammanfattning: Weak Harnack estimates can be regarded as generalizations of submeanvalue and supermeanvalue properties of classical subharmonic and superharmonic functions. These estimates are important tools in the nonlinear potential theory. For the  $p$ -Laplace type equations, weak Harnack estimates can be obtained either by Moser's iteration method or by De Giorgi's method. DiBenedetto and Trudinger extended the latter method in 1980's to a very general class of quasisub- and quasiperminimizers. The purpose of the talk is to give an overview on results related to quasiminimizers of the variational integrals with non-standard growth. We focus on the variable exponent growth but also comment on the recent development related to generalized Orlicz spaces.

## 2 Onsdag 18 januari 2017, Alexander I. Nazarov, Saint Petersburg, Ryssland

*The influence of geometry of a manifold on attainability of the norm of the Sobolev trace embedding operator*

Sammanfattning: Let  $\Omega$  be a smooth compact Riemannian  $n$ -dimensional manifold with smooth boundary, and let  $1 < p < n$ . We consider the trace Sobolev embedding

$$\lambda_p \equiv \inf \frac{\|v\|_{W_p^1(\Omega)}}{\|v\|_{L_{p^*}(\partial\Omega)}} > 0 \quad (1)$$

(here  $p^* = (n-1)p/(n-p)$ ). Similarly to the case of conventional Sobolev embedding (see the survey [3] and references therein), the attainability of the infimum in (1) heavily depends on the geometry of  $\Omega$ . This talk is partially based on the following joint papers with Alexander Reznikov.

1. A.I. Nazarov, A.B. Reznikov, On the existence of an extremal function in critical Sobolev trace embedding theorem, J. Funct. Anal. 258 (2010), N11, 3906--3921.
2. A.I. Nazarov, A.B. Reznikov, Attainability of infima in the critical Sobolev trace embedding theorem on manifolds, AMS Transl. Series 229 (2010), 197--210.
3. A.I. Nazarov, Dirichlet and Neumann problems to critical Emden-Fowler type equations, J. Global Optim. 40 (2008), 289--303.

## 2 Onsdag 25 januari 2017, Adrian Muntean, Karlstads universitet

*Reaction-diffusion systems with distributed microstructures: well-posedness and homogenization asymptotics*

Sammanfattning: The discussion will address the multiscale modeling, analysis and approximation of a class of reaction-diffusion systems posed simultaneously on both macroscopic and microscopic space scales. The coupling between the scales is done via micro-macro transmission conditions. Our target system has a typical structure for reaction-diffusion-flow problems in media with distributed microstructures (also called, double porosity materials), reminding the work of G. Barenblatt on flows through fractured media. The talk will focus on an elementary derivation of the two-scale structure based on formal homogenization asymptotics. Then I will give a hint on how can one handle the weak solvability of the system and will point out a route to justify the homogenization asymptotics rigorously. A numerical example will illustrate how the weak solution "communicates" between the space scales.

## 2 Onsdag 1 februari 2017, Clas Rydgergren, ITN, Norrköping

*New sources of input data for travel demand estimation models*

Sammanfattning: Forecasts of how journeys are made, from where and to where, is usually done using simple mathematical models. The results from the models are used as estimates of the load on the transport system. Estimates are made for the load today, in the future, and to estimate the change in travel demand when changes in the infrastructure are made. The model result is critical input to traffic planners. Different types of models are used depending on, among other things, if the analysis requires the results to include all modes of transport, or not, and the length of the forecast horizon. Models for long-term forecasts (several years) often contain components to describe the travelers' values and perceptions whereas models for the current situation or with a very short forecast horizon, often is based exclusively a network model and data from dedicated traffic measurements. During this seminar, I will present an example of a traditional demand model, and exemplify how this model is used. The ongoing digitization have led to new sources of input data for this type of models. I will exemplify this by presenting inputs used in a couple of research projects ongoing at the Division of Communications and Transport System (KTS) at ITN.

## 2 Onsdag 8 februari 2017, Viktor Kolyada, Karlstads universitet

*On Gagliardo-Nirenberg type inequalities*

Sammanfattning: We prove a Gagliardo-Nirenberg type multiplicative estimates for Lorentz norms of a function. These estimates are expressed in terms of norms of derivatives of a function and its homogeneous Besov norms of negative order. One of our main results is a refinement of the well-known strong version of the Sobolev embedding theorem involving Lorentz norms. Our methods are based on estimates of nonincreasing rearrangements of functions in terms of heat kernels. These methods enable us to cover also the case of Sobolev norms with  $p = 1$ . This is joint work with F.J. Pérez Lázaró (Universidad de La Rioja, Spain).

## 2 Onsdag 15 februari 2017, Maria Przybylska, Zielona Góra, Polen

*Analytical method of spectra calculations for quantum optics systems in the Bargmann representation*

Sammanfattning: The fundamental problem of quantum mechanics is solving the eigenvalue problem for a given Hamilton operator, i.e. determination of eigenfunction called a wave function and corresponding eigenvalue called the energy spectrum. We show that for a certain class of quantum mechanics problems one can use the so-called Bargmann representation which allows one to rewrite the eigenvalue equation as a system of linear equations with one independent complex variable. Using this representation we distinguish three types of conditions determining the spectrum: local conditions around each singularity, compatibility condition joining local solutions around different singularities and normalization condition related to the proper growth order of solutions. A few examples of quantum optics systems describing the interaction of one mode of electromagnetic radiation with two-level atom will be considered and obstructions on spectrum given by these conditions will be presented. In some cases, one can find closed form expression on spectrum formulated by means of transcendental functions of parameters of the systems such as confluent Heun functions or generalised Heun functions.

## 2 Onsdag 1 mars 2017, Milagros Izquierdo, MAI

*On the Connectivity of Branch Loci of Spaces of Curves*

Sammanfattning: Since the 19th century the theory of Riemann surfaces has a central place in mathematics putting together complex analysis, algebraic and hyperbolic geometry, group theory and combinatorial methods. Since Riemann, Klein and Poincaré among others, we know that a compact Riemann surface is a complex curve, and also the quotient of the hyperbolic plane by a Fuchsian group. In this talk we study the connectivity of the moduli spaces of Riemann surfaces (i.e in spaces of Fuchsian groups). Spaces of Fuchsian groups are orbifolds where the singular locus is formed by Riemann surfaces with automorphisms: the branch loci: With a few exceptions the branch loci is disconnected and consists of several connected components. This talk is a survey of the different methods and topics playing together in the theory of Riemann surfaces.

## 2 Onsdag 8 mars 2017, Natan Kruglyak, MAI

*Theory of Interpolation (review)*

Sammanfattning: A year ago I gave a talk during which I have discussed what was done in interpolation theory before 1980. Now I plan to remind (shortly) what was discussed last year and will focus on some results which were obtained after 1980.

## 2 Onsdag 15 mars 2017, Håkan Hedenmalm, KTH

*Bloch functions, asymptotic variance, and geometric zero packing*

Sammanfattning: In connection with the study of the universal integral means spectrum for quasiconformal mapping, it turns out that the main term for small exponents and small Beltrami coefficients is governed by the asymptotic variance introduced by McMullen for a dynamical situation. This follows from work of Oleg Ivrii. The fact that this universal asymptotic variance is less than 1 is shown here. This then leads to the unexpected result that the quasiconformal universal variance is not of the form assumed so far. To obtain the result, we use duality to turn the problem into a problem of analyzing an improvement in the Cauchy-Schwarz inequality. The resulting dual problem has geometric interpretation in terms of "zero packing". In the planar case this is related with Abrikosov's analysis of superconductivity for which he obtained the Nobel prize.

## 2 Onsdag 22 mars 2017, Thomas Geisser, Rikkyo University, Tokyo, Japan, och Institut Mittag-Leffler

*Special values of zeta-functions*

Sammanfattning: One can associate to an algebraic variety over  $\mathbb{Z}$ , i.e. the solution set of multi-variable polynomial equations with integer coefficients, the so-called zeta-function, which encodes the number of solutions of the equations with entries in finite fields. This generalizes, for example, the Riemann zeta function. Expressing the value of the zeta-function at integers in terms of other invariants often gives deep arithmetic formulas. We give an introduction to zeta-functions and examples of such formulae.

## 2 Onsdag 29 mars 2017, Panu Lahti, MAI

*Fine boundaries and Federer's characterization of sets of finite perimeter in metric spaces*

Sammanfattning: Functions of bounded variation (BV functions) are a class functions that is somewhat more general than Sobolev functions, in that they may have discontinuities and even "jumps", but are nonetheless differentiable in a very weak sense. Various minimization problems are natural to formulate for the BV class, due to its good compactness properties. In this talk I focus on sets of finite perimeter, which are sets whose characteristic functions are BV functions. In the Euclidean setting, the so-called Federer's characterization states that a set is of finite perimeter if and only if its measure theoretic boundary has finite "surface area". In the more general setting of a metric measure space, the characterization remains an open problem. In the talk I will show how we can obtain a slightly different characterization by replacing the measure theoretic boundary with a new concept, the so-called fine boundary.

**Onsdag 12 april 2017, Sergey Nazarov, MAI**

*Singularities caused by kissing balls*

Sammanfattning: It will be demonstrated that the gradient of eigenfunctions of the Neumann problem for the Laplace operator at the tangency point of two balls in dimension  $d > 2$  get a singularity  $r^{-0.586\dots}$  which is even higher than the classical square-root singularity of stresses in the theory of cracks. At the same time, in dimension  $d = 2$  all eigenfunctions are infinitely differentiable at the tangency point of two discs. Furthermore, for the Dirichlet problem, eigenfunctions are smooth in any dimension. The Steklov problem related to water-waves, will be discussed as well. This work is done in cooperation with Jari Taskinen, University of Helsinki.

**Onsdag 19 april 2017, Vladimir Guletskii, University of Liverpool, UK, och Institut Mittag-Leffler**

*Motivic obstructions to rationality of a very general cubic fourfold in  $P^5$*

Sammanfattning: After a short overview of intersection multiplicities, rational equivalence of algebraic cycles and motives, the notion of integral (in)decomposability of the transcendental motive of a smooth projective surface  $S$  over a field will be introduced. As an example, I will explain how to prove that the transcendental motive is integrally indecomposable when  $S$  is the self-product of a smooth projective curve having enough morphisms onto an elliptic curve with complex multiplication. This applies, for instance, if the curve is the Fermat sextic in  $P^2$ . Our main result to be explained in the talk asserts that if the transcendental motive is finite-dimensional and integrally indecomposable, for any smooth projective surface  $S$  over the field of complex numbers, then the famous irrationality conjecture for a very general cubic fourfold in  $P^5$  is true.

**Onsdag 26 april 2017, Carl Johan Casselgren, MAI**

*Completing partial Latin squares*

Sammanfattning: Consider an  $n \times n$  array  $P$  where each cell contains a symbol from the set  $\{1, \dots, n\}$ . If each symbol occurs at most once in every row and column of  $P$ , then  $P$  is a partial Latin square, and if in addition, no cell in  $P$  is empty, then  $P$  is a Latin square. An  $n \times n$  partial Latin square  $P$  is completable if it is possible to fill the empty cells of  $P$  so that each of the symbols  $1, \dots, n$  occurs exactly once in each row and exactly once in each column of the array; that is, if there is an  $n \times n$  Latin square  $L$  such that for any nonempty cell  $(i, j)$  of  $P$ ,  $L$  and  $P$  contains the same symbol in position  $(i, j)$ . In this talk I shall discuss some classic results on completing partial Latin squares, and also mention some recent progress in this area.

**Onsdag 3 maj 2017, David Rule, MAI**

*The global boundedness of Fourier integral operators on local Hardy spaces*

Sammanfattning: The question of the local  $L^p$ -boundedness of Fourier integral operators when  $p \neq 2$  was answered in work of Seeger-Sogge-Stein in the early nineties. But only recently have Ruzhansky-Sugimoto found sufficient conditions to prove global  $L^p$ -boundedness. We build on their methods to prove the global boundedness of Fourier integral operators in the (mostly) quasi-Banach setting of local Hardy spaces  $h^p$  in the range  $n/(n+1) < p \leq 1$ . This is joint work with Salvador Rodríguez-López and Wolfgang Staubach.

**Onsdag 10 maj 2017, Nageswari Shanmugalingam, MAI**

*Notions of quasiconformality in non-smooth setting*

Sammanfattning: The aim of this talk is to give an overview of different notions of quasiconformality that are equivalent in the Euclidean setting, and the relationships between them under certain geometric assumptions on the non-smooth metric measure spaces.

**Onsdag 17 maj 2017, Oscar Perdomo, Central Connecticut State University, New Britain, CT, USA**

*Embedded constant mean curvature hypersurfaces on spheres*

Sammanfattning: In this talk we will discuss hypersurfaces of the  $(n+1)$ -dimensional unit sphere with exactly two principal curvatures and constant mean curvature -cmc-. Besides providing an explicit construction for these hypersurfaces, we will show that for every positive integer  $m > 1$  and any  $H$  between  $\cot(\pi/m)$  and  $b_{mn} = (m^2 - 2)((n-1)/(n^2(m^2 - 1)))^{1/2}$ , there exists an embedded hypersurface with cmc  $H$  and with group of isometries invariant under the cyclic group  $Z_m$ . When  $H$  is close to  $\cot(\pi/m)$ , the hypersurface looks like a necklace made out of  $m$  spheres and  $m+1$  catenoid necks attached. When  $H$  is close to  $b_{mn}$ , the hypersurface looks like the cartesian product of an  $(n-1)$ -dimensional sphere with a circumference. Several images of these examples will be shown. When  $n=2$ , this is, for surfaces in the three dimensional sphere, we have that for every  $H$  between  $\cot(\pi/m)$  and  $(m^2 - 2)/2(m^2 - 1)^{1/2}$ , there exists an embedded surface with cmc  $H$ . Andrews and Li showed that these surfaces are the only embedded tori in the sphere with cmc. We will finish the talk by doing some comments on Andrews and Li's proof.

**Torsdag 1 juni 2017, Sony Chan, Hun Kanal, Ngonn Seam and Lin Sok, Phnom Penh, Kambodja**

Abstract Sony Chan:

## A weighted Fourier series with signed good kernels

Sony Chan

### Abstract

It is natural to try to find a kernel such that its convolution of integrable functions converges faster than that of the Fejér kernel. In this thesis, we introduce weighted Fourier partial sums which are written as the convolution of signed good kernels and prove that  $L^2$  convergence of the weighted Fourier partial sums is much faster than that of the Cesàro means. In addition, we present two numerical experiments.

Abstract Hun Kanal:

Title: Probabilistic analysis of a digital search tree in compression algorithms.

Hun Kanal, Lecturer at the department of Mathematics, RUPP.

The digital search tree (dst) plays a central role in compression algorithms, of Lempel-Ziv type. This structure can be viewed as a mixing of a digital structure (the trie) with a binary search tree. Its probabilistic analysis is thus involved, even in the case when the text is produced by a simple source (a memoryless source, or a Markov chain). The analysis for memoryless sources was first performed in the paper of Flajolet and Sedgewick (1986), then Jacquet, Louchard, Prodinger Szpankowski, Tang (between 1990 and 2001) deal with general memoryless sources or Markov chains, and perform the analysis of the main parameters of Digital Search Trees (DST) –namely, internal path length, profile, typical depth-. This talk performs probabilistic analyses of the depth of a dst, when it is built on words emitted by a general source. We prove that for a large class of sources, the typical depth follows an asymptotic gaussian law, with an optimal speed of convergence. The finding is established by analytic and probabilistic methods (i.e. generating functions, Poisson and Mellin transform, Rice formula and asymptotic analyses). The methods involve also dynamical analysis which mixes in an original setting methods from analytic combinatorics and methods from dynamical system theory (namely transfer operators, and their spectral properties).

Abstract Ngonn Seam:

**ABSTRACT.** This work deals with the study of a nonlinear pseudo parabolic problem: Find  $u \in H^1(0, T; H_0^1(\Omega))$  such that

$$f(\partial_t u) - \operatorname{div}[a(u) \nabla u + b(u) \nabla \partial_t u] = g \quad \text{on } Q$$

with the initial condition

$$u(0, \cdot) = u_0 \text{ on } \Omega$$

By the way of an implicit time discretisation, we would prove the existence of a solution to the problem. We would illustrate numerical solution by finite elements method.

Abstract Lin Sok:

## Orthogonal group and Boolean functions

Lin Sok

Department of Maths, Royal University of Phnom Penh,  
12156 Phnom Penh, Cambodia , [sok.lin@rupp.edu.kh](mailto:sok.lin@rupp.edu.kh)  
(joint work with Minjia Shi and Patrick Solé)

### Abstract

In this talk, we study orthogonal group over finite fields. We show how to construct self-dual codes and linear complementary dual (LCD) codes over large finite fields from the elements in the group and explore the connections with the generalized  $Z_{2^m}$  self-dual bent functions. We prove existence of optimal LCD codes of some certain lengths over large finite fields. We prove non-existence of the generalized  $Z_{2^m}$  regular bent functions in odd variables and classify them in low even variables.

#### 2 Onsdag 21 juni 2017, Anita Rojas, Universidad de Chile

*A Sage package for equisymmetric stratification and applications*

Sammanfattning: In this talk we present an algorithm running over the mathematical software Sage, which allows users to compute group actions on Riemann surfaces up to topological equivalence, in terms of generating vectors. Moreover, it allows to compute induced generating vectors when considering subgroups of a given group, hence to classify the corresponding action up to topological equivalence. The main motivation to develop this algorithm is to study the equisymmetric stratification of the moduli space of Riemann surfaces of genus  $g$  defined by Broughton in 1990.

#### 2 Onsdag 23 augusti 2017, Petros Petrosyan, Yerevan State University, Armenien

*Number of palettes in edge-colorings of graphs*

Sammanfattning: A proper edge-coloring of a graph  $G$  is a mapping  $\alpha: E(G) \rightarrow \mathbb{N}$  such that  $\alpha(e) \neq \alpha(e')$  for every pair of adjacent edges  $e, e' \in E(G)$ . If  $\alpha$  is a proper edge-coloring of a graph  $G$  and  $v \in V(G)$ , then the palette of a vertex  $v$ , denoted by  $S(v, \alpha)$ , is the set of all colors appearing on edges incident to  $v$ . For a proper edge-coloring  $\alpha$  of a graph  $G$ , we define  $S(G, \alpha)$  as follows:  $S(G, \alpha) = \{S(v, \alpha): v \in V(G)\}$ . For every graph  $G$  and its proper edge-coloring  $\alpha$ , we have  $1 \leq |S(G, \alpha)| \leq |V(G)|$ . In 1997, Burris and Schelp introduced the concept of vertex-distinguishing proper edge-colorings of graphs. A proper edge-coloring  $\alpha$  of a graph  $G$  is a vertex-distinguishing edge-coloring if for every pair of distinct vertices  $u$  and  $v$  of  $G$ ,  $S(u, \alpha) \neq S(v, \alpha)$ . This means that if  $\alpha$  is a vertex-distinguishing edge-coloring of  $G$ , then  $|S(G, \alpha)| = |V(G)|$ . On the other hand, recently Horňák, Kalinowski, Meszka and Woźniak initiated the study of the problem of finding proper edge-colorings of graphs with the minimum number of distinct palettes. For a graph  $G$ , they define the palette index  $\hat{s}(G)$  of a graph  $G$  as follows:  $\hat{s}(G) = \min_{\alpha} |S(G, \alpha)|$ , where minimum is taken over all possible proper edge-colorings of  $G$ . In this talk we will give a survey of the topic and present a recent progress in the study of palette indices of graphs.

#### 2 Onsdag 30 augusti 2017, Jari Taskinen, Helsingfors Universitet, Finland

*Band-gap spectra of some elliptic equations and systems on waveguides*

Sammanfattning: We consider the band-gap structure of the essential spectrum of some elliptic spectral problems on periodic 2- and 3-dimensional waveguides. In the recent paper with S. Nazarov [1] we study the linearized piezoelectricity system on waveguides with thin structures, which are created by thin ligaments connecting (infinitely many, translated copies of) bounded cells. We establish the existence of an arbitrary number of gaps, if the connecting ligaments of the cells are thin enough. The problem is non-selfadjoint, thus we apply a self-adjoint reduction scheme; also the mere existence of the band-gap structure for the essential spectrum needs a new proof, which we able to provide. In the work [2] with F. Bakharev we study the linearized elasticity system for waveguides, the geometry of which is similar to the above situation. We perform an asymptotic analysis to obtain quite precise information on the position of the spectral bands. Finally, in the project [3] we study the Laplace-Dirichlet problem in the plane which is perforated by a periodic lattice of discs with radius  $r > 0$ . Applying Floquet-Bloch-Gelfand-techniques we show that the FBG-eigenvalues depend real analytically on the geometric parameter  $r$ . This leads to a non-existence result for eigenvalues of infinite multiplicity.

[1] S. Nazarov, JT: Spectral gaps for periodic piezoelectric waveguides, Z. Angew. Math. Phys. 66, 6 (2015), 3017-3047.

[2] F. Bakharev, JT: Bands in the spectrum of a periodic elastic waveguide. To appear in Z. Angew. Math. Phys.

[3] M. Lanza de Cristoforis, P. Musolino, JT: work in preparation.

#### 2 Onsdag 6 september 2017, Ugo Gianazza, University of Pavia, Italien

*A self-improving property of degenerate parabolic equations of porous medium-type*

Sammanfattning: We show that the gradient of solutions to degenerate parabolic equations of porous medium-type satisfies a reverse Hölder inequality in suitable intrinsic cylinders. We modify the by-now classical Gehring lemma by introducing an intrinsic Calderón-Zygmund covering argument, and we are able to prove local higher integrability of the gradient of a proper power of the solution  $u$ . This is a joint work with Sebastian Schwarzacher of Charles University.

#### 2 Onsdag 13 september 2017, Nikolay Kuznetsov, Russian Academy of Sciences, St. Petersburg, Ryssland

*The Legacy of Vladimir Andreevich Steklov in Mathematical Physics: Work and School*

Sammanfattning: Steklov's major contributions mathematical physics will be described along with those of his students (Friedmann, Tamarkin, Smirnov, Shohat) and of students of his students and so on. Steklov's role in making the Russian Academy of Sciences survive during the post-1917 years will be also outlined as well as further development of the Academy in which he participated as its vice president.

#### 2 Onsdag 4 oktober 2017, Samuel Bengmark, Chalmers och Göteborgs universitet

*Success-factors in Transition to University Mathematics*

Sammanfattning: We search for the relative importance of different factors influencing students' performance in first year university mathematics. Characteristics of students at three universities in Sweden ( $n=1007$ ) were measured twice; when entering the university and at the end of the first year. Four important constructs were found which we label Self-efficacy, Motivation type, Study habits and Views of mathematics. Together the predictive power of these four constructs, on the students' success in university mathematics, was in the same range as the grades from upper secondary school. Individually each of the constructs had little predictive power but some aspects seemed more important for students with low grades from secondary school.

#### 2 Onsdag 11 oktober 2017, Mikael Hansson, MAI

*A word property for twisted involutions in Coxeter groups*

Sammanfattning: Coxeter groups are important objects in, e.g., combinatorics and geometry. The word property for Coxeter groups solves the problem of deciding whether two reduced words in the generators represent the same group element. In the last few years, several people have proved word properties for twisted involutions in various Coxeter groups. I will review these results, and then present a word property which holds in the general case. If there is time, I will outline the proof. This is joint work with Axel Hultman.

#### 2 Onsdag 18 oktober 2017, Sergey Vakulenko, St Petersburg, Ryssland

*Biodiversity, extinctions and limit evolution structures in foodwebs*

Sammanfattning: This talk is devoted to the problem of large food web stability and extinctions, mass extinctions under climate changes in such webs. The foodweb model describes ecosystems, where a number of species share a few of resources. The model extends previous ones and takes into account extinctions and self-limitation effects. Under some conditions, we prove a theorem about global convergence of solutions for two cases: for models without extinctions and with ones. The main difference between those cases is that,

when extinctions are taken into account, evolution is fundamentally non-predictable. We estimate a limit level of biodiversity and obtain an explicit estimate involving system parameter, that estimate is valid for any kinds of ecosystem dynamics. The is joint work with V. Kozlov and U. Wennergren.

📅 **Onsdag 15 november 2017, Jan-Åke Larsson, ISV**

*Efficient simulation of some quantum computer algorithms*

Sammanfattning: A long-standing aim of quantum information research is to understand what gives quantum computers their advantage. Such an understanding would be of great benefit when attempting to build a quantum computer. Here we present a framework that uses classical resources but still is able to efficiently run, for example Deutsch-Jozsa and Simon's algorithms, and also can run Shor's factoring algorithm with some systematic errors. We also perform an experiment factoring 15 using classical pass-transistor logic at room temperature, with smaller systematic errors than any former experimental implementation, and the same amount of resources in time and space as a scalable quantum computer. Our results give further insight into the resources needed for quantum computation, aiming for a true understanding of the subject.

📅 **Onsdag 22 november 2017, Filippo Remonato, Norges teknisk-naturvitenskapelige universitet, Trondheim**

*Water waves solutions of the Euler equations with affine vorticity*

Sammanfattning: I will present several solutions of the Euler equations with affine vorticity. We start by reviewing the bifurcation theory for the water wave problem, following in the footsteps of Ehrnstrom-Escher-Wahlén, and then turn our attention to the numerical approach, where the combination of standard Finite Elements and B-splines basis functions, recently known as Isogeometric Analysis, is used to solve the Euler equations in their full free-boundary setting, without any reduction to a fixed domain. Periodic travelling waves solutions are found bifurcating from the line of trivial solutions in accordance with the theory, and we will look at several branches for both small and large amplitude waves with particular attention to the internal critical layers structure. These preliminary results are part of a joint work with the University of Pavia, Italy.

📅 **Onsdag 29 november 2017, Maarit Järvenpää, Oulu University, Finland, och Institut Mittag-Leffler**

*Introduction to random covering sets*

Sammanfattning: Limsup sets, defined as upper limits of various sequences of sets, play an important role in different areas of mathematics. Random covering sets are a class of limsup sets defined by means of a family of randomly distributed subsets of the  $d$ -dimensional torus. We discuss various problems related to random covering sets with special emphasis to their dimensional properties.

📅 **Torsdag 30 november 2017, Dmitri Shakhmatov, Ehime University, Matsuyama, Japan**

*Abelian groups admitting a minimally almost periodic or a connected Hausdorff group topology*

Sammanfattning: In 1934 von Neumann introduced a notion of a minimal almost periodic topological group. A topological group is minimally almost periodic if every non-trivial homomorphism from it into any compact group is discontinuous. These groups are notoriously difficult to construct, yet many classical groups (such as the permutation group of the integers or the group of isomorphisms of the universal separable metric space endowed with the pointwise convergence topology) are minimally almost periodic. In this talk, we overview the complete solutions of both the problem of the existence of minimally almost periodic group topologies and the problem of the existence of connected group topologies on abelian groups, as well as the relation of these two long-standing problems to the Markov-Zariski topology of the group in question.

📅 **Onsdag 6 december 2017, Esa Järvenpää, Oulu University, Finland, och Institut Mittag-Leffler**

*Non-uniform random covering sets*

Sammanfattning: While uniform random covering sets are quite well understood, the theory of non-uniform random covering sets is in its beginning. I will review the results known in non-uniform case and explain the essential differences and complications compared to the uniform case.

# The Mathematical Colloquia held in 2016 at Linköping University

Organized by **Anders Björn**, **Milagros Izquierdo Barrios**, **Vladimir Kozlov**, and **Hans Lundmark**.

## 2 Onsdagen 20 januari 2016, Eero Saksman, Helsingfors universitet, Finland

*On the Gaussian multiplicative chaos*

Sammanfattning: We try to explain what Gaussian multiplicative chaos is, and recall its basic properties. In addition, we revisit the uniqueness problem in the case of critical chaos, and if time permits, some of the (partly heuristic) connections to e.g. Liouville quantum gravity or the Riemann zeta function will be discussed.

## 2 Onsdagen 27 januari 2016, Vitalij Tjatyрко, MAI

*A compact space with non-coinciding dimensions  $\dim$  and  $\text{ind}$*

Sammanfattning: In the talk I will describe a relatively old and simple example of a compact chainable separable 1st countable space having  $\text{ind} = 2$  at each point. All announced properties will be motivated.

## 2 Onsdag 3 februari 2016, Alexander I. Nazarov, Saint Petersburg, Ryssland

*On the Zaremba-Hopf-Oleinik lemma*

Sammanfattning: We discuss the Hopf-Oleinik lemma for solutions to elliptic and parabolic equations. For the Laplace operator this property is well known for more than one hundred years, starting from a pioneer paper of Zaremba (1910), and reads as follows. Let  $\partial\Omega \in \mathcal{C}^2$  and let  $\mathcal{L} = -\Delta$ . Then, if  $0 \in \partial\Omega$ , we have

$$\mathcal{L}u = f \geq 0 \text{ in } \Omega; \quad u(x) > u(0) \text{ in } \Omega \implies \frac{\partial u}{\partial \mathbf{n}}(0) < 0.$$

For general operators of non-divergence type with bounded measurable coefficients this result was established in the elliptic case independently by E.~Hopf and O.A.~Oleinik (1952) and in the parabolic case by L.~Nirenberg (1953). Later the efforts of many mathematicians were aimed at the extension of the class of admissible operators and at the reduction of the boundary smoothness. We consider generalized (strong) solutions for non-divergence type equations

$$\mathcal{L}u \equiv -a_{ij}(x)D_iD_ju + b_i(x)D_iu = f(x); \quad (\text{NDE})$$

$$\mathcal{M}u \equiv \partial_t u - a_{ij}(x;t)D_iD_ju + b_i(x;t)D_iu = f(x;t), \quad (\text{NDP})$$

i.e. we assume  $D(Du) \in L_{n,loc}(\Omega)$  in (NDE) and  $\partial_t u, D(Du) \in L_{n+1,loc}(Q)$  in (NDP) (in the parabolic case also some anisotropic spaces are admissible). We suppose that the operators under consideration are uniformly elliptic (parabolic). The problem of our interest is how "bad" the lower-order coefficients  $b_i$  may be to ensure the Hopf-Oleinik lemma to hold true. We provide sharp conditions for this. Also we provide a new sharp counterexample for this lemma.

## 2 Onsdag 10 februari 2016, Natan Kruglyak, MAI

*Real interpolation: theory and applications*

Sammanfattning: I plan to give a review of the theory of real interpolation with focus on the results in which I was involved.

## 2 Onsdag 17 februari 2016, Søren Eilers, University of Copenhagen, Denmark, & Institut Mittag-Leffler

*The complete classification of Cuntz-Krieger algebras*

Sammanfattning: In 1980, Joachim Cuntz and Wolfgang Krieger showed how to associate a functional analytic object, more precisely a  $C^*$ -algebra, to any finite graph with no stranded vertices in a way reflecting the dynamics of the shift space associated to such a graph, and from the early days of the classification theory of  $C^*$ -algebras this class was under scrutiny to establish classification results by  $K$ -theory. Indeed, Mikael Rørdam in 1995 completed the classification of those Cuntz-Krieger algebras that are simple, by employing in a groundbreaking way that the  $K_0$ -group of such a  $C^*$ -algebra is precisely the Bowen-Franks group well-known in the classification of shift spaces. I will report on how recent understanding of the larger class of graph  $C^*$ -algebras has recently allowed for a complete solution of the classification problem for Cuntz-Krieger algebras in a way which is not at all reflected in the dynamics. The part of the work which I have contributed is joint with Restorff, Ruiz and Sørensen.

## 2 Onsdag 24 februari 2016, Réamonn Ó Buachalla, Polish Academy, Warszawa, Polen

*A  $C^*$ -algebraic introduction to noncommutative geometry*

Sammanfattning: I begin by recalling the definition of a  $C^*$ -algebra, along with Gelfand and Naimark's characterisation of commutative  $C^*$ -algebras as function algebras on compact Hausdorff spaces. I then explain how this motivates us to think of noncommutative  $C^*$ -algebras as "noncommutative topological spaces". Building on this, I introduce Woronowicz's notion of a compact quantum group and show that it reduces to the definition of a compact topological group in the commutative case. I will also discuss quantum-SU(2), the theory's prototypical example. Finally, I present Connes' notion of a spectral triple, together with his commutative classification theorem. This is the result that motivates us to think of spectral triples as noncommutative Riemannian manifolds. For an example we take the Podleś sphere, a quantum homogeneous space of quantum-SU(2), and present its standard spectral triple which is a direct  $q$ -deformation of the standard Dirac operator on the 2-sphere.

## 2 Onsdag 2 mars 2016, Jens Hoppe, KTH

*Minimal hypersurfaces*

Sammanfattning: I will discuss explicit solutions, techniques to obtain them, and closely related questions.

## 2 Fredag 4 mars 2016, Luciano Mari, Universidade Federal do Ceará, Brasilien

Seminarier är ett gemensamt seminarium med Analysseminarier.

*The Ahlfors-Khas'minskii duality for fully nonlinear PDEs, and geometric applications*

Sammanfattning: Maximum principles at infinity (or "almost maximum principles") are a powerful tool to investigate the geometry of Riemannian manifolds. Among them, we stress the Ekeland, the Omori-Yau principles and their weak versions, in the sense of Pigola-Rigoli-Setti. These last have nice probabilistic counterparts in terms of stochastic and martingale completeness, which in turn are related to potential theory and parabolicity. The validity of such principles is usually granted via suitable exhaustion functions called Evans-Khas'minskii potentials. In this talk, I discuss an underlying, unifying duality that allows to uncover relations between the principles. Indeed, duality holds for a broad class of fully nonlinear operators of geometric interest. Our methods use the approach to nonlinear PDEs pioneered by Krylov ('95) and Harvey-Lawson ('09-), and involve the study of viscosity "almost solutions" of obstacle type problems. This is joint work with Leandro F. Pessoa.

## 2 Onsdag 9 mars 2016, Allen Broughton, Rose Hulman, Indiana, USA

*Riemann surfaces: A playground for analysis, topology, geometry, group theory, and Galois theory*

Sammanfattning: The study of Riemann surfaces began in the last half of the nineteenth century and continues as a very active research area today and in the foreseeable future. During its history, the study of Riemann surfaces has used all the tools of the topic areas of the title. In this talk we will explore how Riemann surfaces interact with these topic areas. Special attention will be paid to hyperelliptic surfaces and their generalizations, super-elliptic surfaces.

## 2 Fredag 11 mars 2016, Juha Lehrbäck, Jyväskylä, Finland

Seminarier är ett gemensamt seminarium med Analysseminarier.

*Whitney covers, Minkowski dimensions, and the size of the  $r$ -boundaries*

Sammanfattning: In a doubling metric space, the complement of a closed set  $E$  can always be covered with balls in such a way that the balls have uniformly bounded overlap and the radius of each ball is comparable to the distance from the ball to  $E$ . These are called Whitney covers due to obvious connections with the classical Whitney cubes. In this talk, I will explain how the amount of Whitney balls (or cubes) of certain size is related to the Minkowski dimensions of  $E$ , and how this connection can be used in the  $n$ -dimensional Euclidean space to obtain estimates for the  $(n-1)$ -dimensional Hausdorff measure ("surface measure") of the  $r$ -boundary of  $E$ , i.e. the set of points having distance  $r$  to  $E$ . In particular, I will show how the Minkowski dimensions and also the so-called spherical dimensions of  $E$  can be characterized using the amount of Whitney balls (under some mild conditions on  $E$ ). This talk is based on my joint work with Antti Käenmäki and Matti Vuorinen (Indiana Univ. Math. J. 62 (2013), No. 6, 1861-1889).

## 2 Onsdag 30 mars 2016, Yuri Karlovich, Universidad Autónoma del Estado de Morelos, Mexiko

*One-sided invertibility of functional operators*

Sammanfattning: Let  $\mathbb{R}_+ = (0, +\infty)$  and let  $\alpha$  be an orientation-preserving homeomorphism of  $\mathbb{R}_+$  onto itself with  $\log \alpha' \in L^\infty(\mathbb{R}_+)$  and only two fixed points 0 and  $\infty$ . Given  $p \in (1, \infty)$ , we study Wiener's type functional operators of the form



$$A = \sum_{k \in \mathbb{Z}} a_k U_\alpha^k : L^p(\mathbb{R}_+) \rightarrow L^p(\mathbb{R}_+), \quad (1)$$

where the coefficients  $a_k$  are in  $L^\infty(\mathbb{R}_+)$ , the isometric shift (composition) operator  $U_\alpha$  is given by  $U_\alpha f = (\alpha')^{1/p}(f \circ \alpha)$ , and  $\|A\|_W = \sum_{k \in \mathbb{Z}} \|a_k\|_{L^\infty(\mathbb{R}_+)} < \infty$ . Under assumption that  $\alpha'$  and all  $a_k$  are continuous on  $\mathbb{R}_+$  and slowly oscillating at 0 and  $\infty$ , criteria for the one-sided invertibility of the operator  $A$  on the spaces  $L^p(\mathbb{R}_+)$  are established. First, the one-sided invertibility of the binomial functional operators  $aI - bU_\alpha$  is studied on the basis of some algebraic methods, passing to discrete operators on the space  $\ell^p$  and applying the limit operators techniques. The study and results essentially depend on the oscillation of the shift derivative and coefficients. We then study the one-sided invertibility of operators of the form (1). The invertibility of functional operators associated with subexponential and amenable groups that act topologically freely and the one-sided invertibility of functional operators on rearrangement-invariant spaces are also discussed.

📅 **Onsdag 6 april 2016, Antonio F. Costa, UNED, Madrid**

*Riemann surfaces of genus  $g$  and  $4g$  automorphisms*

Sammanfattning: In 1895 Hurwitz showed that a hyperbolic Riemann surface of genus  $g$  has at most  $84(g-1)$  automorphisms. The surfaces with  $84(g-1)$  automorphisms are called Hurwitz surfaces. It is still an open problem for which genera there are Hurwitz surfaces. There's a difficult problem to identify the surfaces admitting a determined number of automorphisms. However there are very early results: In 1895 Wiman produced Riemann surfaces having an automorphism of order  $4g+2$  and surfaces having an automorphism of order  $4g$ . In 1968 Accola and MacLachlan (independently) gave for each genus surfaces with  $8(g+1)$  automorphisms. In 1996 Kulkarni showed that the Accola-MacLachlan surface is the only surface with  $8(g+1)$  automorphisms. And Wiman surfaces are, with a few exceptions, the only surfaces with an automorphism of order  $4g+2$  and  $4g$  respectively. He also showed that in the second case the surface has  $8g$  automorphisms. Riemann surfaces admitting more than  $4g$  automorphisms are isolated in the corresponding moduli space. In this talk we we talked on Riemann surfaces with  $4g$  automorphisms.

📅 **Onsdag 13 april 2016, Thomas Kragh, Uppsala universitet**

*Symplectic topology and stable homotopy theory*

Sammanfattning: In this talk I will outline some basic concepts in symplectic geometry, and describe how Gromov-Witten invariants of symplectic manifolds are defined by counting solutions to certain non-linear elliptic PDEs. These invariants can be used to define Quantum homology of the symplectic manifold, and I will try and describe how one might go about encoding this structure into a refined object in stable homotopy theory (i.e. a spectrum).

📅 **Onsdag 20 april 2016, Mattias Jonsson, University of Michigan, Ann Arbor, USA, och Göteborg**

*Degenerations of amoebae and tropical varieties*

Sammanfattning: A collection of polynomials with complex coefficients defines a submanifold of complex Euclidean space, which in turn casts a "shadow" on real Euclidean space, under the map that takes each complex coordinate to the logarithm of the absolute value. This shadow is called the amoeba of the submanifold and shows up in various branches of mathematics. I will explain that a suitable rescaling limit of the amoeba coincides with a combinatorial object, the tropical variety associated to the submanifold. The proof involves non-Archimedean geometry, but no knowledge of this will be assumed.

📅 **Onsdag 27 april 2016, Mieczysław Mastyło, University of Poznań, Polen**

*On the multilinear and polynomial inequalities*

Sammanfattning: We will present several new multilinear and polynomial inequalities. The first part of the talk is based on a joint work with Andreas Defnat. We show an exact inequality between the  $L^p$ -norm of a polynomial  $P$  on the  $n$ -dimensional torus  $\mathbf{T}^n$  and its Mahler measure  $M(P)$ . Using extrapolation we transfer this estimate into a Khinchin-Kahane type inequality, which relates a certain exponential Orlicz norm and Mahler's measure. In the second part we will discuss new variants of Kahane-Salem-Zygmund inequalities for the expectation of the supremum norm of homogeneous Bernoulli polynomials on the unit ball of a Banach space. We combine ideas from stochastic processes and interpolation theory to control increments of a Rademacher process in an Orlicz space via entropy integrals.

📅 **Onsdag 4 maj 2016, Sergey Nazarov, MAI och St Petersburg State University, Ryssland**

*Wandering eigenvalues of the Laplacian with an improper Robin conditions*

Sammanfattning: The spectrum of the boundary value problem

$$-\Delta u(x) = \lambda u(x), \quad x \in \Omega, \quad a(x) \partial_n u(x) + u(x) = 0, \quad x \in \partial\Omega,$$

in a smooth bounded domain  $\Omega \subset \mathbb{R}^2$  will be considered under the condition

$$a(s) = a_0 s + O(s^2), \quad s \in \partial\Omega, \quad a_0 \neq 0, \quad a(s) \neq 0 \text{ for } s \neq 0.$$

In other words, the coefficient  $a$  of the normal derivative  $\partial_n$  in the Robin condition changes sign on the boundary of the domain. It will be demonstrated that the spectrum of this problem is residual and covers the whole complex plane  $\mathbb{C}$ . All self-adjoint extensions of the operator  $A_0$  with the domain

$$\mathcal{D}(A_0) = \{u \in H^1(\Omega) : \Delta u \in L^2(\Omega), |a|^{-1/2} u \in L^2(\partial\Omega)\}$$

will be described. They have the discrete spectrum but no appropriate choice of the extension is available. Also a skew-symmetric extension of  $A_0$  can be constructed and it will be shown that this one describing wave processes in a finite volume has a clear physical sense. Namely, the problem

$$-\Delta u^\varepsilon(x) = \lambda^\varepsilon u^\varepsilon(x), \quad x \in \Omega, \quad a^\varepsilon(x) \partial_n u^\varepsilon(x) + u^\varepsilon(x) = 0, \quad x \in \partial\Omega,$$

with the perturbed coefficient

$$a^\varepsilon(s) = a(s) + \varepsilon \operatorname{sign} a(s) \neq 0 \text{ for all } s \in \partial\Omega$$

gets the discrete spectrum but its eigenvalues  $\lambda_k^\varepsilon$  depend periodically on  $\ln \varepsilon$ , i.e., wander (walk aimlessly like a drunkard or insane) when  $\varepsilon \rightarrow +0$ . Such kind of behavior is usually attributed to a skew-symmetric operator with some radiation conditions at the point  $s = 0$  on the boundary  $\partial\Omega$ . My work in this direction has been done in cooperation with the French mathematicians Lucas Chesnel, Xavier Clayes and Nicolas Popoff.

📅 **Onsdag 11 maj 2016, Nancy Abdallah, MAI**

*Milnor Algebra and Singularities of Plane Curves*

Sammanfattning: Let  $C = \mathbb{P}^2$  be a curve given by  $f = 0$  where  $f \in S = \mathbb{C}[x, y, z]$ . Denote by  $J_f$  the Jacobian ideal of  $f$ , i.e. the ideal generated by the partial derivatives of  $f$ . We describe the relations between the Milnor algebra  $M(f) = S/J_f$  of  $f$  and the singularities of  $C$ . In particular, we give the Poincaré series of the Milnor algebra in terms of local invariants of  $C$ . This can be done by a study of the cohomology of the Koszul complex of the partial derivatives of  $f$ . Relations to the syzygies of the Jacobian ideal will also be given.

📅 **Onsdag 18 maj 2016, Johan Thim, MAI**

*Hadamard type asymptotics for elliptic operators*

Sammanfattning: We consider how the eigenvalues of the Neumann and Dirichlet problems for an elliptic operator (mainly the Laplacian) depend on the domain. The two problems require different notions of closeness between the reference and the perturbed domain and abstract frameworks in Hilbert space are presented. The main result is an asymptotic formula where the remainder is expressed in terms of the proximity quantities described above when these are relatively small. We also consider applications to the Laplacian in  $C^{1,\alpha}$ ,  $C^1$  and Lipschitz domains. For the  $C^{1,\alpha}$  case, an asymptotic result for the eigenvalues is given together with estimates for the remainder. In the Neumann case, we also consider an example where  $\alpha = 0$ .

📅 **Onsdag 25 maj 2016, Lukáš Malý, University of Cincinnati, USA**

*Trace and extension theorems for BV and Sobolev functions in metric spaces*

Sammanfattning: In the general Dirichlet problem, one starts with a domain, prescribes boundary values, and looks at the set of functions on the interior of the domain whose trace on the boundary matches the prescribed boundary values. For domains in metric measure spaces, we investigate the class of functions defined on the boundary that can be extended to functions of some specified regularity on the interior. Under some rather mild requirements on regularity of the boundary, we find a linear extension operator from a Besov class on the boundary to BV class (or to the Newton-Sobolev class  $N^{1,p}$ ) on the interior of the domain. This operator can then be used to find BV extensions of  $L^1$  boundary data. Hence, the trace class of  $BV(\Omega)$  is  $L^1(\partial\Omega)$  provided that the ambient metric space admits a 1-Poincaré inequality. We will also look into analogous questions for domains with a thick or a thin boundary (i.e., with a boundary of Hausdorff co-dimension other than 1). This is joint work with N. Shanmugalingam and M. Snipes.

📅 **Onsdag 1 juni 2016, Nikolai Nadirashvili, Aix-Marseille Université, Frankrike**

*Hessian equations and minimal cones*

Sammanfattning: We give a brief introduction to viscosity solutions of fully nonlinear elliptic equations, discuss questions of regularity of viscosity solutions. We also discuss applications of noncommutative and nonassociative algebras to classification of homogeneous solutions of fully nonlinear equations and its connections with minimal cones.

📅 **Torsdag 9 juni 2016, Roy Skjelnes, KTH**

*Hilbert schemes*

Sammanfattning: In algebraic geometry the Hilbert schemes are fundamental objects. These objects are characterized by parametrizing subschemes in projective space. Moduli problems are often solved by relating them to the existence of the Hilbert schemes. Hilbert schemes that possess specific properties are of particular interest. The fact that the Hilbert

scheme of points in the plane is itself smooth has lead to a vast of research activity, and surprising discoveries. In the talk I want to describe some of the interesting advances about Hilbert schemes, but I will also focus on their definition and their basic properties. Instead of presenting results in their most general form I will try to give simplified versions and avoid using the abstract machinery that alienates non-experts from algebraic geometry.

**2 Onsdag 17 augusti 2016, Petros Petrosyan, Yerevan State University, Armenien**

*Cyclic interval edge-colorings of graphs*

Sammanfattning: An edge-coloring of a graph  $G$  with colors  $1, \dots, t$  is called a cyclic interval  $t$ -coloring if the edges incident to each vertex of  $G$  are colored by consecutive colors, under the condition that color 1 is considered as consecutive to color  $t$ . The concept of cyclic interval edge-coloring of graphs was introduced by de Werra and Solot in 1991 and was motivated by scheduling problems arising in flexible manufacturing systems, in particular the so-called cylindrical open shop scheduling problem. In this talk we will give a survey of the topic and present a recent progress in the study of cyclic interval edge-colorings of graphs. In particular, we will present new results on some open problems concerning cyclic interval edge-colorings of graphs. This is joint work with A.S. Asratian and C.J. Casselgren.

**2 Tisdag 23 augusti 2016, Yasunao Hattori, Shimane University, Japan**

*A survey on topologies on the posets of formal balls in metric spaces -A bridge between the theory of computation and topology*

Sammanfattning: The set of formal balls in a metric space was introduced by Weihrauch and Schreiber in [2] to represent a metric space in a domain, and several authors studies it as a computational model for a metric space. Let  $\mathbb{R}$  and  $\mathbb{R}_+$  denote the set of real numbers and non-negative real numbers, respectively. Let  $(X, d)$  be a metric space and  $\mathbf{B}^+(X, d) = X \times \mathbb{R}_+$ . An element of  $\mathbf{B}^+(X, d)$  is called a (f)ormal ball. In (1), Tsuki and Hattori extended the notion of formal balls to balls having negative radii, say generalized formal balls, i.e., let  $\mathbf{B}(X, d) = X \times \mathbb{R}$  and we call an element of  $\mathbf{B}(X, d)$  a generalized formal ball. We induce a partial order in  $\mathbf{B}^+(X, d)$  ( $\mathbf{B}(X, d)$ ) as  $(x, r) \sqsubseteq (y, s)$  if  $d(x, y) \leq r - s$ . Then  $(\mathbf{B}^+(X, d), \sqsubseteq)$  and  $(\mathbf{B}(X, d), \sqsubseteq)$  are continuous posets, and they have the Scott, bi-Scott, Lawson and the Martin topologies from domain theoretical point of view. I shall discuss the topologies above in the posets of formal balls. In this talk, firstly, we introduce fundamental notions from domain theory, say way-below relation, domain, continuous domain, Scott topology, bi-Scott topology, Lawson topology, and Martin topology etc. Then we shall discuss on the relations between the topological structures and the order-theoretic structures on the posets of formal balls in metric spaces from the topological point of view. References: [1] H. Tsuki and Y. Hattori, Lawson topology of the space of formal balls and the hyperbolic topology of a metric space, Theoret. Computer Sci., 405 (2008), 198-205. [2] K. Weihrauch and U. Schreiber, Embedding metric spaces into cpo's, Theoret. Computer Sci. 16 (1981), 5-24.

**2 Onsdag 24 augusti 2016, Anita Rojas, Universidad de Chile**

*Completely decomposable abelian varieties, the case of Jacobians*

Sammanfattning: The action of a finite group  $G$  on an abelian variety  $A$  induces a decomposition of  $A$  into  $G$ -invariant factors, called the isotypical decomposition of  $A$ . It comes from the decomposition of the group algebra  $\mathbb{Q}[G]$  of  $G$  over the rationals, into simple algebras. Hence each factor corresponds to a rational irreducible representation of  $G$ . A completely decomposable Abelian variety is an abelian variety which is isogenous to a product of elliptic curves. In 1993, Ekedahl and Serre asked several questions about completely decomposable Jacobian varieties, some of them are still open. In particular they asked if there are completely decomposable Jacobian varieties in any dimension  $g \geq 2$ . In the same work, the authors presented a list of dimensions in which there are completely decomposable Jacobian varieties. Nevertheless, besides stopping in dimension 1297 leaving open the question whether there are higher dimensional completely decomposable Jacobian varieties, their list has some gaps. These questions have motivated several articles approaching their answers through different methods. We use group actions as the main tool. In this talk we will discuss fundamental results, some recent advances, as well as some work in progress, regarding these questions.

**2 Tisdag 30 augusti 2016, Justyna Signerska-Rynkowska, Gdansk University of Technology, Polen**

*Complex oscillations in a nonlinear neuron model with resets*

Sammanfattning: We investigate complex oscillations in a class of bidimensional nonlinear hybrid dynamical systems modeling neuronal voltage dynamics with adaptation and spikes emission. We show that these models can generically display mixed-mode oscillations (MMOs), i.e. trajectories featuring an alternation of small oscillations with spikes or bursts (multiple consecutive spikes). The mechanism by which this behaviour is generated relies fundamentally on the hybrid structure of the flow: small oscillations are governed by invariant manifolds of the underlying continuous dynamical system consisting of two non-linear ODEs, while discrete resets control the emission of spikes or bursts. The decomposition into these two mechanisms reveals their geometrical origin, allowing a relatively simple classification of points at the reset line associated to specific inter-spike trajectories. Spike patterns and MMOs are thus related to the sequence of consecutive locations of the resets, that we analyze by considering these as points on the orbits of the so-called adaptation map. We demonstrate that this map can be seen as a lift of a discontinuous degree-one circle map with diverging left- and right- derivatives at the discontinuity points. Notwithstanding, in certain cases such a map falls into the framework of either the non-overlapping lifts or the so-called "old heavy maps", which can be studied in detail through the means of rotation theory, with the univocal bidirectional link between the rotation number of the trajectory and the signature of the generated MMOs. In contrast to more classical frameworks in which MMOs were evidenced, the present geometric mechanism neither requires more than two dimensions nor necessitates a separation of timescales and complex return mechanism. The talk is based on a joint work with J. Rubin (University of Pittsburgh), J. Touboul (College de France & INRIA) and A. Vidal (Universite d'Evry-Val-d'Essonne & INRIA).

**2 Onsdag 31 augusti 2016, Tomas Sjödin, MAI**

*Stationary boundary points for a Laplacian growth problem in higher dimensions*

Sammanfattning: This talk will concern the behaviour of corners for certain Laplacian growth processes driven by source terms in higher dimensions. In two dimensions this process corresponds to Hele-Shaw flow, and it is known that corners of interior angle less than  $\pi/2$  in the boundary of a plane domain are initially stationary for such growth processes. The aim here is to present analogous results in higher dimensions.

**2 Onsdag 7 september 2016, Mikko Parviainen, Jyväskylä, Finland**

*Nonlinear stochastic games and normalized  $p$ -Laplacian*

Sammanfattning: In this colloquium, we discuss a class of stochastic games. A particular example of such a stochastic game is the tug-of-war with noise. Passing to the limit with the step size, this game gives rise to the normalized or game theoretic  $p$ -Laplace equation and  $p$ -parabolic equation. After the introduction, we consider regularity for value functions for these stochastic games. We also discuss these equations from the PDE point of view.

**2 Fredag 9 september 2016, Maria Przybylska, Zielona Góra, Polen**

*Applications of Darboux polynomials to integrability studies*

Sammanfattning: In the theory of ordinary differential equations first integrals play a crucial role. Constant value levels of these functions are invariant with respect to the flow generated by the considered system. However there exist also functions such that only their zero level is invariant. Polynomial functions with this property for polynomial vector fields are called Darboux polynomials. They were originally introduced by G. Darboux for construction of first integrals of planar polynomial differential systems in  $\mathbf{R}^2$  and  $\mathbf{C}^2$  and later generalised and analysed by many authors. In the talk, properties of Darboux polynomials and their applications to integrability studies will be presented. Detailed construction of first integrals using Darboux polynomials for non-linear 3-dimensional stretch-twist-fold (STF) flow depending on nine parameters will be shown. In addition to presentation of some integrable cases also a non-integrability proof for a four-parameter family of STF flows by means of differential Galois approach will be given. Also the application of Darboux polynomials for proving non-integrability in the class of polynomial first integrals for the Halphen system will be explained.

**2 Onsdag 14 september 2016, Klas Markström, Umeå universitet**

*Voting and Condorcet domains*

Sammanfattning: From the earliest days of the mathematical study of voting and democracy it has been known that there are "paradoxical" situations in which all candidates of an election loose a majority vote. In this talk I will give an introduction to some of the theory of voting and the study of so called Condorcet domain. The latter are collections of preferences which avoid voting paradoxes in a specific sense. Condorcet domains have connections to combinatorics, order theory, group theory and geometry and I will introduce some of these, including some of my own recent work in this area.

**2 Onsdag 21 september 2016, Nikolay Kuznetsov, Russian Academy of Sciences, St Petersburg, Ryssland**

*Direct and inverse spectral problems for sloshing of a two-layer fluid*

Sammanfattning: Direct and inverse eigenvalue problems for a pair of harmonic functions with a spectral parameter in boundary and coupling conditions will be considered. The direct problem is relevant to sloshing frequencies of free oscillations of a two-layer fluid in an open container. The upper fluid occupies a layer bounded above by a free surface and below by a layer of fluid of greater density. Both fluids are assumed to be inviscid, incompressible and heavy, whereas the free surface and the interface between fluids are supposed to be bounded. The inverse problem deals with recovering of the densities ratio and the interface's depth.

**2 Onsdag 28 september 2016, Michel Chipot, Universität Zürich, Schweiz**

*Asymptotic Issues in Cylinders*

Sammanfattning: We would like to present some results on the asymptotic behaviour of different problems set in cylindrical domains of the type  $\ell\omega_1 \times \omega_2$  when  $\ell \rightarrow \infty$ . For  $i = 1, 2$   $\omega_i$  are two bounded open subsets in  $\mathbb{R}^{d_i}$ . To fix the ideas on a simple example consider for instance  $\omega_1 = \omega_2 = (-1, 1)$  and  $u_\ell$  the solution to

$$-\Delta u_\ell = f \text{ in } \Omega_\ell = (-\ell, \ell) \times (-1, 1), \quad u_\ell = 0 \text{ on } \partial\Omega_\ell.$$

It is more or less clear that, when  $\ell \rightarrow \infty$ ,  $u_\ell$  will converge toward  $u_\infty$  solution to

$$-\Delta u_\infty = f \text{ in } \Omega_\infty = (-\infty, \infty) \times (-1, 1), \quad u_\infty = 0 \text{ on } \partial\Omega_\infty.$$

However this problem has infinitely many solutions since for every integer  $k$

$$\exp(k\pi x_1)\sin(k\pi x_2)$$

is solution of the corresponding homogeneous problem. Our goal is to explain the selection process of the solution for different problems of this type when  $\ell \rightarrow \infty$ .

**2 Onsdag 5 oktober 2016, Jürgen Rossmann, Universität Rostock, Tyskland**

*On the instationary Stokes system in an angle*

Sammanfattning: The talk deals with the first initial-boundary value problem for the Stokes System

$$u_t - \Delta u + \nabla \cdot p = f, -\nabla \cdot u = g$$

in a (2-dimensional) angle  $K$ . The goal is to obtain existence and uniqueness results for solutions in weighted Sobolev spaces. To this end, the parameter-depending problem

$$(s - \Delta)U + \nabla P = F, \quad -\nabla \cdot U = G \text{ in } K, \quad U = 0 \text{ on } \partial K$$

for the Laplace transforms  $U(x, s)$  and  $P(x, s)$  is studied, where  $s$  is an arbitrary complex number with nonnegative real part. Solvability and regularity results are presented both for weak and strong solutions of the parameter-depending problem.

**2 Måndag 10 oktober 2016, Enrique Artal, University of Zaragoza, Spanien**

*Topology, Combinatory and Arithmetic for Line Arrangements in the Projective Plane*

Sammanfattning: A line arrangement is a set of lines in a (projective or affine) plane (over some field). The combinatorics of an arrangement consists of its intersection pattern; this concept induces an abstract definition of combinatorics in an obvious way. Several problems arise: the existence of arrangements with a given combinatorics or the irreducibility of the realization space. The answers to these questions depend on the ground field. If we fix the complex numbers as ground field, topological problems come in, since we are in a "codimension-2 situation", close to knot theory. Orlik and Solomon proved that the cohomology ring of the complement of an arrangement is completely determined by the combinatorics, and a natural question arises: how far combinatorics determines topology. Rybnikov proved that fundamental group of the complement is not determined by the combinatorics. At the end, we will discuss also the relationship between combinatorics and arithmetics.

**2 Onsdag 12 oktober 2016, Sergey Vakulenko, St Petersburg, Ryssland**

*New mathematical approach to morphogenesis: complex attractors and patterns in reaction-diffusion systems*

Sammanfattning: In this talk, we first consider the classical ideas of A. Turing on the morphogenesis problem, namely, how a form of organism can be created and supported stably by a system of chemical reagents. The second part of talk is a statement of new ideas suggested in works of J. Reintz et al. and the author. We consider semiflows generated by initial boundary value problems for reaction-diffusion systems. In these systems, reaction terms satisfy general conditions, which admit a transparent chemical interpretation. It is shown that the semiflows generated by these initial boundary value problems exhibit a complicated large time behavior. Any structurally stable finite dimensional dynamics can be realized by these semiflows by a choice of appropriate external sources and diffusion coefficients (nonlinear terms are fixed). The results can be applied to the morphogenesis and pattern formation problems. They show that chemical systems with fast and slow reagents under fairly general conditions can generate unboundedly complicated spatio-temporal patterns.

**2 Fredagen 14 oktober 2016, Alexandre Krassev, Nipissing University, Kanada**

*Infinite-dimensional spaces and continuous selections*

Sammanfattning: This talk is an overview of several results relating dimension theory and the theory of continuous selections. The focus will be on those results pertaining to infinite-dimensional spaces. In particular, several important theorems involving C-spaces, weakly infinite-dimensional spaces, and Ernest Michael's G-delta problem, will be discussed.

**2 Onsdag 26 oktober 2016, German Zavorokhin, Steklov Math. Institute, St. Petersburg, Ryssland**

*On the space-time ray method for fluid-saturated porous Biot media*

Sammanfattning: This talk is concerned with wave propagation in fluid-saturated porous media in the frame of the Biot model. The space-time ray expansion of the solutions of dynamical equations for the Biot media is constructed (in anisotropic inhomogeneous case). In inhomogeneous isotropic case the Rytov law analogue is deduced like in elasticity theory.

**2 Torsdag 3 november 2016, Gaetano Zampieri, Università degli Studi di Verona, Italien**

*Local and nonlocal constants of motion in Lagrangian dynamics*

Sammanfattning: We give a recipe to generate nonlocal constants of motion for ODE Lagrangian systems and we apply the method to find useful constants of motion which permit us to prove global existence and estimates of solutions to dissipative mechanical systems. We show examples where our recipe can be used to find genuine first integrals too. Our applications are the mechanical systems with homogeneous potential of degree -2, and the conservative MaxwellBloch system with RWA.

**2 Onsdag 9 november 2016, Thomas Kaijser, MAI**

*On products of matrices, asymptotic stability and Blackwell's ergodicity problem*

Sammanfattning: In the early 1970ies I was able to partly solve a problem raised by David Blackwell in 1957 concerning partially observed Markov chains. In recent years I have returned to Blackwell's ergodicity problem and have tried to generalise my earlier results to Markov chains on general state spaces. Roughly speaking, Blackwell's ergodicity problem concerns the existence of a unique stationary measure associated to a transition function induced by a given Markov kernel (stochastic matrix) and observation system, and which takes its values in the set of probabilities on the state space of the given Markov kernel. In this talk I plan to have three intertwining parts. The first part consists of presenting some simple but for me useful inequalities for products of matrices and iterations of integral kernels going back to Perron(1907) and Hopf(1963). The second part consists of presenting some limit theorems concerning the existence of limit measures for iterations of transition kernels on general measurable spaces. The purpose of the third part is to present Blackwell's ergodicity problem in more detail and show how the limit theorems of part two can be applied to Blackwell's problem.

**2 Onsdag 23 november 2016, Lydie Mpinganzima, Chalmers/Göteborgs universitet och University of Rwanda**

*Optimization algorithm for the construction of nanophotonic structures*

Sammanfattning: The talk concerns the problem of the construction of the nanophotonic structures of arbitrary geometry with prescribed desired properties. These properties can be: reduction of reflections, construction of nano-structure with desired dielectric permittivity function. Our problem is formulated as the coefficient inverse problem (CIP) for the hyperbolic problem to find the unknown dielectric permittivity function.

**2 Tisdag 29 november 2016, Lan Anh Pahn, Umeå universitet**

*Structure of classes of graphs defined by constraints on chords*

Sammanfattning: A long unichord in a graph is an edge that is the unique chord of some cycle of length at least 5. A graph is long-unichord-free if it does not contain any long-unichord. We prove a structure theorem for long-unichord-free graphs. We give an  $O(n^4m)$ -time algorithm to recognize them, where  $n$  is the number of vertices and  $m$  is the number of edges. We show that any long-unichord-free graph  $G$  can be colored with at most  $O(\omega^3)$  colors, where  $\omega$  is the maximum number of pairwise adjacent vertices in  $G$ .

**2 Onsdag 30 november 2016, Hans Feichtinger, Universität Wien, Österrike**

*An alternative approach to convolution and the Fourier transform*

Sammanfattning: When one looks at the Fourier transform in the mathematical literature the description starts usually with Fourier Series for periodic functions or right away with the Fourier transform as an integral transform. In either case the transform requires to use integrals, and of course the Lebesgue integral appears to provide the natural domain, namely the space  $L^1$  of integrable functions. Similar arguments apply to the convolution integral. Combining the two concepts one can then derive the all-important convolution theorem, Fourier inversion and Plancherel's theorem, showing that the "complicated convolution" is turned into easy pointwise multiplication. But why should we be interested in convolution? Is it a natural product for integrable functions? And which functions do "have a Fourier transform"? Aside from heuristic manipulations, leading to the forward and inverse Fourier transform the above results are certainly important to (electrical) engineers, when they deal with translation invariant systems, which are usually described by black boxes. They correspond to convolution operators with the so-called impulse response, which is the output of the system to a "Dirac delta-function", and can be described alternatively by their transfer function. We will describe a mathematically correct approach to convolution and Fourier transform which is based on simple functional analytic principles and encompasses the two aspects of basic Fourier analysis in a way (hopefully well) understandable for both sides (mathematicians and engineers). Furthermore, many aspects of this approach can be supported by a set of simple MATLAB experiments, which connects the material with basic concepts from linear algebra and polynomials with complex coefficients.

**2 Måndag 5 december 2016, Zeinab Badreddine, University of Nice, Frankrike**

*Mass transportation on sub-Riemannian structures of rank two in dimension four*

Sammanfattning: This talk is concerned with the study of the Monge optimal transport problem in sub-Riemannian manifolds where the cost is given by the square of the sub-Riemannian distance. Our aim is to extend previous results on existence and uniqueness of optimal transport maps to cases of sub-Riemannian structures which admit many singular minimizing geodesics. We treat here the case of sub-Riemannian structures of rank two in dimension four.

**2 Onsdag 7 december 2016, Venuste Nyagahakwa, MAI**

*Sets with the Baire property in topologies formed from a given topology and ideals of sets*


Sammanfattning: Let  $X$  be a set,  $\tau_1, \tau_2$  topologies on  $X$  and  $\mathcal{B}_p(X, \tau_i)$  the family of all subsets of  $X$  possessing the Baire property in  $(X, \tau_i)$ ,  $i = 1, 2$ . We study conditions on  $\tau_1$  and  $\tau_2$  that imply a relationship (for example, inclusion or equality) between the families  $\mathcal{B}_p(X, \tau_1)$  and  $\mathcal{B}_p(X, \tau_2)$ . We are mostly interested on the case where the topology  $\tau_2$  is formed with the help of a local function defined by the topology  $\tau_1$  and an ideal of sets on  $X$ .

**2 Tisdag 13 december 2016, Pauline Bailet, Universität Bremen, Tyskland**



#### *Monodromy of Milnor fibers of hyperplane arrangements*

Sammanfattning: First I will recall some geometrical and combinatorial objects associated to an hyperplane arrangement such as complement, Milnor fiber, OrlikSolomon algebra and Aomoto complex. Then I will talk about local system cohomology of complements, cohomology of Milnor fibers and monodromy. Finally, I will introduce a graph which is determined by the arrangement's combinatorics and whose connectivity implies the vanishing of the monodromy eigenspaces for certain eigenvalues. We will end up by some open questions and directions of research in the vein of the latter graph.

 **Fredag 16 december 2016, Leslie Jiménez, MAI**

#### *Non-topologically equivalent actions of dihedral groups on Riemann surfaces and their Jacobians*

Sammanfattning: Given a compact Riemann surface  $X$ , we define the group of automorphisms  $\text{Aut}(X)$  of  $X$  as the analytical automorphism group of  $X$ . We say that a finite group  $G$  acts on  $X$  if there is a monomorphism  $\sigma : G \rightarrow \text{Aut}(X)$ . Any curve  $X$  of genus  $g$  has associated a principally polarized abelian variety  $JX := H^{1,0}(X, \mathbb{C})^*/H_1(X, \mathbb{Z})$ , where  $H^{1,0}(X, \mathbb{C})^*$  is the dual of the complex vector space of holomorphic forms of  $X$ , and  $H_1(X, \mathbb{Z})$  is the first homology group of  $X$ . This variety is called the Jacobian variety of  $X$  and has complex dimension  $g$ . In this talk, we will classify actions of certain dihedral groups on a Riemann surface  $X$  using topological equivalence. We use this classification to study the group algebra decomposition of  $JX$ .

# The Mathematical Colloquia held in 2015 at Linköping University

Organized by **Anders Björn**, **Milagros Izquierdo Barrios**, **Vladimir Kozlov**, and **Hans Lundmark**.

## 2 Onsdag 14 januari 2015, Mitsuru Wilson, Western University, London, Ontario, Kanada

*Noncommutative geometry and curvature*

Sammanfattning: In the celebrated 1943 paper, Gelfand and Naimark proved the contravariant equivalence between the category of compact Hausdorff spaces  $X$  and the category of commutative  $C^*$ -algebras  $A$ . In the construction,  $A$  is nothing but the algebra  $C(X)$  of continuous functions on  $X$ . This correspondence can be used to lift the notion of spaces by replacing the spaces by not necessarily commutative  $C^*$ -algebras, which generalizes the algebra  $C(X)$  of continuous functions. Alain Connes introduced a more potent idea in order to implement geometry into noncommutative spaces. It is a very fruitful and beautiful subject where many fields of mathematics merge together in sophistication. In my talk, I will introduce the basic ideas of Noncommutative Geometry and discuss key examples briefly to present our joint work with Joakim Arnlind on the computation of curvatures for noncommutative spaces.

## 2 Onsdag 21 januari 2015, Kristian Bjerklov, KTH

*Strange attractors arising from linear difference equations*

Sammanfattning: The intriguing computer generated pictures of so-called strange attractors, for example the Lorenz attractor, have popularized the field of dynamical systems. Linear difference equations do not seem to have much in common with such pictures. We all know how to solve linear difference equations with constant coefficients. However, if we move away from the constant case, things easily turn very difficult. One important case is when the coefficients of the equation are so-called quasi-periodic, that is, they are not periodic, but in some sense "close" to being periodic. In this talk we will focus on the most studied class of such an equation: the discrete Schrödinger equation with a quasi-periodic potential. Recent results, as well as open problems, will be presented. In particular we will see that a special type of strange attractor actually dwells deep inside this equation.

## 2 Onsdag 28 januari 2015, George Baravdish, ITN, LiU, Norrköping

*Inversa problem för bildförbättring*

Sammanfattning: Inversa problem uppkommer när utdata för en matematisk modell är givet men indata eller parametrar/koefficienter i modellen är okända. Begrepp som rekonstruera och identifiera förknippas därför ofta med inversa problem då okända storheter behöver beräknas. Om indata och modell istället är kända kallas problemen för direkta. Inversa problem är till skillnad från direkta oftast illa-ställda i den meningen att små ändringar i indata ger stora störningar i utdata och är därmed svåra att lösa. En tillämpning inom inversa problem som på senare år har intresserat mig är att från en given suddig eller brusig bild rekonstruera en originalbild. Bilder är numera ett viktigt instrument för visualisering av komplexa fenomen med typiska exempel inom medicin, satellit- och kamerasytem samt tryck. Defekter i sensorer, brus, och överföringsproblem medför att bilderna kan vara skadade eller sakna information. Efter en kort introduktion till inversa problem kommer jag under den senare delen av presentationen att diskutera nya matematiska modeller för rekonstruktion av bilder. Avslutningsvis kommer jag att lyfta fram utblickar för fortsatt arbete för rekonstruktion och parameteridentifikation inom bildbehandling. I föreläsningen är förhoppningen att med dessa tekniker och med den nya information som kan tas fram kunna förbättra diagnostiseringen av vissa typer av tumörskjdomar.

## 2 Måndag 2 februari 2015, Petros Petrosyan, Yerevan State University, Armenien

*Interval edge-colorings of graphs*

Sammanfattning: An edge-coloring of a graph  $G$  with colors  $1, \dots, t$  is called an interval  $t$ -coloring if all colors are used and the colors of edges incident to each vertex of  $G$  are distinct and form an interval of integers. The concept of interval edge-coloring of graphs was introduced by Asratian and Kamalian in 1987 and was motivated by the problems in scheduling theory. For example, it is known that the problem of finding an interval edge-coloring of bipartite graph corresponds to the problem of constructing a timetable without a "gap", in which every class and (or) every teacher conducts lessons in a continuous period of time. In this talk we give a survey of the topic and present a recent progress in the study of interval edge-colorings of graphs. In particular, we present new bounds for the number of colors in interval edge-colorings of some classes of graphs.

## 2 Onsdag 4 februari 2015, Agnieszka Kałamańska, University of Warsaw, Polen

*On one variant of an interpolation inequality and its applications to nonlinear eigenvalue problems*

Sammanfattning: We are interested in a certain type of interpolation inequality, estimating the term  $\int_{\Omega} |f'(x)|^p h(f(x)) dx$  by a quantity which involves the function  $f$  and its gradient of second order on the right hand side. The function  $h$  is supposed to be continuous. Such inequalities imply the classical Gagliardo-Nirenberg interpolation inequalities in the case  $h \equiv 1$ , while in the case of general  $h$  they seem to be unknown. The problem when the function  $f$  depends on one variable is essentially simpler than the one in  $n$  dimensions. To construct such inequalities we apply Hardy inequalities with best constants, as well as knowledge about constants in inequalities which involve the vectorial Riesz transforms. Applications to PDEs will also be discussed. The lecture will be based on a series of recent joint works with Tomasz Choczewski, Jan Peszek and Katarzyna Mazowiecka.

## 2 Onsdag 11 februari 2015, Egmont Porten, Mittuniversitetet

*Polynomial hulls on analytic varieties*

Sammanfattning: Polynomial hulls and polynomial convexity of compact subsets of  $C^n$  are among the fundamental concepts of complex analysis. Whereas their relevance for polynomial approximation is not always explicitly mentioned in one complex variable (since polynomial hull admit a topological description), polynomial convexity becomes pivotal for the development of pseudoconvexity theory in several variables. After a general introduction, we will consider the thickening problem, raised by John Erik Fornæss in the seventies. If  $K$  and  $L$  are compact subsets of  $C^n$  such the first is contained in the interior of the second, then the same relation holds for their polynomial hulls. Now the problem is whether this thickening property generalizes to compacts contained in a complex subvariety  $X$  of  $C^n$ . One encounters serious additional difficulties if  $X$  has singularities. Even for isolated singularities the problem is still wide open. The talk will present recent progress obtained jointly with Andreas Lind. The main topics are an affirmative result for isolated singularities of quotient type and a result on thickening of hulls with holomorphic structure, which rules out "algebraic" counter-examples.

## 2 Onsdag 18 februari 2015, Lydie Mpinganzima, University of Rwanda

*Robin-Dirichlet algorithms for the Cauchy problem for the Helmholtz equation*

Sammanfattning: We consider the Cauchy problem for the Helmholtz equation. It was demonstrated in our previous paper that the alternating algorithm suggested by V.A. Kozlov and V.G. Maz'ya does not converge for large wavenumbers  $k$  in the Helmholtz equation. We present some modifications of the algorithm that may restore the convergence. They consist of the replacement the Neumann-Dirichlet iterations by the Robin-Dirichlet ones and they repair the convergence for  $k^2$  less than the first Dirichlet-Laplacian eigenvalue. In order to treat large wavenumbers, we present an algorithm based on iterative solution of Robin-Dirichlet boundary value problems in a sufficiently narrow strip. Numerical implementations obtained using the finite difference method are presented. The numerical results illustrate that these algorithms, produce a convergent iterative sequences.

## 2 Onsdag 25 februari 2015, Elijah Liflyand, Bar-Ilan University, Israel

*Fourier transform versus Hilbert transform*

1. In the '50s (Kahane, Izumi-Tsuchikura, Boas, etc.), the following problem in Fourier Analysis attracted much attention: Let  $\{a_k\}$  ( $k = 0, 1, 2, \dots$ ) be the sequence of the Fourier coefficients of the absolutely convergent sine (cosine) Fourier series of a function  $f: \mathbb{T} = [-\pi, \pi) \rightarrow \mathbb{C}$ , that is  $\sum |a_k| < \infty$ . Under which conditions on  $\{a_k\}$  will the re-expansion of  $f(t)$  ( $f(t) - f(0)$ ), respectively) in the cosine (sine) Fourier series also be absolutely convergent?
2. We solve a similar problem for functions on the whole axis and their Fourier transforms. Generally, the re-expansion of a function with integrable cosine (sine) Fourier transform in the sine (cosine) Fourier transform is integrable if and only if not only the initial Fourier transform is integrable but also the Hilbert transform of the initial Fourier transform is integrable.
3. The following result is due to Hardy and Littlewood: If a (periodic) function  $f$  and its conjugate  $\tilde{f}$  are both of bounded variation, their Fourier series converge absolutely. We generalize the Hardy-Littlewood theorem (joint work with U. Stadtmüller) to the Fourier transform of a function on the real axis and its modified Hilbert transform. The initial Hardy-Littlewood theorem is a partial case of this extension, when the function is taken to be with compact support.
4. These and other problems are integrated parts of harmonic analysis of functions of bounded variation. We have found the maximal space for the integrability of the Fourier transform of a function of bounded variation. Along with those known earlier, various interesting new spaces appear in this study. Their inter-relations lead, in particular, to improvements of Hardy's inequality. There are multidimensional generalizations of these results.

## 2 Fredag 27 februari 2015, Antonio F. Costa, UNED, Spanien

*Limit points of equisymmetric 1-dimensional families of Riemann surfaces*

Sammanfattning: We describe the limit surfaces of some special types of 1-dimensional equisymmetric families of Riemann surfaces in the Deligne-Mumford compactification of moduli space. We provide a description of such nodal Riemann surfaces in terms of the deck group of the coverings defining the family. We apply our method to some well known examples. This is joint work with Victor Gonzalez-Aguilera.

## 2 Onsdag 4 mars 2015, Fredrik Viklund, Uppsala universitet och KTH

*Self-avoiding walks*

Sammanfattning: A self-avoiding random walk is a random walk with no self-intersections. The Self-Avoiding Walk (SAW) model is the uniform measure of self-avoiding walks of a given length. It was introduced by Flory in the 1940s as a model for polymers. Since then, it has been studied intensely by physicists and mathematicians and there are many beautiful predictions and conjectures. Despite much effort, however, there are few rigorous results. Loop-erased random walk (LERW) is another probability measure on self-avoiding walks about which more is known. In the talk I will explain some of the natural questions about these models, along with corresponding predictions and (in a few cases) answers.

**2 Onsdag 11 mars 2015, Pär Kurlberg, KTH**

*Nodal length statistics for arithmetic random waves*

Sammanfattning: The Laplacian acting on the standard two dimensional torus has spectral multiplicities related to the number of ways an integer can be written as a sum of two integer squares. Using these multiplicities we can endow each eigenspace with a Gaussian probability measure. This induces a notion of a random eigenfunction (aka "random wave") on the torus, and we study the statistics of the lengths of nodal sets (i.e., the zero set) of the eigenfunctions in the "high energy limit". In particular, we determine the variance for a generic sequence of energy levels, and also find that the variance can be different for certain "degenerate" subsequences; these degenerate subsequences are closely related to circles on which lattice points are very badly distributed. Time permitting we will discuss which probability measures on the unit circle that "come from" lattice points on circles.

**2 Onsdag 18 mars 2015, Hojoo Lee, Seoul, Sydkorea**

*Sweeping out minimal cones in Euclidean space*

Sammanfattning: The theory of minimal submanifolds has its origin in the theory of calculus of variations developed by Euler and Lagrange in the 18th century and in later investigations by Schwarz, Riemann and Weierstrass in the 19th century. It has very recently seen remarkable advances that have solved lots of long standing open problems. We explicitly construct generalized helicoids in odd dimensional Euclidean spaces, and minimal cones in even dimensional Euclidean spaces. Our minimal varieties unify various interesting examples: classical helicoids foliated by straight lines, Choe-Hoppe's minimal hypersurfaces foliated by Clifford's minimal cones, Barbosa-Dajczer-Jorge's ruled minimal submanifolds, and Harvey-Lawson's twisted normal cone over Clifford torus. This work is joint with E. Lee.

**2 Onsdag 25 mars 2015, Berkant Savas, ITN**

*Clustering based low rank matrix approximations*

Sammanfattning: In this talk we will present, discuss, and motivate clustering based low rank matrix approximation methods. These methods are particularly suited for problems with large scale sparse matrices that represent graphs and/or bipartite graphs from information science applications. Our approach has a number of benefits: (1) the approximation preserves important structure present in the original matrix (graph); (2) the approximation contains both global-scale and local-scale information; (3) the procedure is efficient both in computational speed and memory usage; (4) the obtained low rank approximations are highly accurate; and (5) the method outperforms regular low rank approximations in various application problems. Our approach is quite flexible and may be combined with different clustering methods and different algorithms for low rank approximations, e.g. the singular value decomposition, stochastic methods for matrix approximations, non-negative matrix factorizations. We will show deterministic and probabilistic approximation error bounds obtained by our method, and a number of experiments that exemplify various benefits in our approach.

**2 Onsdag 8 april 2015, Evgeniy Lokharu, MAI**

*Uniqueness and stability results on steady water waves with vorticity*

Sammanfattning: We consider the two-dimensional nonlinear problem describing steady gravity water waves with vorticity in a channel of finite depth. The water motion is assumed to be unidirectional and the surface tension is neglected. It is well known that among small-amplitude waves only Stokes and solitary waves exist provided the Bernoulli constant is close to its critical value. We complete this result by proving that all near-critical waves are necessarily small. Furthermore, we prove a stability estimate that imply uniqueness for small-amplitude waves with a prescribed Cauchy data of the profile at some point and provide a parametrization by the amplitude for the family of waves with near-critical values of the Bernoulli constant. Using similar methods, we study solitary-type waves for arbitrary Bernoulli constants and prove that they are necessarily supported by sub-critical shear flows. In particular, this means that there are no waves that oscillate and decay at the same time. This is joint work with Vladimir Kozlov and Nikolay Kuznetsov.

**2 Onsdag 15 april 2015, Valov Vesko, Nipissing University, North Bay, Kanada**

*Homogeneous ARN compacta*

**2 Onsdag 22 april 2015, Sergey Nazarov, MAI / Russian Academy of Sciences, Sankt Petersburg, Ryssland**

*Stabilizing solutions at a threshold of the continuous spectrum and scattering anomalies in a wave guide*

Sammanfattning: A criterion for the existence of stabilizing solutions of the homogeneous Dirichlet and Neumann spectral problems at thresholds of the continuous spectrum in a perturbed cylindrical or periodic wave guide will be presented on the base of the energy radiation conditions. The relationship of these solutions with the following spectral anomalies will be explained:

- the appearance of near-threshold eigenvalues, isolated in the discrete spectrum and embedded into the continuous spectrum;
- classical Wood's anomalies of the diffraction patterns;
- almost full transmission and reflection at above-threshold frequencies;
- failure of the limit absorption principle at the threshold frequencies.

Some open questions, especially in elasticity, will be formulated.

**2 Onsdag 29 april 2015, Emilio Bujalance, UNED, Spanien**

*Pseudo-real Riemann surfaces*

Sammanfattning: A Riemann surface is called pseudo-real if it admits anticonformal automorphisms but no anticonformal involution. Pseudo-real Riemann surfaces appear in a natural way in the study of the moduli space  $M_g^K$  of Riemann surfaces considered as Klein surfaces. If  $M_g$  is the moduli space of Riemann surfaces of genus  $g$ , then there is a two fold covering  $\pi: M_g \rightarrow M_g^K$ , and the preimage of the branch locus consists of the Riemann surfaces admitting anticonformal automorphisms, which are either real Riemann surfaces admitting anticonformal involutions, or pseudo-real Riemann surfaces. So pseudo-real Riemann surfaces are Riemann surfaces that are equivalent to their conjugate but the equivalence is not realized by an involution. In this talk we will study some results known about these Riemann surfaces.

**2 Onsdag 13 maj 2015, Lars Andersson, KTH och Albert Einstein Institut, Potsdam, Tyskland**

*Geometry and analysis in black hole spacetimes*

Sammanfattning: The dynamical stability of the Kerr rotating black hole spacetime is one of the central open problems in general relativity. The Carter constant plays a key role in understanding the stability problem. I will explain how the Carter constant arises and discuss how it and related symmetry operators enters in the analysis of the dynamics of geodesics, waves and Maxwell fields on the Kerr background.

**2 Onsdagen 20 maj 2015, Dewey Estep, University of Cincinnati, USA**

*Prime Ends on Metric Spaces and the Dirichlet Problem*

**2 Onsdagen 27 maj 2015, Maria Saprykina, KTH**

*Examples of Hamiltonian systems with Arnold diffusion*

Sammanfattning: Here is a heuristic description of the problem setting in "physical terms". Imagine a chain of mathematical pendula attached to the wall in a line, and moving. If they are not coupled, the energy of each pendulum is preserved for all time. Now we join each pair of neighboring pendula by a thin rubber band. Of course, the total energy of the system is still preserved. But what happens with the energy of each individual pendulum? KAM theorem asserts that under some generic assumptions, for "most" initial conditions the energy of each pendulum will stay close to the initial one for all time. But what happens for the "small part" of the initial conditions that are not described by this theorem? One of our results states that there exist initial conditions and a sequence of moments of time, such that at  $j$ -th moment of time the  $j$ -th pendulum moves with almost the total energy of the system. This behaviour is a manifestation of so-called Arnold diffusion. I shall speak about one more example exhibiting Arnold diffusion. These results were obtained in collaboration with Vadim Kaloshin and Mark Levi.

**2 Onsdagen 26 augusti 2015, Nikolay Kuznetsov, Russian Academy of Sciences, St. Petersburg, Ryssland**

*On delusive nodes of free oscillations*

Sammanfattning: Two theorems and one conjecture about nodal sets of eigenfunctions arising in various spectral problems for the Laplacian will be reviewed. It occurs that all these assertions are either incorrect or only partly correct, but their analysis has brought better understanding of the corresponding area of mathematical physics. The contribution made by V. I. Arnold will be emphasized.

**2 Måndagen 31 augusti 2015, Xining Li, Aalto University, Helsingfors, Finland**

*Preservation of bounded geometry under sphericalization and flattening: quasiconvexity and  $\infty$ -Poincare inequality*

Sammanfattning: This is a joint work with Estibalitz Durand-Cartagena. In this work we explore the preservation of quasiconvexity and  $\infty$ -Poincare inequality under sphericalization and flattening in the metric setting. The results developed in our previous work show the preservation of Ahlfors regularity, doubling property and the  $p$ -Poincare inequality for  $p < \infty$  under the sphericalization and flattening transformations provided the underlying metric space has annular quasiconvexity. In this work, we propose a weaker assumption to still preserve quasiconvexity and  $\infty$ -Poincare inequality, called radial starlike quasiconvexity and meridian starlike quasiconvexity extending in particular a result by Buckley, Herron and Xie to a wider class of metric spaces and covering the case  $p = \infty$  in our previous work.

**2 Onsdagen 2 september 2015, Maria Przybylska, University of Zielona Góra, Polen**

*Hunting for integrable systems in curved spaces*

Sammanfattning: The differential Galois theory has been created as a tool for checking that a given linear differential equation has a closed form solution. Quite recently it was discovered that it can be also used for investigation of the integrability of Hamiltonian systems. Namely, if a Hamiltonian system is integrable in the Liouville sense, then the differential Galois group of linear equations obtained from the linear part of the Hamiltonian vector field's expansion along a particular solution has abelian identity component. In the case of

Hamiltonians depending on parameters this implication can be used for searching for new integrable systems. Analysis of the differential Galois group enables one to distinguish values of these parameters such that necessary integrability conditions are satisfied and the system can be integrable. This procedure has appeared very effective in the case of natural Hamiltonian systems with standard kinetic energy and potentials that are homogeneous functions of coordinates. Then integrability obstructions are expressible by means of restrictions on eigenvalues of the Hessian matrix calculated for the potential function in some points, called Darboux points. It means that calculations of differential Galois integrability conditions reduce to linear algebra operations and this method really gave new, previously unknown, integrable cases. In this lecture we will define a class of Hamiltonian systems in curved spaces for which one can also formulate differential Galois integrability obstructions in such a nice form. This is the class of natural Hamiltonian systems with two degrees of freedom with kinetic energy depending on coordinates and appropriate form of potentials. The obtained integrability conditions gave new integrable and superintegrable systems which will be presented.

2 **Onsdagen 9 september 2015, Ugo Gianazza, University of Pavia, Italien**

*The expansion of positivity: old and new*

Sammanfattning: The so-called *Expansion of Positivity* (also known in the literature as Growth Lemma) is an important tool in the proof of regularity results for elliptic and parabolic equations, both in divergence and non-divergence form. After reviewing some well known examples and their applications, I will present and discuss a recent result about singular parabolic equations, both of p-laplacian and porous medium type.

2 **Fredagen 11 september 2015, Francesco Calogero, University of Rome "La Sapienza", Italien**

*On the generations of monic polynomials obtained by replacing the coefficients of the polynomials of the next generation with the zeros of a polynomial of the previous generation*

Sammanfattning: Generations of monic polynomials all of arbitrary degree  $N$  are obtained from a seed polynomial of degree  $N$  by identifying the coefficients of the polynomials of the next generation with the zeros of a polynomial of the previous generation; and Diophantine properties are reported of the zeros of the polynomials thereby obtained when the seed polynomial is the Hermite polynomial of degree  $N$ . This is joint work in progress with Oksana Bihun.

2 **Onsdagen 23 september 2015, Ralf Fröberg, Stockholms universitet**

*Something about numerical semigroups*

Sammanfattning: A numerical semigroup is a subset of the natural numbers which is closed under summation, contains 0, and contains all but finitely many natural numbers. The theory about numerical semigroups has applications to algebraic geometry and coding theory. I will talk about the main definitions and properties, and some applications to algebraic curves. The best reference to the pure theory of numerical semigroups is a book by Garcia-Sanchez and Rosales with the title Numerical semigroups.

2 **Onsdag 30 september 2015, Jürgen Rossmann, Universität Rostock, Tyskland**

*Asymptotics of solutions of the Neumann problem for the Poisson equation near edges*

Sammanfattning: We study the behavior of solutions of the Neumann problem for the Poisson equation near a straight  $(n - m)$ -dimensional edge in the  $n$ -dimensional space. More precisely, we consider the problem

$$-\Delta u = f \text{ in } K \times \mathbb{R}^{n-m}, \quad u = 0 \text{ on } \partial K \times \mathbb{R}^{n-m},$$

where  $K = \{x' \in \mathbb{R}^m : |x'| \in \Omega\}$  is a cone in  $\mathbb{R}^m$ ,  $2 \leq m < n$ , with vertex at the origin,  $\Omega$  denotes a subdomain of the unit sphere with smooth boundary  $\partial\Omega$ . First, we consider the Green function  $G(x, y)$  of this problem. We obtain the asymptotics of the Green function near the edge. Here, precise formulas for the coefficients of all terms in the asymptotics are given. These results are used in order to describe the behavior of the variational solution  $u$  of the problem (1) near the edge  $M$  of the domain  $K \times \mathbb{R}^{n-m}$  if the right-hand side  $f$  belongs to a weighted Sobolev space. The representation of the solution  $u$  in the case of the Neumann problem (1) is very similar to the case of the Dirichlet problem. However, we have an additional quasipolynomial in the decomposition of the solution of the Neumann problem. The coefficients both of the singular functions and of the quasipolynomial are represented in terms of the function  $f$ .

2 **Onsdagen 7 oktober 2015, Thomas Vogel, Ludwig-Maximilians-Universität München & Inst Mittag-Leffler**

*Approximation of foliations by contact structures*

Sammanfattning: In dimension three, foliations (except foliations by spheres) of codimension one can be approximated by contact structures. We will discuss this theorem together with some question about the relationship between properties of these structures (taut foliations, fillable contact structures, etc.) and applications.

2 **Onsdagen 14 oktober 2015, German Zavorokhin, Steklov Math. Institute, St. Petersburg, Ryssland**

*A fractal graph model of capillary type systems*

Sammanfattning: The capillary system is modelled by a fractal graph attached to a blood vessel. It is supplied with differential equations obtained from three-dimensional model by the dimension-reduction procedure. The geometry and physical parameters of this system are described by a finite number of scaling parameters which allows the system to have self-reproducing solutions, solutions which are determined by their values on a certain finite piece of the fractal graph and are continued on the remaining part by using these scaling factors. We describe all self-reproducing solutions, and as a result we obtain a connection between the pressure and the flux at the junction point between the capillary system and blood vessel. This connection gives an artificial boundary condition at the junction in the blood vessel and allows us to solve the problem for the flow in the blood vessel without solving it in the capillary system. This is joint work with Vladimir Kozlov and Sergei Nazarov.

2 **Torsdagen 15 oktober 2015, Alex Karashev, Nipissing University, North Bay, Kanada**

*Inductive dimension with respect to normal base*

Sammanfattning: The dimension-like invariant  $\text{Ind}_F$ , introduced by Illiadis, is a generalization of large inductive dimension  $\text{Ind}$ . It is defined with respect to a normal base  $F$ , which is a base of closed sets with special properties. We will discuss the definition and some properties of this invariant. Further, we briefly discuss applications of  $\text{Ind}_F$  to estimate dimensions of some topological spaces. The second part of this talk will be devoted to the dimension  $\text{Ind}_F$  of  $n$ -dimensional cubes. In particular, we show that, under the appropriate choice of the normal base  $F$ , the values  $\text{Ind}_F$  of  $n$ -cube form the set  $\{n, n+1, n+2, \dots, \infty\}$ .

2 **Onsdagen 21 oktober 2015, Mark Groves, Universität des Saarlandes, Saarbrücken och Lunds universitet**

*Three-dimensional solitary water waves*

Sammanfattning: The existence of solitary-wave solutions to the three-dimensional water-wave problem is predicted by the Kadomtsev-Petviashvili (KP) equation in the case of strong surface tension and the Davey-Stewartson (DS) system in the case of weak surface tension. The term "solitary wave" describes any solution which has a pulse-like profile in its direction of propagation, and these model equations admit three types of solitary waves. A line solitary wave is spatially homogeneous in the direction transverse to its direction of propagation, while a periodically modulated solitary wave is periodic in the transverse direction. A fully localised solitary wave on the other hand decays to zero in all spatial directions. In this talk I outline mathematical results which confirm the existence of the three-dimensional solitary waves for the full gravity-capillary water-wave problem in its usual formulation as a free-boundary problem for the Euler equations; both strong and weak surface tension are treated.

2 **Onsdagen 28 oktober 2015, Sergey Vakulenko, St Petersburg, Russia**

*Replicator stability of ecological systems*

Sammanfattning: We propose a new approach to the problem of stability of large ecological systems. This approach develops some ideas of M. Gromov for cell and molecular biology, and use the known Kolmogorov system for population dynamics. The model takes into account species extinctions and emergence and it is focused on two important problems: invasion of ecosystems by immigrants (that is important in connection with climate warming) and mass-extinctions. We show that stability depends not only on the usual parameters (mortality rates, self-limitation coefficients, and resource abundances), but also on an additional parameter  $N$  ("ecological potential", or the number of non-occupied ecological niches). The main result is as follows. If  $N$  is sufficiently large, then the system is stable with a probability close to 1. Such stability is possible even when for standard models (without species emergence) an ecological equilibrium does not exist and all species are extinct. This is joint work with Vladimir Kozlov and Uno Wennergren.

2 **Onsdagen 4 november 2015, Irina Asekritova, MAI**

*Interpolation of Fredholm operators*

Sammanfattning: It is well known that the class of Fredholm operators is very important for several areas of mathematics including spectral theory of operators on Banach spaces and PDEs. The study of interpolation properties of Fredholm operators was initiated by I. Shneberg in 1974 and continued in works by M. Zafran, Y. Sagher, W. Cao, N. Kalton, M. Mitrea and others. In the talk we will discuss some recent results on interpolation of Fredholm operators and their applications to the famous Lions-Magenes problem on interpolation of closed subspaces.

2 **Fredagen 6 november 2015, Tilahun Abebaw, Addis Ababa University, Etiopien**

*Decomposition factors of  $D$ -modules over a hyperplane arrangement in the plane*

Sammanfattning: Let  $\alpha_1, \alpha_2, \dots, \alpha_m$  be linear forms defined on  $\mathbb{C}^n$  and  $X = \mathbb{C}^n \cap V(\alpha)_{i=1}^m$ , where  $V(\alpha_i) = \{p \in \mathbb{C}^n : \alpha_i(p) = 0\}$ . The coordinate ring  $O_X$  of  $X$  is a holonomic  $A_n$ -module, where  $A_n$  is the  $n$ th Weyl algebra and since holonomic  $A_n$ -modules have finite length,  $O_X$  has finite length. We consider a "twisted" variant of this  $A_n$ -module which is also holonomic. Define  $M_\alpha^\beta$  to be the free rank-1  $\mathbb{C}[x]_\alpha$ -module on the generator  $\alpha^\beta$  (thought of as a multivalued function), where  $\alpha^\beta = \alpha_1^{\beta_1} \dots \alpha_m^{\beta_m}$  and the multi-index  $\beta = (\beta_1, \dots, \beta_m) \in \mathbb{C}^m$ . Our main focus will be the computation of the number of decomposition factors of  $M_\alpha^\beta$  and their description when  $n = 2$ .

2 **Onsdagen 11 november 2015, Johan Björklund, Uppsala universitet**

*Real algebraic knots and knot diagrams*

Sammanfattning: Hilbert's 16th problem concerns possible topological configurations of real algebraic curves (that is, what can a curve that is the zero set of some real polynomial look like?) in the plane (for some given degree). A natural generalization is to examine the same situation in space. Here the real algebraic curves can form knots, and so we need some form of real algebraic knot theory to understand the situation. During the talk I will discuss how to "translate" the classical smooth knot theory to a real algebraic setting (knot

diagrams, invariants, etc.) and in particular describe some new phenomena which appear.

2 **Onsdagen 18 november 2015, Alexander Khludnev, Russian Academy of Sciences, Novosibirsk, Ryssland**

*On the hierarchy of thin inclusions in elastic bodies*

Sammanfattning: In this talk, we consider equilibrium problems for elastic bodies with thin inclusions of different rigidity. It is assumed that a delamination of inclusions takes place, therefore forming a crack between the inclusions and the elastic body. We justify passages to zero and infinity of rigidity parameters and discuss limit problems. A full hierarchy of thin inclusions in elastic bodies is established.

2 **Onsdagen 25 november 2015, Venuste Nyagahakwa, MAI**

*Topology of Vitali selectors on the real line*

Sammanfattning:

# TOPOLOGY OF VITALI SELECTORS ON THE REAL LINE

Let  $\mathcal{F}$  be the family of all dense countable subgroups of the real numbers  $\mathbb{R}$ . Consider  $Q \in \mathcal{F}$ . Recall that a Vitali  $Q$ -selector of  $\mathbb{R}$  is any set which meets every coset  $x + Q, x \in \mathbb{R}$ , precisely in one point. We denote by  $\mathcal{V}(Q)$  the family of all Vitali  $Q$ -selectors of  $\mathbb{R}$  and by  $\mathcal{S}_{\mathcal{V}(Q)}$  the family of all non-empty finite unions of elements of  $\mathcal{V}(Q)$ . Let us recall (see [2] and [1], resp.) that the elements of  $\mathcal{S}_{\mathcal{V}(Q)}$  are non-measurable in the Lebesgue sense and without the Baire property on the real line. We define a new topology  $\tau(Q)$  on  $\mathbb{R}$  generated by the family  $\{\mathbb{R} \setminus S : S \in \mathcal{S}_{\mathcal{V}(Q)}\}$  as a base. One can observe that a subset  $A$  of  $\mathbb{R}$  is closed according to this topology iff  $\sup\{|A \cap (x + Q) : x \in \mathbb{R}|\} < \aleph_0$  or  $A = \mathbb{R}$ . We study topological properties of spaces  $\mathbb{R}(Q) = (\mathbb{R}, \tau(Q)), Q \in \mathcal{F}$ . In particular, we show that each  $\mathbb{R}(Q)$  is a  $T_1$  (not  $T_2$ ) hyperconnected topological space with  $\text{ind } \mathbb{R}(Q) = \text{Ind } \mathbb{R}(Q) = 1$ . Moreover, if  $Q_1, Q_2 \in \mathcal{F}$  then the spaces  $\mathbb{R}(Q_1)$  and  $\mathbb{R}(Q_2)$  are homeomorphic. Let us note that if  $Q_1 \subseteq Q_2$  and  $|Q_2/Q_1| = \aleph_0$  then  $\mathcal{S}_{\mathcal{V}(Q_1)} \cap \mathcal{S}_{\mathcal{V}(Q_2)} = \emptyset$ , and there exists a countable subset  $A$  of  $\mathbb{R}$  such that  $A$  is closed in  $\mathbb{R}(Q_1)$  but  $A$  is neither closed nor open in  $\mathbb{R}(Q_2)$ .

The results above can be extended to abelian Hausdorff topological groups of the second category without isolated points having countable dense subgroups.

This is a joint work with Vitalij A. Chatyrko.

## References

- [1] V. A. Chatyrko, On Vitali sets and their unions, *Matematicki Vesnik*, 63, 2 (2011) 87-92
- [2] A. B. Kharazishvili, Measurability properties of Vitali sets, *Amer. Math. Monthly* 118 (2011), no. 8, 693-703

Venuste Nyagahakwa  
Linköping University

2 **Onsdagen 2 december 2015, Eric Setterqvist, MAI**

*Taut strings and real interpolation*

Sammanfattning: The notion of taut string was introduced by G.B. Dantzig in 1971 and has since then appeared in a broad range of applications including statistics, image processing, stochastic processes and communication theory. In this talk, we present an approach based on the  $K$ -functional of real interpolation that provides a unifying framework of existing theories and extend the range of applications of taut strings. More precisely, we introduce the notion of invariant  $K$ -minimal sets, explain their connection to taut strings and characterize all bounded, closed and convex sets in  $\mathbb{R}^n$  that are invariant  $K$ -minimal with respect to the couple  $(\ell^1, \ell^\infty)$ . Different examples of invariant  $K$ -minimal sets with respect to the couple  $(L^1, L^\infty)$  will also be discussed. In particular, the  $L^1$ -closure of the image of the  $L^\infty$ -ball of smooth vector fields with compact support in  $(0, 1)^m$  under the divergence operator is an invariant  $K$ -minimal set with respect to  $(L^1, L^\infty)$ .

2 **Onsdagen 9 december 2015, Hans Jürgen Engelbert, Friedrich Schiller-University, Jena, Tyskland**

*Stochastic Differential Equations for Sticky Reflecting Brownian Motion*

Sammanfattning: We study the 'ordinary' stochastic differential equation (SDE) with irregular drift and diffusion coefficients

$$X_t = x_0 + \mu \int_0^t 1_{\{0\}}(X_u) du + \int_0^t 1_{(0,+\infty)}(X_u) dB_u, \quad t \geq 0,$$

where  $x_0 \in \mathbb{R}_+$  is the initial state and  $(B, \mathbb{F})$  is a standard Brownian motion. It turns out that the equation above is equivalent to a system of equations involving local time at state 0 and the occupation time of the solution at state 0, and a solution  $(X, \mathbb{F})$  will be called \textit{sticky reflecting Brownian motion} with stickiness parameter  $\mu \in (0, \infty)$ . The limiting cases  $\mu = 0$  and  $\mu = \infty$  correspond to an absorbing Brownian motion and an immediately reflecting Brownian motion, respectively. We prove that the SDE has a jointly unique weak solution but no strong solution. The latter fact verifies Skorokhod's conjecture on sticky reflecting Brownian motion and provides alternative arguments to those given in the literature.

2 **Onsdagen 16 december 2015, Jonas Sjöstrand, KTH**

*Integer-valued games*

Sammanfattning: I will give an introduction to Conway's beautiful theory of combinatorial games and surreal numbers. Then I will present a new class of integer-valued partizan games called element-removal games. In general, these games are PSPACE-hard to play, but in some instances there is a simple optimal strategy. One example is the leaf-removal game, played on a tree whose vertices are colored black or white. The players are called Black and White and in each move the player removes any leaf of their own color. If this is impossible the game is lost.



# The Mathematical Colloquia held in 2014 at Linköping University

Organized by **Anders Björn**, **Milagros Izquierdo Barrios**, **Vladimir Kozlov**, and **Hans Lundmark**.

## 2 Onsdag 29 januari 2014, Jan van den Heuvel, London School of Economics and Political Science, UK

*The complexity of change*

Sammanfattning: Many combinatorial puzzles and problems can be formulated as "Can I transform configuration 1 into configuration 2, if certain transformations only are allowed?". An example of such a question is: given a certain position of the Rubik's Cube, is it possible to go back to the position with all sides of one colour (and without taking the cube apart!)? A more mathematical example is: given two valid assignments of a logical expression, can I transform the first assignment into the second one, by changing the truth value one variable at a time, and always maintaining a solution of the SAT-instance? A final example is: given two  $k$ -colourings of a graph, can I transform the first  $k$ -colouring into the second one, by recolouring one vertex at a time, and always maintaining a proper  $k$ -colouring? In this talk we shall give an overview of some older and more recent work on this type of problem. The emphasis will be on the computational complexity of the problems: how hard is it to decide if a certain transformation is possible or not?

## 2 Onsdag 5 februari 2014, Alexander I. Nazarov, Steklov Institute and Saint Petersburg State University, Russia

*On monotonicity of integral functionals under monotone and symmetric rearrangements*

Sammanfattning:

**On monotonicity of integral functionals  
under monotone and symmetric rearrangements**  
Nazarov A. I. (St. Petersburg Dept of Steklov Institute  
and St. Petersburg State University, Russia)

Let  $u \in W_1^1([-1, 1])$  be a nonnegative function. We define  $\bar{u}$  to be the monotone rearrangement of  $u$  and  $u^*$  to be its Steiner symmetrization. Let  $F : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$  be a continuous function which is increasing and convex in the second argument.

It is well known that for  $I(u) = \int_{-1}^1 F(u, |u'|) dx$  the inequalities  $I(u^*) \leq I(u)$  and  $I(\bar{u}) \leq I(u)$  hold.

In the paper [3] the inequality  $I(u^*) \leq I(u)$  was proved for the functional

$$I(u) = \int_{-1}^1 F(u, |a(x, u)u'|) dx,$$

where a continuous weight function  $a$  is even and convex in  $x$ . Also a multidimensional analog was studied. However, in fact the proof in [3] is correct only for Lipschitz functions  $u$ . We prove this result for a natural class of  $u$ .

Then we consider the more complicated case of the monotone rearrangement. Let  $a$  be even with respect to the first argument and satisfy

$$\forall u \in \mathbb{R}, \forall s, t \in [-1, 1] : 1 + s + t \in [-1, 1] \\ a(s, u) + a(t, u) \geq a(1 + s + t, u).$$

We modify the construction from [4] and prove  $I(\bar{u}) \leq I(u)$  for functions  $u$  lying in the natural functional class.

Note that in the paper [5] the inequality  $I(\bar{u}) \leq I(u)$  was considered under the additional restriction  $u(-1) = \min u$ . However, both the class of functions  $F$  and the class of admissible weights in that paper are non-optimal.

This talk is based on the joint work with Sergey Bankeevich [1]. An earlier version of our result was announced in [2].

## References

- [1] Bankeevich S. V., Nazarov A. I. On monotonicity of some functionals under rearrangements // SPbMS El. Prepr. Archive. N 2013-04. 23p. (Russian).
- [2] Bankeevich S. V., Nazarov A. I. A generalization of the Pólya-Szegő inequality for one-dimensional functionals // Doklady RAN, 438 (2011), N1, 11-13 (Russian). English transl.: Doklady Math. 83 (2011), N3, 287-289.
- [3] Brock F. Weighted Dirichlet-type inequalities for Steiner symmetrization // Calc. Var. 8 (1999), 15-25.
- [4] Alberti G., Serra Cassano F. Non-occurrence of gap for one-dimensional autonomous functionals // Proceedings of "Calc. Var., Homogen. and Cont. Mech.", Bouchitté G., Buttazzo G., Suquet P., ed.: World Sci., Singapore, 1994, 1-17.
- [5] Landes R. Some remarks on rearrangements and functionals with non-constant density // Math. Nachr. 280 (2007), N5-6, 560-570.

## 2 Onsdag 12 februari 2014, Mieczysław Mastyło, University of Poznań, Polen

*On almost everywhere convergent Fourier series*

Sammanfattning: We will discuss some recent work with Maria J. Carro and Luis Rodríguez-Piazza, concerning the fundamental problem of Fourier analysis related to almost everywhere convergent Fourier series.

## 2 Onsdag 5 mars 2014, Alexandr Kostochka, University of Illinois at Urbana-Champaign, USA

*Coloring sparse graphs*

Sammanfattning: Graph coloring is assigning colors to vertices of a graph in such a way that adjacent vertices always have different colors. The problem of coloring vertices of a graph with fewest colors is NP-hard and attracted a lot of attention. Among the most known results are the Four Color Theorem claiming that every planar graph is 4-colorable and Grötzsch's Theorem stating that every planar graph without 3-cycles is 3-colorable. A graph  $G$  is  $k$ -critical if  $G$  cannot be colored with  $k-1$  colors but each of its proper subgraphs can. In order to understand why some graphs need many colors to be colored, one needs to understand the structure of  $k$ -critical graphs. The first main result of the talk is a new lower bound on the number of edges in  $n$ -vertex  $k$ -critical graphs proving a conjecture of Gallai from 1963 (joint with Yancey). On the base of this, we give a very simple proof of Grötzsch's Theorem above and describe the planar graphs with exactly four triangles that are not 3-colorable (joint result with Borodin, Dvorak, Lidicky and Yancey). This answers a question of Erdős from 1990.

## 2 Onsdag 12 mars 2014, Vicenç Torra, IIIA-CSIC, Barcelona, Spanien

*Non-additive measures and integrals*

Sammanfattning: In this talk I will present some results on non-additive measures and integrals. I will review the Choquet integral, a generalization of the Lebesgue integral for non-additive measures, review distorted Lebesgue measures and distorted probabilities, and present our last results on the definition of the  $f$ -divergence and Hellinger distance for this type of measures. I will also discuss applications of non-additive measures and integrals, specially in decision making.

## 2 Onsdag 19 mars 2014, Krzysztof Marciniak, ITN

*Flat coordinates for Stäckel systems*

Sammanfattning: Stäckel separable systems, i.e., a class of quadratic in momenta Hamiltonian systems separable (in the sense of HamiltonJacobi theory) in orthogonal coordinates, are most conveniently obtained from appropriate separation relations. In this talk I use this fact to show how to explicitly construct flat coordinates for those Stäckel systems of the so called Benenti type that are flat from a single generating function. Our construction encompasses the already known cases of flat coordinates for Benenti systems, namely Jacobi elliptic coordinates, Jacobi parabolic coordinates and (in certain cases) coordinates recently discovered by A. Sergueyev and M. Blaszk. Moreover, I will demonstrate the explicit form

(in flat coordinates) of some important geometric objects related with these systems, such as Killing tensors and families of separable potentials.

📅 **Onsdag 26 mars 2014, Mats Andersson, Chalmers och Göteborgs universitet**

*Generalized cycles in projective space and local intersection numbers*

Sammanfattning: Let  $Z$  and  $W$  be two varieties in projective space. Some years ago, Tworzewski, and independently Gaffney and Gassler, introduced for each point on the set-theoretical intersection of  $Z$  and  $W$ , a list of non-negative integers, called the local intersection numbers, that generalize the classical HilbertSamuel multiplicity. We introduce a class of generalized cycles, that contains all analytic cycles. Each generalized cycle has well-defined multiplicity at each point and a well-defined degree. Given two (generalized) cycles  $Z$  and  $W$  we define a product  $Z \bullet W$ , which is a generalized cycle with the property that its multiplicities at each point is precisely the local intersection numbers. Moreover, the product respects Bezout's identity. We also discuss the relation to the classical non-proper intersection product. In particular, from  $Z \bullet W$  one can get the associated cohomology class on the set-theoretical intersection of  $Z$  and  $W$ . This is a joint work in progress with D Eriksson, H Samuelsson, E Wulcan and A Yger.

📅 **Onsdag 2 april 2014, Helge Holden, Norges teknisk-naturvitenskapelige universitet, Trondheim, Norge**

*On the initial-value problem for the CamassaHolm equation*

Sammanfattning: The CamassaHolm (CH) equation reads  $u_t - u_{txx} + \kappa u_x + 3uu_x - 2u_x u_{xx} - uu_{xxx} = 0$  where  $\kappa$  is a real parameter. We are interested in the Cauchy problem on the line with initial data in  $H^1$ . There is a well-known and well-studied dichotomy between two distinct classes of solutions of the CH equation. The two classes appear exactly at wave breaking where the spatial derivative of the solution becomes unbounded while its  $H^1$  norm remains finite. We here survey this problem and introduce a novel solution concept gauged by a continuous parameter  $\alpha$  in such a way that  $\alpha = 0$  corresponds to conservative solutions and  $\alpha = 1$  gives the dissipative solutions. This allows for a detailed study of the difference between the two classes of solutions and their behavior at wave breaking. We also extend the analysis to a two-component CamassaHolm system. This is joint work with Katrin Grunert (NTNU) and Xavier Raynaud (SINTEF).

📅 **Onsdag 9 april 2014, Alain Albouy, Observatoire de Paris, Frankrike**

*An extension of Beltrami's theorem about spaces which are geodesically equivalent to the affine space*

Sammanfattning: The central projection from a plane to a sphere maps geodesics on geodesics without respecting the time parameter. Beltrami's theorem is a characterization of the spaces of constant curvature by such a property. We will present an extension of this theorem to the case of a field of degenerate symmetric bilinear forms on the tangent space, instead of non-degenerate ones. We will meet an interesting algebraic problem: what are the linear maps from a space of bivectors to another space of bivectors which send decomposable bivectors on decomposable bivectors? In 1980, Jaak Vilms used some results on this question to solve problems of local embedding of Riemannian manifolds.

📅 **Onsdag 16 april 2014, German Zavorokhin, Steklov Math. Institute, St. Petersburg, Ryssland**

*The wave field of a point source that acts on the permeable free boundary of a Biot half-plane*

Sammanfattning: The initial boundary value problem of wave propagation in a half-plane filled with a fluid-saturated porous solid is considered. The Biot medium is isotropic, homogeneous, and with open pores on the boundary. Using complex analysis techniques, explicit formulas for the components of displacement vectors in elastic and fluid phases are obtained.

📅 **Tisdag 22 april 2014, Alexandre Krashev, Nipissing University, Kanada**

*Homogeneous compacta and the BingBorsuk conjecture*

Sammanfattning: An absolute neighbourhood retract (ANR) is a topological space which is a retract of some open neighbourhood for any embedding of it into another space. It is well-known that all polyhedra are ANRs, but there are many examples of compact ANRs that exhibit some extraordinary properties different from that of polyhedra. One of the most intriguing open questions about compact ANRs is the following problem, known as the BingBorsuk conjecture: is every finite-dimensional compact homogeneous ANR a manifold? Here a space is called homogeneous if for any points  $x$  and  $y$  in it there exists a homeomorphism of the space onto itself that moves  $x$  to  $y$ . It has been shown that the BingBorsuk conjecture implies the Poincare conjecture. In my talk, I will give an overview of various results related to the BingBorsuk conjecture and homogeneous ANRs.

📅 **Onsdag 23 april 2014, Magnus Herberthson, MAI**

*The Physical Optics approximation for the scattering from a metallic sphere*

Sammanfattning: In electromagnetics, when an incoming wave interacts with and is scattered from an object, the scattered field is often calculated from the induced (surface) currents. For complicated objects, which are also many wavelengths in size, the problem of determining these currents often gets unmanageable, and one way around this is to use the so called Physical Optics approximation. This approximation roughly says that the incoming field pointwise gives rise to the surface current which would result if the wave instead was reflected from an infinite tangent plane at the point in question. Although widely used, there are few (if any) situations where this approximation has been formally validated. I will discuss what this approximation says in the simplest case, i.e., scattering from a sphere. I also claim that the approximation in this case is valid, and I will discuss some parts of the proof.

📅 **Onsdag 7 maj 2014, Juhana Siljander, Helsingfors universitet, Finland**

*A parabolic Harnack inequality for a nonlocal in time diffusion equation*

Sammanfattning: Nonlocal PDEs have gathered a lot of interest during the last years. In particular, the fractional Laplacian and its generalizations have been studied extensively recently. In this talk we will discuss a different kind of nonlocal equation: namely a parabolic diffusion model where the nonlocal operator is in time instead of space. This kind of equations arise in physics as a random walk model for anomalous diffusion. They have also been used to model diffusion on fractals as well as heat conduction with memory. The talk will consider a recent result concerning the Harnack inequality for weak solutions of the so called time-fractional heat equation.

📅 **Onsdag 14 maj 2014, Bas Michielsen, ONERA, Toulouse, Frankrike**

*Stochastic boundary integral equations in electromagnetic scattering*

Sammanfattning: In electromagnetic interaction theory, the scattering of electromagnetic waves by obstacles plays an important role. This scattering process is characterised by a set of "observables", i.e., the scattering coefficients. Each observable is defined by the evaluation of a current distribution on a regular electromagnetic field. The numerical construction of the current distributions and fields needs the solution of boundary value problems for the Maxwell equations. The practical value of a numerical computation of such scattering coefficients depends, of course, on the correspondence between the numerical representation of the obstacle's geometry. Modelling errors can be due to deviations between the nominal geometry and the actual realisation of the obstacle or lack of precision in its discretised representation. It is important to be able to characterise the variability of the observables due to such modelling errors. One strategy for finding such characterisations consists of replacing a deterministic model by a probabilistic model. A probability measure is chosen to parameterise the uncertainty in the geometry of the scattering object. This leads to boundary value problems with stochastic boundaries. Such problems can be solved numerically in many different ways. In this talk, we study the consequences of uncertainties when solving a boundary value problem by means of an equivalent integral equation on the boundary. A standard boundary integral equation is then replaced by a boundary integral on a stochastic surface. In the analysis of this problem, we benefit from the exterior differential calculus. In the first part of the talk, we present some essential relations of this calculus in the context of boundary integral equations in electromagnetic scattering. In the second part of the talk, we derive a first order asymptotic of a stochastic boundary integral equation which gives important estimates of its solution.

📅 **Onsdag 21 maj 2014, Johann Engelbrecht, University of Pretoria, Sydafrika**

*Visualising complex zeroes with sibling curves*

Sammanfattning: A parabola such as  $y = x^2 + 1$  is said to have "imaginary" zeroes. Through centuries, many people have attempted to explain these illusive zeroes. In this talk we trace root calculation and root representation through history including the development of the notion of complex numbers and subsequent graphical representation thereof. We show that by restricting the domain to those complex numbers that map onto real numbers, representations of functions other than the ones in the real plane are obtained. In other words, the well-known curves in the real plane only depict part of a bigger whole, e.g. the graph that we normally use for the parabola above is one of a pair the other sibling up to now totally hidden! This expanded representation brings new insight into visualising complex roots and the graphs of complex functions. In fact, the question immediately arises: when is a parabola a parabola?

📅 **Måndag 26 maj, Luboš Pick, Prag, Tjeckien**

*Sharp iteration principle for higher-order Sobolev embeddings*

Sammanfattning: We survey results from the paper [CPS, arXiv:1311.0153] in which we developed a new sharp iteration method and applied it to show that the optimal Sobolev embeddings of any order can be derived from isoperimetric inequalities. We prove thereby that the well-known link between first-order Sobolev embeddings and isoperimetric inequalities translates to embeddings of any order, a fact that had not been known before. We show a general reduction principle that reduces Sobolev type inequalities of any order involving arbitrary rearrangement-invariant norms on open sets in  $\mathbf{R}^n$ , possibly endowed with a measure density and satisfying an isoperimetric inequality of fairly general type, to considerably simpler one-dimensional inequalities for suitable integral operators depending on the isoperimetric function of the relevant sets. As a direct application of the reduction principle we determine the optimal target space in the relevant Sobolev embeddings both in standard and in non-standard classes of function spaces and underlying measure spaces. In particular, the results apply to any-order Sobolev embedding on regular (John) domains, on Maz'ya classes of (possibly irregular) Euclidean domains described in terms of their isoperimetric function, and on families of product probability spaces, of which the Gauss space and the exponential measure space are classical instances. This is a joint work with Andrea Cianchi of Florence and Lenka Slavíková of Prague.

📅 **Måndag 26 maj, Andrea Cianchi, Florens, Italien**

*Gradient regularity in elliptic boundary value problems via rearrangements*

Sammanfattning: This talk is devoted to some results, in collaboration with V. Maz'ya, on global integrability properties of the gradient of solutions to boundary value problems for nonlinear elliptic equations (or systems, in some cases) in divergence form. Minimal assumptions on the regularity of the ground domain and of the prescribed data for a certain gradient bound are pursued. A distinctive feature of our approach is in the derivation of estimates which are flexible enough to be applied in the proof of gradient bounds for a wide choice of norms. Most of the relevant estimates are formulated in terms of pointwise inequalities for the distribution function of the length of the gradient, or, equivalently, for its

decreasing rearrangement. With this tool at disposal, global bounds for any rearrangement invariant norm of the gradient of solutions to either Dirichlet or Neumann boundary value problems are simply reduced to one-dimensional inequalities for Hardy type operators.

**Onsdag 28 maj 2014, Pekka Koskela, Jyväskylä, Finland**

*Intrinsic Hardy-Orlicz spaces of conformal maps*

Sammanfattning: The theory of Hardy spaces of analytic functions goes back to F. Riesz and G.H. Hardy. In the definition, one considers boundedness of the integrals of the  $p$ -th power of the modulus of  $f(z)$  over circles centred at the origin. The very same definition has also been used for injective analytic functions, that is for conformal maps. However, a conformal map is simply a conformal change of the metric and hence the modulus of  $f(z)$  is not a natural quantity to consider. A much more natural choice is the intrinsic distance to  $f(0)$  in the image domain i.e. the distance between 0 and  $z$  induced by the conformal density associated to the conformal map. Assuming that  $f(0) = 0$ , this intrinsic distance is at least the modulus of  $f(z)$  and can well be substantially larger. Somewhat surprisingly, the conformal map belongs to the Hardy class  $H^p$  if and only if it belongs to the corresponding space associated to the intrinsic distance. This is not anymore true for the case of Hardy-Orlicz classes.

**Måndag 9 juni 2014, Helen Doerr, Syracuse University, USA**

*Engaging students in the Calculus class*

Sammanfattning: How can asking good questions help students learn Calculus? In this talk, I will show how the use of "student response systems" (also known as "clickers") can engage students in making conjectures and arguing about their validity. Good clicker questions can also provide both students and instructors with insights into prior knowledge and misconceptions that can be addressed through peer interactions and feedback. The development of "good" questions and the management of the class discussion will be demonstrated as well.

**Onsdag 11 juni 2014, Jacek Szmigielski, University of Saskatchewan, Kanada**

*An introduction to G-functions and some of their applications*

Sammanfattning: In the first part of the talk I will introduce in a very elementary way Meijer G-functions. This remarkable class of special functions of the hypergeometric type appears in a variety of applications as well as is behind modern on-line depositories of integral formulas and integral identities. In a slightly more technical second part of the talk I will discuss the origin of my own interest in these functions: random two-matrix models of CauchyLaguerre type. For these models many probabilistic questions can be reduced to statements involving a subclass of Meijer G-functions. This talk is based on joint work with M. Bertola, M. Gekhtman and, independently, R. Beals.

**Freitag 13 juni 2014, Jonatan Lenells, Baylor University, USA**

*RiemannHilbert problems and long-time asymptotics for the DegasperisProcesi equation*

Sammanfattning: I will introduce a theory of matrix RiemannHilbert problems for a class of jump contours of very low regularity. As an application, the long-time asymptotics for the DegasperisProcesi equation on the line and on the half-line will be determined.

**Onsdag 20 augusti 2014, Visa Latvala, University of Eastern Finland, Joensuu, Finland**

*Weak Cartan and Cartan properties of the fine topology on metric spaces*

Sammanfattning: We consider the fine topology on a complete metric space equipped with a doubling measure supporting a  $p$ -Poincare inequality. We focus on two versions of the Cartan property and discuss their applications to the non-linear potential theory as well as to the theory of Sobolev functions on metric spaces.

**Onsdag 27 augusti 2014, Axel Hultman, MAI**

*Inversion graphs and rook theory on permutations diagrams*

Sammanfattning: The diagram of a permutation  $\pi$  of  $n$  elements is the subset of the  $n$  by  $n$  grid consisting of the points that are above and to the right of "1" entries in the permutation matrix of  $\pi$ . Let  $B(\pi)$  be the number of permutations  $\sigma$  such that the permutation matrix of  $\sigma$  has only "0" entries on the diagram of  $\pi$ . The inversion graph of  $\pi$  has vertex set  $\{1, \dots, n\}$  and an edge connecting  $i < j$  if  $\pi(i) > \pi(j)$ . Let  $A(\pi)$  be the number of ways to direct the edges of the inversion graph without introducing directed cycles. Remarkably,  $A(\pi) = B(\pi)$  for all  $\pi$ . This was independently proven by Lewis and Morales and by myself. It is an instance of a more general conjecture due to Klein, Lewis and Morales, parts of which are still open. I shall sketch an elementary proof of the asserted identity and discuss what all this has to do with the cohomology of Schubert varieties and point counts of matrix varieties over finite fields.

**Onsdag 3 september 2014, Scott Armstrong, Université Paris Dauphine, Frankrike**

*Stochastic homogenization of elliptic PDEs*

Sammanfattning: The talk will concern stochastic homogenization of elliptic equations. An essential example is

$$-\operatorname{div}\left(A\left(\frac{x}{\epsilon}\right)\nabla u\right)=f$$

where the coefficient matrix  $A$  is independent and identically distributed on unit cubes with integer vertices. The goal is to study the equation in the regime  $0 < \epsilon \ll 1$  (so the coefficients are oscillating very quickly) and to understand in what sense the randomness is averaging out" and we see a homogenized" equation with constant coefficients. This problem has many applications (e.g., to material science) and has equivalent formulations in the probability community (random conductance model, random walks in random environments). I will discuss some recent progress in the quantitative theory for both linear and nonlinear equations.

**Onsdag 10 september 2014, Maria Przytylska, University of Zielona Góra, Polen**

*A differential Galois framework for searching for new integrable systems*

Sammanfattning: The problem of detection of new integrable systems has a long history, but it is only quite recently that some new methods for its (more or less) systematic investigation have appeared. Strong necessary conditions for integrability were formulated in terms of properties of the differential Galois group of equations obtained from linearisation of the investigated system along a non-equilibrium particular solution. As an example, the efficiency of these conditions will be demonstrated on the famous problem of the heavy rigid top. It will be shown that there are no other meromorphically integrable cases apart from those already known: the Euler case, the Lagrange case and the most complicated Kovalevskaya case. A programme for systematic research of integrable homogeneous potentials will be sketched. For this class of systems, integrability conditions obtained from local differential Galois analysis along particular solutions are complemented by other conditions due to some global analysis. These conditions are expressible by means of some Diophantine equations. These equations give the basis for systematic research of integrable homogeneous potentials. Examples of such analysis for potentials with two and three degrees of freedom will be shown.

**Onsdag 17 september 2014, Cornelia Schiebold, Mittuniversitet**

*Soliton equations and operator theory*

Sammanfattning: Soliton equations are an important class of nonlinear partial differential equations, which contain physically relevant equations like the Korteweg-de Vries (KdV) equation, the sine-Gordon equation, and the Nonlinear Schrödinger equation. While these equations govern very different physical phenomena, they have striking common structural properties like the existence of particle-like solutions (solitons) interacting in elastic collisions. The struggle to find a mathematical explanation has led to substantial progress in mathematical physics. Today it is known that the theory of soliton equations is linked to several major branches of mathematics. Our talk will be an introduction to an operator theoretic approach to soliton equations, which may be traced back to work of Marchenkov and enables us to apply Banach geometry in the study of solution families. As a motivation, we will carefully explain this in the most accessible case of the KdV equation. Then we will discuss further developments of the method in the study of matrix equations and hierarchies. In the applications part, we will talk on the asymptotic description of multiple pole solutions, the construction of matrix solitons and countable nonlinear superposition. We will illustrate our results by Mathematica plots.

**Onsdag 24 september 2014, Pencho Petrushev, University of South Carolina, USA**

*Spaces of distributions associated with non-negative self-adjoint operators*

Sammanfattning: Classical and nonclassical Besov and Triebel-Lizorkin spaces are developed in the general setting of a metric measure space with the doubling property and in presence of a non-negative self-adjoint operator whose heat kernel has Gaussian localization and the Markov property. Sub-exponentially localized and compactly supported frames are constructed and utilized for decomposition of Besov and Triebel-Lizorkin spaces. The frames are also applied for Littlewood-Paley decomposition of Hardy spaces. Furthermore, atomic decomposition of Hardy spaces is established in this general setting. This theory allows, in particular, to develop the Littlewood-Paley theory and spaces of distributions on the sphere, cube, ball, and simplex with weights as well as in the context of Lie groups, Riemannian manifolds, and other settings.

**Torsdag 25 september 2014, Sebastian Reyes Carocca, Universidad Autonoma Madrid, Spanien**

*Arithmeticity of Kodaira fibrations*

Sammanfattning: Let  $X$  be an algebraic variety and  $k$  a subfield of field of the complex numbers. We say that  $k$  is a field of definition for  $X$  if there exist polynomials with coefficients in  $k$  so that the variety that they define is isomorphic to  $X$ . If  $k$  is a field of numbers we shall say that the variety is arithmetic. Let  $S \rightarrow C$  be a Kodaira fibration, i.e. a non-trivial holomorphic family of algebraic curves over an algebraic curve  $C$ . In this talk we shall show that whether or not the algebraic surface  $S$  is arithmetic depends only on the biholomorphic class of its universal cover.

**Onsdag 8 oktober 2014, Leif Melkersson, MAI**

*Mittag-Leffler Modules*

Sammanfattning: I will present a class of modules which lies between the projective modules and the flat ones, namely the locally projective modules, also called flat strict Mittag-Leffler modules by Raynaud and Gruson, who introduced Mittag-Leffler modules in their famous and difficult paper in *Inventiones Mathematicae* 13 (1971). In particular I will study certain linear operators on such modules.

**Onsdag 15 oktober 2014, Milagros Izquierdo, MAI**

#### Cyclic Trigonal Maps

Sammanfattning: A complex algebraic curve (or Riemann surface)  $X$  of genus at least two will be considered, following Poincaré's uniformization, as the quotient of the hyperbolic plane by a Fuchsian group. The curve/Riemann surface  $X$  is defined over a number field if and only if there is meromorphic function  $f$  on the Riemann sphere ramified on at most three points, that is the (torsion-free) uniformizing group of  $X$  is a subgroup of a triangle group. The lifting by  $f$  of the segment  $[0, 1]$  is a graph embedded in the surface  $X$ : a **map** or **Grothendieck "dessin d'enfant"**. The map determines the analytical/birational structure of  $X$ . In this talk we will give an introduction to the theory of maps and focus on maps admitting an automorphism of prime order  $p$  whose quotient by  $f$  is an automorphism of the Riemann sphere, so called  $p$ -gonal maps.

#### Onsdag 5 november 2014, Klara Stokes, Höskolan i Skövde

##### Geometric Point-circle Geometries from Moore graphs

Sammanfattning: The study of configurations in projective real or complex plane is a classical subject in geometry. Configurations appear naturally as certain arrangements of lines, planes or circles in a geometric plane or space. A map is a drawing of a graph on a compact Riemann surface such that the complement of the drawing is the disjoint union of finitely many topological discs called faces. It will be explained how to construct geometric point-circle configurations embedded on Riemann surfaces from uniform maps. In particular, geometric realizations of all pentagonal geometries with  $k$  lines through each point and either  $k$  or  $k-1$  points on each line can be obtained in this way. All these pentagonal geometries come from Moore graphs. Therefore this work involves a study of maps of Moore graphs. In particular we give the minimum genus of the Hoffman-Singleton graph.

#### Onsdag 12 november 2014, Tomas Persson, Matematikcentrum, Lunds universitet

##### Random limsup-sets

Sammanfattning: Suppose that we have a sequence of arcs on a circle. We rotate these arcs randomly and form the union  $A$  of the arcs. A classical problem, originating from Dvoretzky, is when almost surely  $A$  is the entire circle. Similarly, one may consider the limsup-set  $B$  of the randomly rotated arcs, that is  $B$  is the set of points that are covered by infinitely many arcs. In fact, when  $A$  is the entire circle, then almost surely  $B$  is the entire circle. Recently, there has been some interest in determining the almost sure Hausdorff dimension of the limsup-set  $B$  as well as other fractal properties. These kind of problems have also been considered in higher dimension. I will talk about old and new results in this area. Necessary concepts such as Hausdorff dimension will be explained.

#### Onsdag 19 november 2014, Sergey Vakulenko, St Petersburg, Russia

##### Mechanisms of catastrophic phenomena in complex ecological webs via Hamiltonian dynamics

Sammanfattning: We investigate global stability and dynamics of large bipartite ecological networks by classical methods of dynamical systems theory, including Hamiltonian methods and averaging. Our analysis exploits the network topological structure, namely, existence of strongly connected nodes (hubs) in the networks. We reveal new relations between topology, interaction structure and network dynamics. We describe mechanisms of catastrophic phenomena leading to sharp changes of dynamics and investigate how these phenomena depend on ecological interaction structure. This is joint work with Vladimir Kozlov and Uno Wennergren.

#### Onsdag 26 november 2014, Anders Björn, MAI

##### Boundary regularity, barriers and Petrovskii's condition for $p$ -parabolic equations

Sammanfattning: Consider the Dirichlet (boundary value) problem for the Laplacian, i.e. take a domain  $G$  in  $\mathbf{R}^n$  and a continuous function  $f$  on the boundary  $\partial G$  and find the harmonic function  $u_f$  which has  $f$  as boundary values. For general domains this problem can't always be solved, and for existence of a solution we have to allow the boundary values to be taken in some weak sense (e.g. using Perron solutions). A boundary point  $x \in \partial G$  is regular if

$$\lim_{G \ni y \rightarrow x} u_f(y) = f(x) \quad \text{for all } f \in C(G).$$

One can similarly define regularity for other equations, e.g. the (elliptic nonlinear)  $p$ -harmonic equation  $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2} \nabla u) = 0$ , the (parabolic linear) heat equation  $\partial_t u = \Delta u$ , and the (parabolic nonlinear)  $p$ -parabolic equation  $\partial_t u = \Delta_p u$ . In the first three cases, boundary regularity can be characterized by the existence of a barrier, whereas in the last case one can use the existence of a barrier family. An open problem for 20 years has been whether the existence of a single barrier can be used to characterize regularity for the  $p$ -parabolic equation. I will show that this is not possible. Petrovskii (1935) showed that the origin is regular for the heat equation with respect to

$$\{(x, t) : |x| < A\sqrt{-t} \sqrt{\log|\log(-t)|} \text{ and } -1 \leq t < 0\},$$

if  $A = 2$ , while it is irregular if  $A > 2$ . I will discuss nonlinear  $p$ -parabolic analogs of this result. This talk is based on joint work with Jana Björn and Ugo Gianazza.

# The Mathematical Colloquia held in 2013 at Linköping University

Organized by Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov and Hans Lundmark.

## 2 Wednesday 23 January 2013, Elizabeth Wulcan, Chalmers

*On the effective membership problem for polynomial ideals*

Abstract: I will discuss how analytic tools, such as multivariable residue theory, can be used to obtain bounds for the degrees of solutions to polynomial ideal membership problems. I will present some extensions of classical results to the setting of singular varieties – in particular a global version of Huneke's singular Briançon-Skoda theorem. This is based on joint work with Mats Andersson.

## 2 Wednesday 30 January 2013, Hugo Parlier, University of Fribourg, Switzerland

*Kissing numbers for surfaces*

Abstract: The classical kissing number problem for sphere packings is the search for an optimal upper bound on the number of  $n$ -dimensional euclidean unit spheres, pairwise disjoint in their interior, that can be tangent to a fixed unit sphere. In the case of lattice sphere packings, one asks that the centers of the spheres be points lying on a lattice. Another classical problem for lattices is the study of Hermite constants which is the problem of finding optimal bounds on the length of a shortest non-trivial vector of a unit volume lattice of dimension  $n$ . Schmutz Schaller introduced a nice parallel between these problems and problems related to systoles on closed hyperbolic surface of genus  $g$ . A systole of a surface is a homotopically non-trivial curve of shortest length and the parallel problems focus on the maximum number of systoles and their maximum length in function of genus. In this talk I'll explain this parallel and why a hyperbolic surface of genus  $g$  cannot have more than roughly  $g^2$  systoles.

## 2 Wednesday 6 February 2013, Annemarie Luger, Stockholms universitet

*On Nevanlinna functions*

Abstract: Nevanlinna functions (these are functions mapping the upper half plane analytically into itself) and their generalizations appear in different areas of analysis as well as in many applications. In this talk I am going to introduce these functions (from my point of view) and present two examples where they play an important role. The first comes from spectral theory of quantum graphs, the second one is connected with applications in the theory of antennas.

## 2 Wednesday 13 February 2013, Jens Jonasson, Försvarsmakten

*How to keep data secret using a small permutation*

Abstract: I will present a modern approach to the problem of obtaining data confidentiality, the most classical goal of cryptography. Keeping data confidential for a long time means protecting against future adversaries with more knowledge and more computational power, placing high demands on the design of new cryptosystems. One way to increase the confidence in the design is to build provable secure constructions, relying only on a few well-studied primitives. During the talk we will see examples of how different branches of mathematics and computer science play a central role in modern cryptography.

## 2 Wednesday 20 February 2013, Sergio Benenti, Università di Torino, Italy

*Analytical cosmology (How a mathematician can understand cosmology: a personal viewpoint)*

Abstract: The aim is to make a well-ordered list of Principles, Definitions and Theorems. In most of the books and papers on cosmology there is not a clear distinction between these three kinds of statements. In this way we get economy, simplicity, and a very nice landscape of the matter.

## 2 Wednesday 27 February 2013, Tudor Ratiu, EPFL, Lausanne, Switzerland

*The Weil-Petersson geodesics on the universal Teichmüller space*

Abstract: The talk will begin with a quick review, from a geometrical point of view, of the classical theory of the universal Teichmüller space. Then, the Nag-Verjovsky formula for the Weil-Petersson metric will be presented. This then motivates the introduction of the Takhtajan-Teo Hilbert manifold structure on which this formula is given by a convergent integral. Properties of the geodesic spray will be discussed as well as questions about the diffeomorphism group on the circle at critical Sobolev index. Time permitting, some comments on the the image recognition problem will be also presented and linked to the geodesic spray of the Weil-Petersson metric.

## 2 Wednesday 6 March 2013, Jacek Szmigielski, University of Saskatchewan, Canada

*The life of peakons*

Abstract: Peakons are non-smooth solutions to a class of nonlinear partial differential equations. They were first discussed by R. Camassa and D. Holm (then both at the Los Alamos National Lab) in the early nineties of the last century. Peakons represent non-smooth nonlinear waves, yet one can think of them as interacting particles. The mathematical theory of peakons, as it turned out, goes back to T. Stieltjes. In particular the first complete construction of peakons was done by adapting Stieltjes' method of continued fractions. In this sense the theory of peakons is intimately connected with the theory of orthogonal polynomials and their generalizations. One such generalization suggested by the theory of peakons is a family of Cauchy biorthogonal polynomials. In this talk I will retrace the main steps in the story of peakons with an emphasis on the connection with orthogonal and biorthogonal polynomials.

## 2 Wednesday 13 March 2013, Klas Nordberg, ISY

*Symmetries in P3P*

Abstract: P3P is classical problem in geometry, in which we want to determine the position and orientation (the pose) of a camera that is observing 3 known 3D points, given the projections of these points in the camera image. The literature offers multiple proposals for how to solve P3P, in most cases based on finding the roots of a quadric polynomial since there are up to 4 poses of the camera that solve the problem. In the talk, some of these methods will be described briefly, but the focus is on a novel method that combines two earlier approaches: (1) a direct parametrisation of the camera pose in terms of a rotation matrix  $R$  and a translation vector  $t$ , and (2) a reduction of the computational cost of P3P to that of finding a single real root of a cubic polynomial with real coefficients. On the way to the cubic, we will stumble across a conjugate reciprocal quadric and also notice that there are several instances of symmetries involved in the proposed method that allow us to reach the cubic.

## 2 Wednesday 10 April 2013, Yuri Safarov, King's College, London

*Almost commuting operators*

Abstract: The talk will review recent results on the following well known problem: if the commutator of two bounded operators is small, are they close to a commuting pair?

## 2 Wednesday 24 April 2013, Sergey Nazarov, Russian Academy of Sciences, Saint Petersburg, Russia

*The spectrum of the thin Dirichlet grate of quantum waveguides*

Abstract: The spectrum of the Dirichlet Laplacian in the union of double-periodically posed thin strips is investigated by means of asymptotic analysis. Position and sizes of spectral gaps are described. In the case of local perturbations the appearance of eigenvalues in the discrete spectrum is verified, either below the essential spectrum, or in gaps. The influence of open subwaveguides is studied, too.

## 2 Wednesday 15 May 2013, Grigori Rozenblioum, Chalmers

*Finite rank Toeplitz operators and related problems in Analysis*

Abstract: A Bergman type space  $B$  is a subspace in  $L^2$  in a domain, consisting of solutions of an elliptic equation or system. The leading example is the classical Bergman space of square integrable functions, analytical in the unit disk. If  $P$  denotes the projection from  $L^2$  onto the disk, the Toeplitz operator  $T$  acts in  $B$  and maps  $u$  to  $PFu$ , where  $F$  is a function called the symbol. We consider the following problem. Suppose that the operator  $T$  has finite rank. What can be said about  $F$ ? We describe some recent result concerning this problem, related questions in real and complex analysis, and some applications of the finite rank results.

## 2 Wednesday 22 May 2013, Klara Stokes, IDA

*An alternative way to generalise the pentagon*

Abstract: A generalised polygon is a partial linear space such that its bipartite incidence graph has girth twice its diameter, just as do ordinary polygons - the incidence graph of the ordinary  $n$ -gon is the cyclic graph on  $2n$  vertices. Generalised polygons were introduced by Jacques Tits in order to describe geometric properties of simple groups of Lie type. By the Feit-Higman Theorem, the only finite examples of generalised polygons are thin (having two points on each line or two lines on each point) or the diameter  $n$  is either 3, 4, 6, or 8. In particular there are no (thick and finite) generalised pentagons or generalised heptagons. In this talk I will describe an alternative way of generalising the pentagon: the pentagonal geometry. A pentagonal geometry is a partial linear space in which for all points  $x$ , the points not collinear with the point  $x$ , form a line. Bounds for their parameters will be given, different constructions and some non-existence results will be presented, and a connection with distance-regular graphs will be described. This is joint work with Simeon Ball, John Bamberg and Alice Devillers.

## 2 Friday 31 May 2013, Francesco Calogero, Physics Department, University of Rome "La Sapienza", Italy

*Isochronous systems are not rare*



Abstract: A survey will be given of *isochronous* systems, i. e. systems that oscillate with a *fixed* period (for largely arbitrary initial data). It will be shown how to manufacture many such models, including "realistic" many-body problems whose time evolution is characterized by Newtonian equations of motion. In particular a fairly general technique will be described to modify fairly general models describing a time evolution so that the modified systems are *isochronous* (with period  $T$ ) yet mimic closely (or even exactly) the behavior of the unmodified system for a time interval  $\tilde{T}$  much smaller (or just smaller) than  $T$ .

As a particularly remarkable example (joint work with **F. Leyvraz**), it will be shown how – given the (autonomous) Hamiltonian  $H$  describing the most general (standard) nonrelativistic many-body problem (*arbitrary* number  $N$  of particles, *arbitrary* masses, *arbitrary* dimensions of ambient space, forces depending *arbitrarily* from all the particle coordinates, with the only restriction that the system be overall translation-invariant, i. e. no external forces) – it is possible to construct another (also autonomous) Hamiltonian  $\tilde{H}$  (in fact, an infinity of such Hamiltonians) featuring the same dynamical variables and parameters as  $H$  and in addition two *arbitrary* positive parameters  $T$  and  $\tilde{T}$  with  $T > \tilde{T}$ , and having the following two properties. (i) The new Hamiltonian  $\tilde{H}$  yields, over the (*arbitrarily* long!) time interval  $\tilde{T}$ , a dynamical evolution *identical* to that yielded by  $H$ . (ii) The Hamiltonian  $\tilde{H}$  is *isochronous*: all its solutions (for *arbitrary* initial data) are *completely periodic* with period  $T$ .

This finding raises (interesting?) questions about the difference among *nonintegrable* and *integrable* dynamics (*all isochronous systems are integrable*, indeed *more than superintegrable*), about the definition of *chaotic* behavior (including the apparent need to invent some such notion for a *finite* time interval), about the validity (say, for  $N \approx 10^{23}$ ) of statistical mechanics and of the second principle of thermodynamics, about cosmology (say, for  $N \approx 10^{85}$ ). It also demonstrates the impossibility to ascertain which dynamical theory is the correct one, out of an infinity of different theories predicting the same (*exactly the same*) evolution over an *arbitrarily long* time interval, but being *qualitatively different* (*isochronous* versus *chaotic*, *integrable* versus *nonintegrable*).

Main references: F. Calogero, *Isochronous systems*, OUP, Oxford, 2008 (paperback, 2012). F. Calogero and F. Leyvraz, "How to extend any dynamical system so that it becomes isochronous, asymptotically isochronous or multi-periodic", J. Nonlinear Math. Phys. **16**, 311–338 (2009); "Isochronous systems, the arrow of time and the definition of deterministic chaos", Lett. Math. Phys. **96**, 37–52 (2011).

## 2 Wednesday 4 September 2013, Frank Wikström, Lund

*Radó's theorem for polyanalytic functions*

Abstract: An old result due to Radó says that if a continuous function is holomorphic outside its zero set, then it is in fact holomorphic everywhere. I will give you a little about the story behind this result and some of its generalizations to several complex variables as well as to the class of polyanalytic functions of one and several variables. I will also mention a few other results on polyanalytic functions. (A function is called polyanalytic if it satisfies  $\frac{\partial^k f}{\partial \bar{z}^k} = 0$  for some  $k > 1$ .)

## 2 Wednesday 11 September 2013, Hiroaki Aikawa, Hokkaido University, Sapporo and Institut Mittag-Leffler

*Intrinsic ultracontractivity and the boundary Harnack principle*

Abstract: The semigroup associated with the Dirichlet heat kernel is said to be *intrinsic ultracontractive* if

- (a) the Dirichlet realization of the associated self adjoint operator has the first positive eigenvalue with positive  $L^2$  eigenfunction;
- (b) the heat kernel is bounded above and below by the product of the eigenfunctions with positive multiplicative constants depending on time.

We study intrinsic ultracontractivity and the boundary Harnack principle in a unified fashion by using capacity width.

First, we give an upper and lower estimate of the first eigenvalue in terms of capacity width, which yields a satisfactory answer to (i). Our parabolic argument also yields an exponential decay property of a certain caloric measure. Secondly, we employ a parabolic box argument to control the caloric measure by the elliptic Green function. This is the key step for (ii). We give a sharp sufficient integral condition for intrinsic ultracontractivity in terms of capacity width. A similar integral condition for the boundary Harnack principle is also obtained. Under geometric specifications, these integral conditions generalize known results and give more precise conditions. Sharpness is examined by an infinite funnel, for which we obtain a complete characterization of intrinsic ultracontractivity. Our method is purely analytic and elementary; it enables us to dispense with logarithmic Sobolev inequalities.

## 2 Wednesday 18 September 2013, Nikolay Kuznetsov, Russian Academy of Sciences, St. Petersburg, Russia

*Steady water waves with vorticity: spatial Hamiltonian structure*

Abstract: Spatial dynamical systems are obtained for two-dimensional steady gravity waves with vorticity on water of finite depth. These systems have Hamiltonian structure and Hamiltonian is essentially the flow-force invariant.

The presented results are obtained in collaboration with Vladimir Kozlov.

## 2 Wednesday 25 September 2013, Jörgen Backelin, Stockholms universitet

*Ramsey theory, e-numbers, and laces.*

Abstract: Ramsey theory is a part of the extremal graph theory, where both the bounds for invariants of graphs having certain properties, and the structure of graphs close to such bounds, are studied.

I first shall present a brief survey of the classical 2-colours Ramsey theory, and of the related e-numbers. I then shall specialise to laces, a kind of graphs defined by "cyclic recursion", and mention their use in determining upper bounds for e-values, and thus sometimes the exact values.

## 2 Wednesday 2 October 2013, Joachim Toft, Linnéuniversitetet, Växjö

*Harmonic oscillator and its inverse*

Abstract: Vi studerar inversen till harmoniska svängningsoperatorn. Speciellt studerar vi Weylsymbolen till denna invers. För jämna dimensioner visar vi hur ett exakt uttryck för denna kan fås fram. För alla dimensioner beskriver vi hur man kan säkerställa att denna operator avbildar analytiska och superanalytiska funktioner till funktioner av samma klass.

## 2 Wednesday 9 October 2013, Peter Lindqvist, NTNU, Trondheim, Norway and Institut Mittag-Leffler

*The infinity-Laplace equation and viscosity solutions*

Abstract: The infinity-Laplace equation and its solutions, the infinity-harmonic functions, have been much studied during the last twenty years. This "fully non-linear equation" is akin to the ordinary Laplace equation, but requires viscosity solutions, a concept that, originally, for first order equations was developed by Lions, Evans, Ishii, Souganidis, Jensen, and others. It appears in calculus of variations, PDEs, game theory, and image processing. It is appropriate to mention that the equation was found more than half a century ago by Aronsson in connexion with Lipschitz extensions.

## 2 Wednesday 16 October 2013, Raphael Stuhlmeyer, Universität Wien, Austria

*Interfacial Gerstner waves*

Abstract: While Gerstner wave theory belongs to the early beginnings of the mathematical treatment of water waves, there has been a recent resurgence of interest in adapting Gerstner's explicit solution – the only such explicit solution for the 2D gravity water-wave problem – to different physical configurations. This talk presents some recent work extending Gerstner's wave to two-fluid interfaces, for both planar waves and waves along a sloping beach.

## 2 Wednesday 23 October 2013, Ugo Gianazza, University of Pavia, Italy and Institut Mittag-Leffler

*Porous medium type equations and potential estimates*

Abstract: The regularity theory for solutions to the porous medium equation

$$u_t - \Delta u^m = 0, \quad m > 1,$$

has seen a tremendous development in the last 30 years, but quite a number of very interesting problems remain open: in particular, very little is known about the properties of solutions, when on the right-hand side there is a non-negative Radon-measure  $\mu$  having finite total mass. In order to have an insight about the kind of estimates one should expect in this situation, I will review two different types of results:

- (a) Regularity estimates for the classical porous medium equation, i.e. when  $\mu = 0$ ;
- (b) Potential estimates for Laplace,  $p$ -Laplace and heat equation.

I will then present linear pointwise estimates for solutions to our problem via Riesz potentials, and discuss future research perspectives. These are results obtained in collaboration with Verena Bögelein (Erlangen) and Frank Duzaar (Erlangen).

## 2 Wednesday 6 November 2013, Nageswari Shanmugalingam, University of Cincinnati, USA and Institut Mittag-Leffler

*Poincaré inequalities in metric measure spaces: some new examples*

Abstract: A brief survey on motivation for Poincaré inequalities based on the upper gradient approach will be given, followed by a discussion of some example metric measure spaces that support a Poincaré inequality, with focus on a recent work on sphericalization and flattening procedures that produce new examples.

## 2 Wednesday 13 November 2013, Sergey Vakulenko, Institute of Mechanical Engineering Problems and University of Technology and Design, Saint Petersburg, Russia

*Evolution as a hard combinatorial problem*

Abstract: We consider the key question of biology: how to explain complexity emergence and formation of complex organs. In biology, the concept of "complexity" is not transparently defined. Different approaches to complexity were developed. We exploit here some recent ideas from theoretical computer science. This allows us to formulate the problem in a rigorous mathematical way.

The great difficulty in organ evolution problem was well understood still by Ch. Darwin. He noted that if we could not explain how complex organs (for example, eyes) can appear as a

result of small, slight modifications, then the evolution theory "absolutely breaks down". To shed a light on this problem, we use an analogy between these evolution processes and hard-combinatorial problems, which have received great attention from mathematicians and theoretical physicists.

The main result is that a genetic redundancy, when  $K$  different genes encode the same trait, provides an exponential effectiveness of organism morphogenesis and evolution: with  $n$  genes one can obtain approximately  $O(n2^K)$  traits. We also show, analytically and numerically, that special genes (capacitors) can turn on or turn off evolution, and stabilize trait pattern when the environment is stable, and engines evolution when the environment is unstable. They are hubs in genetic networks.

(Joint work with D. Grigoriev, J. Reinitz, and A.Weber.)

2 **Wednesday 20 November 2013, Paul Andrews, Stockholms universitet**

*Coorganized with Ämnesdidaktiskt seminarium.*

*The cultural construction of school mathematics and student achievement*

Abstract: In this talk I will consider the extent to which school mathematics is a cultural construction. Curricula may be similar, at least as far as their content is concerned, but the underlying expectations and the extent to which systems mandate particular didactics vary considerably. Schools are located in cultural contexts that determine the ways in which they are structured and participants behave. Teachers themselves are, typically, products of the system in which they teach. Consequently their actions tend to be governed by hidden assumptions and beliefs about what constitutes good practice. Students also are conditioned to behave in ways that identify them with their cultural backgrounds. Drawing on a range of sources, but mostly my own research and video-recordings of European mathematics lessons, I will show how mathematics teaching is conceptualised in different cultural contexts and ask the question; can analyses of classroom practice explain students' performance on international tests of achievement like TIMSS and PISA?

2 **Wednesday 27 November 2013, Estibalitz Durand Cartagena, National University of Distance Education (UNED), Spain and Institut Mittag-Leffler**

*Rectifiable curves in Sierpiński carpets*

Abstract: In the last years, there has been an intensive research on the setting of metric measure spaces, where a first order differential calculus has been developed. In this talk we focus our attention on a particular case of doubling metric measure space: the Sierpiński carpet endowed with its associated Hausdorff measure. We will first review some of the latest results which have contributed to understanding the geometrical structure of metric measure spaces supporting a  $p$ -Poincaré inequality and motivate why the families of curves that live in the Sierpiński carpet are not enough for our purposes; that is, in terms of Poincaré inequalities. In the second part, we will characterize the slopes of nontrivial line segments contained in self-similar Sierpiński carpets. The set of slopes will be related to Farey sequences and the dynamics of punctured square toral billiards.

2 **Wednesday 4 December 2013, Leslie Jimenez, Universidad de Chile**

*On Jacobian varieties with group action*

Abstract: Given a compact Riemann surface  $X$  (curve), we study its Jacobian variety  $JX$ . The action of a finite group  $G$  on  $X$  induces an action of  $G$  on  $JX$ . Using the irreducible rational representations of  $G$  we obtain a decomposition of  $JX$  in lower dimension subvarieties (factors). In fact, we have the isogeny (epimorphism with finite kernel denoted by  $\sim$ )

$$B_1^{n_1} \times \cdots \times B_r^{n_r} \sim JX$$

where  $r$  is the number of irreducible rational representations of  $G$  and the  $n_j$ 's values are well known. It is called *the isogeny decomposition of  $JX$  with respect to  $G$* . In this talk we will see how to get geometric information of the factors in this decomposition. We will give examples about it for some family of curves and different groups  $G$ .

2 **Wednesday 11 December 2013, Maciej Błaszak, Adam Mickiewicz University, Poznań, Poland**

*Classical and quantum separability of Stäckel systems*

Abstract: The Stäckel separability of a Hamiltonian system is well known to ensure existence of a complete set of Poisson commuting integrals of motion quadratic in the momenta. I consider a class of Stäckel separable systems where the entries of the Stäckel matrix are monomials in the separation variables. I also show that the only systems in this class for which the integrals of motion arising from the Stäckel construction keep commuting after quantization are, up to natural equivalence transformations, the so-called Benenti systems. Moreover, it turns out that the latter are the only quantum separable systems in the class under study. Some discussion about admissible quantization procedures will also be presented.

2 **Monday 16 December 2013, Vince Geiger, Australian Catholic University, Brisbane, Australia**

*Coorganized with Ämnesdidaktiskt seminarium.*

*Numeracy across the curriculum: An Australian curriculum perspective on mathematics on preparing students to use mathematics in the world of work and for participatory citizenship*

Abstract: Numeracy, or mathematical literacy as it is also known, is a major educational goal internationally, and as such, is addressed in the curriculum documents of educational jurisdictions and in national and international testing regimes. Numeracy is increasingly seen as fundamental to developing students' capacities to use mathematics to function as informed and reflective citizens, to contribute to society through paid work, and in other aspects of community life (Steen, 2001). This presentation will outline and describe a series of projects conducted across three different educational jurisdictions in Australian that aimed to enhance teachers' instruction practice through a rich model of numeracy. Outcomes of the projects include evidence of teachers' natural and effective realisation of numeracy opportunities in a range of school subjects, as well as a mapping of their personal professional trajectories in understanding the nature of numeracy teaching and learning.

2 **Wednesday 18 December 2013, Bruce Sagan, Michigan State University, USA**

*Factoring rook polynomials*

Abstract: A board  $B$  is a subset of the squares of an  $n \times n$  chess board. Let  $r_k(B)$  denote the number of ways of placing  $k$  nonattacking rooks on  $B$  which means that every row and column has at most one rook. Various specializations of  $r_k(B)$  count permutations, derangements, and set partitions. One nice set of boards are the *Ferrers boards*  $B = (b_1, \dots, b_n)$  where the  $b_j$  are a weakly increasing sequence of nonnegative integers and the corresponding board is obtained by choosing the lowest  $b_j$  squares in column  $j$  of the chess board for  $1 \leq j \leq n$ . In a landmark paper, Goldman, Joichi, and White showed that if  $B$  is a Ferrers board then an appropriately chosen generating function for the  $r_k(B)$  factors over the integers. They also gave various applications of this result, such as a new proof of a theorem of Foata and Schützenberger. In the first half of this lecture we will provide an introduction to these beautiful results. The second half will be devoted to recent research concerning a generalization of rook placements where the rows of a board are grouped into levels and one can have at most one rook in any level or any column. This part is joint work with Kenneth Barrese, Nicholas Loehr and Jeffrey Remmel.

# The Mathematical Colloquia held in 2012 at Linköping University

Organized by Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov and Hans Lundmark.

## 2 Wednesday 25 January 2012, Stefan Rauch, MAI

*Triangular systems of Newton equations*

Abstract: Triangular form of Newton equations is a strong property. Together with the existence of a single quadratic (with respect to velocities) integral of motion, it usually implies the existence of  $n - 1$  further integrals that are also quadratic. These integrals make the triangular system separable in a new type of coordinates. The separation coordinates are built of quadric surfaces that are nonorthogonal and nonconfocal and can intersect along lower dimensional singular manifolds. We present here the main structural theorems of the theory for  $n$ -dimensional triangular systems and discuss the structure of separation coordinates in two and three dimensions.

## 2 Wednesday 1 February 2012, Alexander I. Nazarov, Saint Petersburg State University, Russia

*Qualitative properties for solutions to elliptic and parabolic equations with divergence-free lower-order coefficients*

Abstract: We consider uniformly elliptic and uniformly parabolic equations of divergence type:  $Lu \equiv -D_i(a_{ij}(x)D_ju) + b_i(x)D_iu = 0$ ,

$Mu \equiv \partial_t u - D_i(a_{ij}(x; t)D_ju) + b_i(x; t)D_iu = 0$ , with additional structure condition  $\operatorname{div}(b_i) \leq 0$  (\*) in the sense of distributions. The equations with the lower-order coefficients satisfying this structure condition arise in some applications, in particular in hydrodynamics. We deal with classical properties of solutions, namely, strong maximum principle, Hölder estimates, the Harnack inequality and the Liouville Theorem. We show that under condition (\*) the assumptions on  $(b_i)$  which ensure these properties can be considerably weakened in the scale of Morrey spaces. The talk is based on a joint paper with N. N. Ural'tseva.

## 2 Wednesday 8 February 2012, Joakim Arnlind, MAI

*Poisson algebraic and non-commutative geometry*

Abstract: Non-commutative geometry has been a fruitful field, both in pure mathematics and in its applications to physics. Ordinary (commutative) geometry can be studied in terms of the algebra of functions on, for instance, a manifold, and one tries to extend the algebraic formulation to non-commutative algebras.

In the context of mechanics, the algebra of functions is endowed with another structure – the Poisson bracket. When trying to consider a quantum mechanical analogue of the system, one maps functions to operators such that the Poisson bracket corresponds to the commutator of operators. Thus, it becomes important to understand how geometry can be described in terms of the Poisson algebra of smooth functions (on a manifold).

In this talk, I will give an overview of an approach to non-commutative geometry in terms of matrix limits, together with some physical motivation and general ideas of non-commutative geometry, and explain how one can formulate Riemannian geometry in a Poisson algebraic way.

## 2 Wednesday 15 February 2012, Irina Asekritova, MAI

*On invertibility of linear operators in interpolation spaces*

Abstract: Let  $A$  be a linear bounded operator from a Banach couple  $\vec{X} = (X_0, X_1)$  to a Banach couple  $\vec{Y} = (Y_0, Y_1)$  such that the restrictions of  $A$  to the spaces  $X_0$  and  $X_1$  have bounded inverses. This condition does not imply that the restriction of the operator  $A$  to the real interpolation space  $(X_0, X_1)_{\theta, q}$  has a bounded inverse for all values of the parameters  $\theta$  and  $q$ . I plan to discuss the following problem: how can we describe all spaces  $(X_0, X_1)_{\theta, q}$  such that the operator  $A : (X_0, X_1)_{\theta, q} \rightarrow (Y_0, Y_1)_{\theta, q}$  is invertible? The talk is based on joint work with N. Kruglyak.

## 2 Wednesday 29 February 2012, Niklas Lundström, Umeå universitet

*Estimates for  $p$ -harmonic functions vanishing on a flat*

Abstract: We study  $p$ -harmonic functions in a domain  $\Omega \subset \mathbb{R}^n$  near an  $m$ -dimensional plane (an  $m$ -flat)  $\Lambda_m$ , where  $0 \leq m \leq n - 1$ . In particular, let  $u$  be a positive  $p$ -harmonic function, with  $n < p \leq \infty$ , vanishing on a portion of  $\Lambda_m$ , and suppose that  $\beta = (p - n + m)/(p - 1)$ , with  $\beta = 1$  if  $p = \infty$ . We prove, using certain barrier functions, that

$$u \approx d(x, \Lambda_m)^\beta \quad \text{near } \Lambda_m.$$

The lower bound holds also in the range  $n - m < p \leq \infty$ .

## 2 Monday 5 March 2012, Per Enflo, Kent State University, USA

*Orbits of diagonal operators*

Abstract: We will discuss hyperful orbits of operators, i.e. orbits where every subsequence of the orbit spans the whole space. A cyclic vector is a vector whose orbit spans the whole space. Among other things we will show that for diagonal operators on Hilbert space either every cyclic vector has a hyperful orbit or no cyclic vector has a hyperful orbit.

## 2 Wednesday 7 March 2012, Sergey Vakulenko, Institute of Mechanical Engineering Problems and University of Technology and Design, Saint Petersburg, Russia

*Flexibility and robustness under fluctuations of genetic networks*

Abstract: We consider networks with two types of nodes. The  $v$ -nodes, called centers, are hyperconnected and interact one to another via  $u$ -nodes, called satellites. This centralized architecture realizes a bow-tie scheme and possesses interesting properties. Namely, this organization creates feedback loops that are capable to generate any prescribed patterning dynamics, chaotic or periodic, and create a number of equilibrium states. We show that activation or silencing of a node can sharply switch the network attractor, even if the activated or silenced node is weakly connected. Centralized networks can keep their flexibility, and still be protected against environmental noise. Finding an optimized network that is both robust and flexible is a computationally hard problem in general, but nonetheless this problem is feasible when the number of satellites is large. This finding reduces to a minimization of some spin glass Hamiltonian. This is a joint work with Ovidiu Radulescu (Montpellier, France).

## 2 Wednesday 14 March 2012, Andrés Navas, Universidad de Santiago de Chile

*On groups generated by elements near rotations*

Abstract: We will show that for a group of circle diffeomorphisms, having a system of generators close (in the  $C^2$  topology) to rotations imposes several dynamical restrictions. The most important one goes back to Dumny: such a group cannot be of "second kind" (it cannot admit a minimal invariant Cantor set). We will see that among subgroups of the Möbius group, the critical case corresponds to the classical  $(2,3)$  group.

## 2 Wednesday 21 March 2012, Rögnvaldur G. Möller, University of Iceland

*Symmetry in the theory of infinite graphs*

Abstract: Symmetry has a strong hold on the human mind and is also a fundamental concept in mathematics. In this talk I will discuss symmetry in the context of infinite graphs. More specifically I want to describe various classes of graphs possessing a very high degree symmetry and constructions and classification results of such graphs. The study of these classes of graphs and their automorphism groups has connections to logic, group theory (relate to many different aspects of group theory), graph theory, probability theory and analysis.

## 2 Wednesday 28 March 2012, Ryszard Rubinsztajn, Uppsala universitet

*Knots, quandles and connections*

Abstract: Quandles are spaces acting on themselves according to certain rules. Examples are given by symmetric manifolds and conjugacy classes in groups. I shall explain how one can use quandles to construct invariants of knots. I shall then discuss how, on the other hand, these invariants can, in some cases, be interpreted in terms of moduli spaces of flat connections on a punctured 2-dimensional sphere.

## 2 Wednesday 4 April 2012, Michelle Bucher, University of Geneva, Switzerland

*Title: Volumes in geometry and topology*

Abstract: The simplicial volume of manifolds was introduced by Gromov in the beginning of the 80's to give a topological description of the volume of (families of) Riemannian manifolds. Applied to hyperbolic manifolds, this led Gromov to a new proof of Mostow rigidity. In fact the simplicial volume of any Riemannian manifold is proportional to its Riemannian volume by a constant depending only on the universal cover. This phenomenon is reminiscent of the Hirzebruch proportionality principle between Euler characteristic and Riemannian volume, and in fact Euler characteristic and simplicial volume share important properties such as that their positivity implies the positivity of the minimal volume. In this talk, I will review positivity results for the simplicial volume and its relations to Riemannian volume and Euler characteristic.

## 2 Wednesday 11 April 2012, Sergey Nazarov, Russian Academy of Sciences, Saint Petersburg, Russia

*Spectral gaps for periodically perturbed cylindrical waveguides*

Abstract: The band-gap structure of the spectrum in a periodic waveguide permits for the opening of a spectral gap that is an interval of the real positive semi-axis which is free of the spectrum but has both the endpoints in it. The simplest way to indicate spectral gaps is to consider periodic perturbations of a cylindrical waveguide and to apply asymptotic methods

for studying eigenvalues of the model problem in the periodicity cell. In the talk some new approaches will be demonstrated to detect spectral gaps and open questions will be formulated as well.

**Wednesday 18 April 2012, Martin Bridson, University of Oxford, UK**

*Rigidity, mapping class groups and automorphism groups of free groups*

Abstract: I shall begin with a discussion about the universe of discrete groups and explain why lattices in semisimple Lie groups, mapping class groups of surfaces, and automorphism groups of free groups have a special place in it. Then, developing the deep analogy between these three types of groups, I shall describe several results that extend ideas of rigidity (à la Mostow and Margulis) from the classical setting to the setting of mapping class groups and automorphism groups of free groups. For example, if  $n$  is at least 3, the  $SL(n, \mathbb{Z})$  cannot act with infinite image on a compact surface or on a non-abelian free group, nor can it act by homeomorphisms on a sphere of dimension less than  $n-1$ .

**Wednesday 25 April 2012, Montserrat Casals-Ruiz, University of Oxford, UK**

*First-order theories and Tarski problems*

Abstract: Equations are present, implicitly or explicitly, in most branches of mathematics. The first ones to be formalised were the diophantine equations – equations with integer coefficients and integer solutions. Hilbert's tenth problem asks to construct an algorithm to decide whether or not a diophantine equation is compatible. In 1970, combined work of Davis, Putnam, Robinson and Matiyasevich culminated in a proof of the algorithmic undecidability of this problem.

Nowadays, Hilbert's tenth problem can be formulated for arbitrary structures and in a more general setting in terms of first-order theories. In the case when the structures under consideration are free groups, this problem is known as Tarski's problem. In contrast to the diophantine case, Makanin devised an algorithm to solve the compatibility problem for systems of equations with coefficients and solutions over a free group. The theory developed to solve Tarski problems has established different connections between model theory, geometry and group theory. In this talk, we will present these connections, introduce some of the key techniques and discuss some new directions in this area.

**Wednesday 2 May 2012, Sebastian Hensel, Universität Bonn, Germany**

*Geometry of mapping class groups*

Abstract: The mapping class group of a surface is one of the central objects in low dimensional topology and geometry. As a finitely generated group, it carries a natural metric whose geometry is by now well-understood. The geometry of other mapping class groups, however, is much less studied. In this talk we present joint work with Ursula Hamenstädt on the mapping class groups of handlebodies and doubled handlebodies and their relations.

**Wednesday 9 May 2012, Ruth Kellerhals, University of Fribourg, Switzerland**

*Minimal volume tessellations in hyperbolic space*

Abstract: After a short introduction to hyperbolic tessellations, orbifolds, simple constructions and properties, we consider those with many symmetries and try to rank them by means of their volumes. We discuss known results in dimensions below five and present then new developments in hyperbolic 5-space by restricting ourselves to the arithmetic, oriented case. This is joint work with Vincent Emery (MPI Bonn).

**Friday 11 May 2012, Maria del Carmen Reguera Rodriguez, Lunds universitet**

*Weights that avoid the cancellative properties of singular integrals*

(Joint with the **Analysis seminar series**.)

Abstract: In this talk, we will present a family of weights that avoid the cancellative properties of Singular Integrals. These weights first appeared in the speaker's thesis to provide a counterexample to a dyadic version of the so called Muckenhoupt-Wheeden Conjecture, a weighted weak type estimate for Singular Integrals at the end point  $p = 1$ . The construction presented in this talk is a simplified version of the original one and it allows to disprove the full Conjecture. This is joint work with C. Thiele. In recent work with J. Scurry, we find applications to another question of Muckenhoupt and Wheeden in the two weight setting.

**Wednesday 16 May 2012, Sergey Nazarov, Russian Academy of Sciences, Saint Petersburg, Russia**

*Spectral gaps for double periodic perforated media*

(Joint with the **Analysis seminar series**.)

Abstract: It will be shown that both, the Dirichlet and the Neumann problems for the Laplace operator in the plane perforated with the double periodic family of circular holes may have spectral gaps in their spectra. The method of investigation has, necessarily, crucial difference with the case of a waveguide. The famous golden section  $0.6180 \dots$  appears in several related calculations. Some other geometries and open questions will be demonstrated.

**Wednesday 30 May 2012, Hjalmar Rosengren, Chalmers**

*Three-coloured chessboards*

Abstract: I will discuss some mathematics and physics related to what I call three-coloured chessboards. These seemingly simple combinatorial objects turn out to have intriguing relations to topics such as elliptic functions and modular forms, solvable models of statistical mechanics, affine Lie algebras and Painlevé equations. The lecture should be accessible to a wide audience.

**Wednesday 29 August 2012, Anders Björn, MAI**

*The Perron method for  $p$ -harmonic functions: New resolvitivity and invariance results*

Abstract: In the Dirichlet problem one looks for a  $p$ -harmonic function  $u$  on some domain  $\Omega \subset \mathbb{R}^n$  which takes prescribed boundary values  $f$ . A  $p$ -harmonic function  $u$  is a continuous weak solution of the equation

$$\operatorname{div}(|\nabla u|^{p-2} \nabla u) = 0.$$

(And thus for  $p = 2$  we obtain the usual harmonic functions.) Here  $1 < p < \infty$  is fixed.

If  $f$  is not continuous, then there usually is no  $p$ -harmonic function  $u$  which takes the boundary values as limits (i.e. such that  $\lim_{y \rightarrow x} u(y) = f(x)$  for all  $x \in \partial\Omega$ ), and even for continuous  $f$  this is not always possible. One therefore needs some other precise definition of what is a *solution* to the Dirichlet problem. For  $p$ -harmonic functions there are at least 4 different definitions, of which the *Perron method* is the most general.

For any boundary function  $f : \partial\Omega \rightarrow [-\infty, \infty]$ , the Perron method produces an upper and a lower Perron solution. When these coincide it gives a reasonable solution to the Dirichlet problem, called the *Perron solution*  $Pf$ , and  $f$  is said to be *resolutive*.

In 2003 Björn-Björn-Shanmugalingam showed the following invariance result: If  $f \in C(\partial\Omega)$  and  $h = f$  outside a set of  $p$ -capacity zero, then  $h$  is resolutive and  $Ph = Pf$ .

We will look at recent improvements of this result. Some of these will be related to the prime end boundary, in the sense of the recent definition of prime ends introduced by Adamowicz-Björn-Björn-Shanmugalingam. Note that for our results we *cannot* use Carathéodory's classical definition, not even in simply connected planar domain.

**Wednesday 12 September 2012, Nikolay Kuznetsov, Russian Academy of Sciences, St. Petersburg, Russia**

*No steady water waves of small amplitude are supported by a shear flow with still free surface*

Abstract: The two-dimensional free-boundary problem describing steady gravity waves with vorticity on water of finite depth is considered. It is proved that no small-amplitude waves are supported by a horizontal shear flow whose free surface is still in a coordinate frame such that the flow is time-independent in it. The class of vorticity distributions, for which shear flows with still free surface exist, includes any positive constant vorticity, as well as linear and quadric vorticity with arbitrary positive coefficients.

**Wednesday 19 September 2012, Pekka Koskela, University of Jyväskylä, Finland**

*Gromov hyperbolicity and quasihyperbolic geodesics*

**Wednesday 26 September 2012, Dimitri Yafaev, Université Rennes, France**

*Hankel operators and moment problems*

Abstract: We find explicit necessary and sufficient conditions for a Hankel operator to be positive. In view of the Hamburger theorem, this yields necessary and sufficient conditions for the solvability of a moment problem. A generalization of Bernstein's theorem on exponentially convex functions to distributions is established.

**Wednesday 3 October 2012, Vladimir Tkatchev, MAI**

*Minimal cones, Jordan algebras and isoparametric hypersurfaces*

Abstract: By exploiting equivariant properties of quadratic Clifford-Simons cones, Bombieri, De Giorgi and Giusti were able to construct in 1969 non-affine entire minimal graphs over  $\mathbb{R}^8$ , thereby settling the celebrated Bernstein problem. The construction and understanding of the inner structure of minimal cones of degree higher than 2 remains a long-standing difficult problem. We give an excursion into the first non-trivial case, the cubic minimal cones, which is shown to be very related to Jordan algebras and isoparametric hypersurfaces. We shall also discuss some connections of this to recent progress in construction of non-classical solutions of certain elliptic PDEs.

**Friday 5 October 2012, Britt-Marie Stocke, Umeå universitet**

*Matematikerna – ett broderskap?*

(Seminarium inom **Kvinnor inom matematik**.)

Sammanfattning: Genom tiderna har kvinnor haft mycket skiftande tillgång till studier, akademisk undervisning och yrkesverksamhet. Under vissa tider och i vissa kulturer tillåts inte kvinnor att lära sig läsa, skriva och räkna. Men det finns en obruten tradition av kvinnor i matematiken, från kvinnorna i Pythagoras akademi till vår första kvinnliga professor i

matematik, Sofia Kovalevskaja. I vår egen tid finns exempel som Emmy Noether och Ingrid Daubechies.

Föredraget kommer att handla om kvinnor som haft betydelse för matematikens utveckling.

**Wednesday 10 October 2012, Yuri Brudnyi, Technion, Haifa, Israel**

*Interpolation of compact operators*

(Joint with the **Analysis seminar series**.)

Abstract: We begin with the still unsolved Calderón problem (1964) on preserving compactness of linear operators under the complex interpolation method. The presented partial solution for a more general problem deals with quasi-Banach lattices as target spaces, arbitrary interpolation methods and Lipschitz (in particular linear) operators. The key point of the proof is a new compactness criterion for Fréchet lattices that will be presented as well.

**Wednesday 17 October 2012, Qimh Xantcha, Uppsala universitet**

*Binomial rings: axiomatisation, transfer and classification*

Abstract: The topic of the talk is binomial rings, rings with binomial co-efficients, which were introduced by Hall in 1954 in connexion with his ground-breaking work on nilpotent groups. We shew how binomial rings admit an elegant axiomatisation, implying their equivalence with the class of numerical rings studied by Ekedahl. A binomial transfer principle is described, enabling combinatorial proofs of algebraical identities. Finally, we provide a complete classification of the finitely generated binomial rings.

**Wednesday 24 October 2012, Peter Hästö, Uleåborgs universitet, Finland**

*The strong minimum principle for quasisuperminimizers of non-standard growth*

Abstract: I will discuss the strong minimum principle for non-negative quasisuperminimizers of the variable exponent Dirichlet energy integral. With Harjulehto, Latvala and Toivainen, we proved the SMP under the assumption that the exponent has modulus of continuity slightly more general than Lipschitz. The proof is based on a new version of the weak Harnack estimate.

**Friday 26 October 2012, Alex Karashev, Nipissing University, North Bay, Canada**

*Spans of continua*

(Joint with the **Analysis seminar series**.)

Abstract: The concept of span was introduced by Andrew Lelek in 1964 and played a substantial role in continuum theory since then. Roughly speaking, the span of a space is the largest distance two travelers can keep between them while traversing the whole space. The talk will be an overview of various versions of span and relations between them, as well as other results in continuum theory, related to the concept of span. Most of the spaces under consideration will be graphs. Some open problems will be posed.

**Wednesday 7 November 2012, Carl Johan Casselgren, MAI**

*Coloring graphs from random lists*

Abstract: The topic of this talk is list colorings of graphs. In this model each vertex of a graph is assigned a list (set) of colors and the task is then to construct a proper coloring of the graph such that each vertex gets a color from its list. Usually, for a given graph, one is interested in determining the minimum number  $k$ , such that if each vertex gets a list of size  $k$ , then this is always possible. I will review some basic facts about list coloring and then discuss a variation on list coloring where each vertex receives a random list: let  $G = G(n)$  be a graph on  $n$  vertices and assign to each vertex  $v$  of  $G$  a list  $L(v)$  of colors, by choosing each list uniformly at random from all  $k$ -subsets of a color set of size  $\sigma(n)$ . I will discuss various conditions which imply that with probability tending to 1 as  $n \rightarrow \infty$ ,  $G$  has a proper coloring from the random lists.

**Wednesday 14 November 2012, Juha Lehtinen, Jyväskylä University, Finland**

*Hardy inequalities and uniform fatness*

Abstract: There is a well-known connection between the validity of the  $p$ -Hardy inequality in a domain and the uniform  $p$ -fatness of the complement of the domain, due to Ancona, Lewis, and Wannebo. However, when the  $p$ -Hardy inequality is replaced by a pointwise variant of the inequality, we even obtain an equivalence between the two concepts. I will discuss this and related results in the setting of metric spaces, based mainly on joint work with Riikka Korte and Heli Tuominen.

**Wednesday 21 November 2012, Sergei Vakulenko, Russian Academy of Sciences, St. Petersburg, Russia**

*Chaos for infinite-dimensional dissipative systems*

Abstract: In the 1990s, P. Polacik proposed the method of realization of vector fields (RVF). This method allows us to find infinite-dimensional dissipative dynamical systems with chaotic attractors. The talk is a review of results in this field. The following topics will be considered: chaos for parabolic equations, for neural and genetic networks and reaction-diffusion systems.

**Wednesday 28 November 2012, Ari Laptev, MAI, Institut Mittag-Leffler and Imperial College London**

*On some spectral inequalities for Schrödinger operators on graphs*

Abstract: We shall discuss Lieb-Thirring inequalities for a 1D Schrödinger operator on the semi-axes with Robin boundary conditions and then apply them to the study of the spectrum on star graphs.

**Wednesday 5 December 2012, Visa Latvala, University of Eastern Finland, Joensuu, Finland**

*Two minimization problems related to image restoration*

Abstract: We prove the existence of the solutions of a variant of the Geman-McClure-model for image restoration. This extends the one-dimensional existence result due to Chipot, March, Rosati and Vergara Caffarelli to higher dimensions. We also discuss a related variable exponent model in the case when the exponent attains the critical value one.

**Wednesday 12 December 2012, David Rule, MAI**

*Weighted norm inequalities for linear and multi-linear pseudo-differential operators*

Abstract: I will give an overview of some boundedness results for pseudo-differential operators on weighted Lebesgue spaces. We will start by trying to understand linear operators

$$T_a(f)(x) = \int a(x, \xi) \hat{f} e^{2\pi i x \cdot \xi} d\xi$$

whose symbols  $a(x, \xi)$  are only assumed to be measurable in the  $x$ -variable. We will then see that this knowledge can help us understand other operators, for example, multi-linear versions of  $T_a$ . If there is time we will also consider operators where we replace the oscillatory factor  $e^{2\pi i x \cdot \xi}$  with the more general factor  $e^{i\varphi(x, \xi)}$ . This is joint work with Nick Michalowski, Salvador Rodríguez-López and Wolfgang Staubach. The talk will be aimed at a general (mathematical) audience.

**Tuesday 18 December 2012, Antonio F. Costa, UNED, Madrid, Spain**

*Klein Doubles*

Abstract: A Klein surface is a (real) surface with (dimension one) dianalytic structure, i. e. the changes of coordinates between different charts are analytic or anti-analytic. Klein surfaces can be non-orientable and have boundary. The term Klein surface goes back to Felix Klein and the modern concept appeared in a Lecture Notes by Alling and Greenleaf in 1971. A double of a Klein surface  $X$  is a Klein surface  $X'$  such that there is a degree 2 morphism  $X \rightarrow X'$ . When the genus of the Klein surface or the number of boundary connected are large, there are many possible doubles of a given Klein surface. We shall study the most important ones by historical, algebraic or geometrical reasons, including the complex double, the orienting double or the Schottky double.



# The Mathematical Colloquia held in 2011 at Linköping University

Organized by Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov and Hans Lundmark.

## 2 Wednesday 19 January 2011, Jana Björn, MAI

*Quasiminimizers – pros and cons*

Abstract: We define quasiminimizers of the  $p$ -energy integral as more robust generalizations of  $p$ -harmonic and harmonic functions. They share many useful properties with  $p$ -harmonic functions, such as maximum principle, Harnack inequality and some regularity. Other properties are much less understood or even fail, as shown by some counterexamples.

## 2 Wednesday 26 January 2011, Pär Kurlberg, KTH, Stockholm

*Point count statistics for families of curves over finite fields*

Abstract: We investigate the distribution of the number of  $F_p$ -points of curves in various families. (Here  $F_p$  is the finite field with  $p$  elements.) If we consider a family of curves having fixed genus  $g$  and let  $p$  tend to infinity the situation is fairly well understood – the distribution of the point count fluctuations are given by the (generalized) Sato–Tate distribution, which in turn is closely related to random matrix theory. On the other hand, if  $p$  is fixed and we let  $g$  tend to infinity (or taking  $p, g$  to infinity in some arbitrary way), the situation is less clear, e.g., since the number of points on a curve cannot be negative, the random matrix theory model is not valid in this setting. However, for certain families of curves, certain "coin flip models" can be used to describe the fluctuations; using this we can show that the point count fluctuations are Gaussian in the large genus limit.

## 2 Wednesday 2 February 2011, Rosa M. Miró-Roig, Universitat de Barcelona, Spain

*Cohomological characterization of vector bundles*

Abstract: In my talk, I will address the problem of giving a cohomological characterization of vector bundles on algebraic varieties. This is a longstanding problem in Algebraic geometry which has its roots in an old paper by Horrocks where he gave a cohomological characterization of line bundles on projective spaces  $\mathbb{P}^n$ . In my talk, I will give a cohomological characterization of the bundle of  $p$ -differential forms on multiprojective spaces  $\mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_s}$  and a cohomological characterization of Steiner bundles on algebraic varieties. As a main tool I will use a generalized version of Beilinson's spectral sequence. This is joint work with Costa and Soares.

## 2 Wednesday 9 February 2011, Bernd Sturmfels, University of California, Berkeley, USA

*Quartic curves and their bitangents*

This is a joint arrangement with The Royal Swedish Academy of Sciences (Kungliga Vetenskapsakademien, KVA); see p. 7 of their Spring Programme 2011 (pdf file).

Abstract: This will be a lecture where classical mathematics from the 19th century is treated with modern methods. An exciting journey into the wonderful world of geometry spiced with a great deal of visual illustrations.

## 2 Wednesday 16 February 2011, Bernard Mourrain, INRIA, University of Nice Sophia-Antipolis, France

*Border basis, Hilbert scheme and tensor decomposition*

Abstract: The commutativity is a natural property that we expect in the context of algebraic geometry. Surprisingly, this simple property is also enough to characterize the solution of several problems. We will illustrate it on three apparently disconnected topics: the construction of border basis and the solution of polynomial equations by eigenvector computation; the equations of the Hilbert scheme of points; the decomposition of tensors in relation with truncated moment problems. The approach will be detailed on some typical examples and applications.

## 2 Wednesday 23 February 2011, Klas Nordberg, Computer Vision Laboratory, ISY

*Tensors in computer vision*

Abstract: The concept of tensors have been around in computer vision and image processing for a few decades. Tensors have two main applications in computer vision, as representations of local image features, e.g., orientation of lines and edges, and in geometry where they are used to represent mappings and constraints on projective spaces. This seminar presents an overview of these two application areas of tensors in computer vision, with examples that illustrate how they are derived and used.

## 2 Wednesday 2 March 2011, Andreas Nilsson, SAAB, Linköping

*Invariant multipliers*

Abstract: Multipliers correspond to translation invariant operators. Sometimes they can satisfy more invariance conditions and this talk will be about such multipliers. For example Stein has shown that the Riesz transforms can be characterized as being invariant under dilations and satisfying a certain invariance condition under rotations. In this talk I will give a survey on this topic.

## 2 Wednesday 9 March 2011, Erik Wahlén, Lunds universitet

*Existence and stability of solitary water waves with surface tension*

Abstract: Solitary water waves with surface tension on water of infinite depth are constructed by minimising the energy subject to the constraint of fixed momentum. The stability of the set of minimisers follows by a standard principle since the energy and momentum are conserved quantities. The stability must however be understood in a qualified sense due to the lack of a global well-posedness theory for the initial value problem. The variational method relies on the concentration-compactness principle and a penalisation argument, which is needed because of the quasilinear structure. To leading order the solitary waves take the form of a periodic wave train modulated by a small-amplitude decaying envelope described by the focusing nonlinear Schrödinger equation.

## 2 Wednesday 16 March 2011, Prof. Giorgio Ottaviani, University of Firenze, Italy

*On the rank of real polynomials*

Abstract: A Waring decomposition of a polynomial is a sum of powers of polynomials of degree one expressing it. The rank of a polynomial is the minimal number of summands in a Waring decomposition. For example the rank of  $x^d + y^d$  is 2. The interest on this notion is motivated by applications to communication theory and other fields. We discuss about joint work with P. Comon about the rank of real polynomials in one variable.

## 2 Wednesday 23 March 2011, Nageswari Shanmugalingam, University of Cincinnati

*A geometric characterization of planar BV extension domains*

Abstract: The class of functions of bounded variation (BV) embodies the geometry of hypersurfaces and of boundaries of sets. It is therefore interesting to know whether a Euclidean domain is a BV extension domain, that is, a domain whose BV functions extend as BV functions to the entire Euclidean space with controlled BV energy. Burago and Maz'ya gave a characterization of BV extension domains in terms of extendability of sets of finite perimeter in the domain. In this talk we will discuss a very simple geometric characterization of bounded simply connected planar BV extension domains. The results pertinent to this talk are based on joint work with Pekka Koskela and Michele Miranda.

## 2 Wednesday 30 March 2011, Zohra Farnana, Aalto University, Helsinki, and MAI,

*Stability of solutions of the double obstacle problem on metric spaces*

Abstract: We study the regularity properties of solutions to the double obstacle problem in metric spaces. Our main results are a global reverse Hölder inequality and stability of solutions. We assume that the space supports a weak Poincaré inequality and a doubling measure. Furthermore we assume that the complement of the domain is uniformly thick in the capacitary sense.

## 2 Wednesday 6 April 2011, Prof. Bharath Sriraman, The University of Montana

*Mathematical Giftedness and Models of Talent Development – Issues, Practices and Challenges*

Abstract: In recent years, gifted education/ giftedness has become an object of focus in Sweden. There is a tendency for researchers to transpose models and/or theories situated in different (national) contexts. i.e. from elsewhere, into their local context. However this creates conflict of these models clash with the local culture and the larger ideology that forms the basis of the educational system. In this lecture, an overview of issues and practices in gifted education will be presented with an emphasis on cultural norms and a focus on "mathematical" giftedness. Political, sociological and cultural issues in gifted education in the U.S and elsewhere is provided. Finally, different techniques and programs devised for identification and meeting the needs of mathematically gifted students are addressed.

## 2 Wednesday 27 April 2011, Prof. Sergey Nazarov, Institute of Mechanical Engineering Problems, St. Petersburg

*On spectra of water-waves problems: Trapped modes and comparison principles*

Abstract: A new approach in the linear theory of water-waves is described. It is based on the notion of a trace operator and elementary issues in the theory of self-adjoint operators in Hilbert space. By means of simple calculations and arguments, it gives direct proofs of many known facts and also new sufficient conditions for the existence of trapped modes that are eigenfunctions, solutions to the homogeneous problem with a finite energy in unbounded basins.

## 2 Wednesday 4 Maj 2011, Axel Hultman, MAI

*Schubert varieties and inversion hyperplane arrangements*

Abstract: With a permutation of a finite set, we can associate an arrangement of hyperplanes called the inversion arrangement. In the real setting, it cuts the ambient space into connected pieces, or chambers. Studying cell decompositions of totally positive Grassmannians, A. Postnikov discovered an intriguing enumerative identity which led to his 2007 conjecture relating the number of said chambers and the number of permutations smaller than the chosen one in Bruhat order. The entities related by Postnikov's, now proven, conjecture can be interpreted as Betti numbers of complexified inversion arrangements and Betti numbers of Schubert varieties, respectively. This suggests underlying geometric connections that are not yet fully understood. In this talk, I will give an elementary account of recent work in this area.

**2 Wednesday 11 Maj 2011, Johan Andersson, Uppsala University**

*On the power sum problem*

Abstract: I will discuss for what choices of  $m$  and  $n$  the quantity

$$\min_{|z_k|=1} \max_{\nu=1 \dots m} \left| \sum_{k=1}^n z_k^\nu \right|$$

can be exactly determined, when asymptotic estimates can be found and when the right order of magnitude can be obtained. Methods used include the non negativity of the Fejér kernel, the Newton–Girard identities, as well as estimates for character sums. I will also mention recent applications of these results on the explicit construction of RIP-matrices, which are useful for compressed sensing.

**2 Wednesday 18 Maj 2011, Dustin Cartwright, University of California, Berkeley**

*Interference alignment*

Abstract: Interference alignment is a technique in wireless communications for allowing increased capacity across a communications channel. In order to realize these gains it is necessary to find vector spaces satisfying certain containment conditions. I will explain how these solutions can be investigated using both elementary linear algebra techniques and the more sophisticated machinery of Schubert calculus.

**2 Wednesday 25 Maj 2011, Prof. Joseph Landsberg, Texas A&M University, College Station**

*The geometry of tensors*

Abstract: I will begin by reviewing basic results from linear algebra and discuss the corresponding issues in multilinear algebra – many of them translate to open questions! Even a notion as simple as the rank of a matrix becomes a subtle concept when discussing higher dimensional matrices, i.e., tensors. Some of these open questions are central to issues in areas such as computer science (complexity theory), engineering (signal processing) and physics (quantum information theory). The second part of my talk will discuss questions arising in these application areas. Recently many researchers in geometry have become interested in these open questions and I will conclude with a survey of recent progress.

**2 Wednesday 8 June 2011, Henrik Branden, MAI (Joint Seminar with the Didactics Group)**

*Teaching Higher Order Thinking Skills in Scientific Computing*

Abstract: In this talk I will present a teaching project that I did during the fall of 2011. The goal was to encourage students in scientific computing courses to train higher order thinking skills and to increase both the number of students with a passing grade and the number of students with higher grades. The main tool for this was constructive alignment, which required a major revision of the courses. The new versions was given with good outcome on three different programs.

**2 Thursday 9 June 2011, K. Kozlov, Moscow State University, Russia**

*Compacta as equivariant compact extension of rationals*

Abstract: The question what compact spaces can be equivariant extensions of the space of rational numbers  $\mathbb{Q}$  with transitive action of some group  $G$  will be discussed. Let  $K$  be a homogeneous CDH (countable dense homogeneous) metrizable compactum. Then there is a Polish group  $G$  which acts transitively on  $\mathbb{Q}$  with  $K$  a unique equivariant compact extension of  $\mathbb{Q}$ .

**2 Wednesday 31 August 2011, Hans Lundmark, MAI**

*Cauchy biorthogonal polynomials*

Abstract: The theory of Cauchy biorthogonal polynomials was developed recently by M. Bertola, M. Gekhtman and J. Szmigielski as a general framework for understanding certain formulas that appear in so-called peakon solutions to some integrable PDEs (in particular the Degasperis–Procesi equation that I have studied together with J. Szmigielski). In this talk I will concentrate on these polynomials, and not say very much about the PDEs. After reviewing the basic facts about ordinary orthogonal polynomials, I will describe Cauchy biorthogonal polynomials and explain differences and similarities to the classical theory. Previous acquaintance with orthogonal polynomials might be helpful, but shouldn't really be necessary.

**2 Wednesday 7 September 2011, David Singerman, University of Southampton, UK**

*The Geometry of Galois' final theorem".*

Abstract: In Galois' last letter he found the values of the primes  $p$  for which the group  $\text{PSL}(2,p)$  acts transitively on less than  $p+1$  points. (It always acts transitively on the  $p+1$  points of the projective line.) He found that these values of  $p$  are 2,3,5,7,11. The cases  $p=7, p=11$  have much geometric interest.  $\text{PSL}(2,7)$  is the automorphism group of the simplest projective plane, the Fano plane on 7 points. The simplest biplane is on 11 points, and  $\text{PSL}(2,11)$  is its automorphism group. The Fano plane can be embedded in Klein's Riemann surface of genus 3. We find an interesting surface of genus 70, in which we can embed the biplane on 11 points. This surface also contains truncated icosahedra or buckyballs and so is called the buckyball curve.

**2 Wednesday 14 September 2011, Ignacio Uriarte-Tuero, Michigan State University, USA**

*Two conjectures of Astala on distortion of sets under quasiconformal maps and related removability problems.*

Abstract: Quasiconformal maps are a certain generalization of analytic maps that have nice distortion properties. They appear in elasticity, inverse problems, geometry (e.g. Mostow's rigidity theorem)... among other places. In a celebrated paper (Acta 1994), Astala gave sharp distortion estimates for the area under planar quasiconformal mappings, and sharp dimension distortion estimates for sets of dimension smaller than 2. He conjectured an even sharper distortion estimate for the corresponding Hausdorff measure.

UT showed that Astala's conjecture is sharp in the class of all Hausdorff gauge functions (IMRN, 2008).

Lacey, Sawyer and UT jointly proved completely Astala's conjecture in all dimensions (Acta, 2010). The proof uses Astala's 1994 approach, geometric measure theory, and new weighted norm inequalities for Calderón–Zygmund singular integral operators which cannot be deduced from the classical weighted theory.

These results are related to removability problems for various classes of quasiregular maps. I will mention sharp removability results for bounded  $K$ -quasiregular maps (i.e. the quasiconformal analogue of the classical Painlevé problem) recently obtained jointly by Tolosa and UT.

I will further mention recent results related to another conjecture of Astala on Hausdorff dimension of quasicircles obtained jointly by Prause, Tolosa and UT.

The talk will be self-contained and should be accessible to graduate students.

**2 Friday 16 September 2011, Rubén Hidalgo, Universidad Técnica Federico Santa María, Valparaíso, Chile**

*The full automorphism group of a family of generalized Fermat curves.*

Abstract: In this talk we will be concerned with a certain family of closed Riemann surfaces which are the highest (branched) abelian covers of orbifolds with signature  $(0;k,\dots,k)$ . We will provide simple algebraic curves of these surfaces and discuss the group of conformal automorphisms of them. In the particular case that the signature is  $(0;k,k,k,k)$ , we will be able to obtain the full group of automorphisms as a consequence of Singermann's list of maximal signatures and also to compute the field of moduli. We also obtained that the field of moduli is in fact a field of definition.

This is a joint work with Y. Fuentes, G. Gonzalez and M. Leyton.

**2 Wednesday 21 September 2011, Alexander Vasiliev, Universitetet i Bergen**

*Evolution of smooth shapes and integrable systems*

Abstract: We consider a homotopic evolution in the space of smooth shapes starting from the unit circle. Based on the Loewner–Kufarev equation we give a Hamiltonian formulation of this evolution and provide conservation laws. The symmetries of the evolution are given by the Virasoro algebra. The

positive' Virasoro generators can be recovered by an iterative method making use of the canonical Poisson structure. We study an embedding of the Loewner–Kufarev trajectories into the Segal–Wilson Grassmannian, construct the tau-function, the Baker–Akhiezer function, and finally, give a class of solutions to the KP equation. Joint work with Irina Markina.

**2 Wednesday 28 September 2011, Magnus Jacobsson, Uppsala University**

*New Invariants in Knot Theory*

Abstract: New invariants in knot theory have been found the last decades which generalize and enlight the well-known Jones polynomial and its associated polynomials. I will describe the simplest of the new invariants, Khovanov homology, together with some of its properties.

**2 Wednesday 5 October 2011, John Lewis, University of Kentucky, Lexington**

*Harmonic measure and p-harmonic measure*

Abstract: Let  $\Omega \subset \mathbb{R}^n$ ,  $n \geq 2$ , be a bounded domain. The Dirichlet problem for Laplace's equation in  $\Omega$  can be stated as follows: Given a continuous function  $f$  on  $\partial\Omega$ , find a harmonic function  $u$  in  $\Omega$  with continuous boundary values equal to  $f$ . If  $\partial\Omega$  is smooth and  $x_0 \in \Omega$  one can use the Riesz representation theorem to show the existence of a measure  $\omega = \omega(\cdot, x_0)$  on  $\partial\Omega$  satisfying

$$u(x_0) = \int_{\partial\Omega} f d\omega(\cdot, x_0).$$

$\omega$  is called harmonic measure with respect to  $x_0$ . During the first part of my talk I will give an outline of some of the many results for harmonic measure which have been of interest to me during my career. During the second part of the talk I will indicate some recent results of myself and coauthors which generalize results for harmonic measure to measures associated with a positive solution to a nonlinear partial differential equation (called the  $p$  Laplacian) that vanishes on a portion of  $\partial\Omega$ .

**2 Wednesday 12 October 2011, Andreas Rosén, MAI, N.B.** *This talk was a joint seminar with the didactics group.*

*Riemann eller Lebesgue?*

Sammanfattning: Målet med detta föredrag är att väcka tankar och en debatt om hur det är lämpligt att undervisa integrationsteorin i de grundläggande kurserna i envariabel- och flervariabelanalys för våra studenter. Under året 2006 undervisade jag flervariabelanalys i Lund, där jag inspirerad av Claesson–Boiers kompendium i flervariabelanalys skrev ihop ett kompendium där jag vidareutvecklade deras idé om att undervisa en form av pre-Lebesgueintegral istället för den traditionella Riemannintegralen. Både mina studenter och jag tyckte att detta pedagogiska experiment föll ut mycket väl, vilket inspirerade mig att fortsätta utveckla dessa idéer. Mitt kompendium och lite fler tankar om denna pre-Lebesgueintegral finner ni på min hemsida <http://www.mai.liu.se/~anaxe/> under länken **Integrationsteori för kontinuerliga funktioner**.

Jag planerar att tala i 30–45 minuter med en efterföljande diskussion. Dels kommer jag att förklara uppbyggnaden av pre-Lebesgueintegralen, och dels kommer jag att gå igenom fördelar gentemot Riemannintegralen. Alla på MAI som är inblandade i envariabel- och flervariabelanalysundervisningen bör ha behållning av att delta.

**2 Wednesday 19 October 2011, Nikolay Kuznetsov, Russian Academy of Sciences, St. Petersburg**

*Loads on marine structures in extreme waves*

Abstract: The impact of extreme waves (also referred to as freak waves) on ships and off-shore structures is an important question from different points of view (environmental, engineering etc.). The aim of this talk is to present heuristic considerations on how to simplify calculations of loads due to freak waves. One way of doing this is to combine a nonlinear model of such waves (Zakharov et al.) with an asymptotic approach to brief wave disturbances developed in the framework of the linear theory of water waves (Kuznetsov & Maz'ya).

**2 Wednesday 9 November 2011, Olli Martio, Helsingfors universitet and Finnish Academy of Science and Letters**

*Riccati equations*

Abstract: In the classical sense a solution to a differential equation is well understood. However, modern mathematics has created solution classes that considerably extend the classical concept. These generalized solutions also play an important role in applications. Riccati type second order partial differential equations

$$-\nabla \cdot (|\nabla u|^{p-2} \nabla u) = |\nabla u|^q, \quad p > 1, q > 0,$$

and their solutions offer an interesting platform for various solution classes.

In the talk the classes of ordinary, weak, very weak and superharmonic solutions of the Riccati equation and some of their properties are considered and the effects of the exponents  $p$  and  $q$  are discussed.

**2 Wednesday 16 November 2011, Kristian Seip, Norges teknisk-naturvitenskapelige universitet, Trondheim**

*The Bohnenblust–Hille inequality*

Abstract: The Bohnenblust–Hille inequality, proved in 1931, says that the  $\ell^{2m/(m+1)}$ -norm of the coefficients of an  $m$ -homogeneous holomorphic polynomial  $P$  on  $\mathbf{C}^n$  is bounded by  $\|P\|_\infty$  times a constant independent of  $n$ , where  $\|\cdot\|_\infty$  denotes the supremum norm on the polydisc  $\mathbf{D}^n$ . The result is sharp in the sense that the independence of  $n$  fails if the exponent  $2m/(m+1)$  is replaced by a smaller number. The talk will present the historical background for this result, a few applications, the notion of polarization (the basic idea of the proof), and the hunt for the best constant which was essentially completed in 2011.

**2 Wednesday 23 November 2011, James Brennan, University of Kentucky, Lexington**

*The Cauchy integral and certain of its applications*

Abstract: The Cauchy integral plays a fundamental role in almost every area of complex analysis. In this talk it is my intention to describe, in outline, the manner in which the Cauchy integral enters into certain problems in approximation theory. In the process, I will also give an introduction to three seemingly disparate but, nevertheless, interrelated topics:

1. The invariant subspace problem for subnormal operators on a Hilbert space;
2. Thomson's theorem on mean-square polynomial approximation;
3. Tolsa's work on the semiadditivity of analytic capacity.

**2 Wednesday 30 November 2011, Irina Markina, Universitetet i Bergen**

*Sub-Riemannian geometry and principal bundles*

Abstract: It will be an introductory talk to the subject that in the last decade received the name sub-Riemannian geometry. All necessary definitions and examples will be presented. We also illustrate how the geometry of principal bundles and sub-Riemannian geometry are related. In particular, we give the description of sub-Riemannian geodesics on principal bundles and their interesting properties.

**2 Wednesday 7 December 2011, Leif Melkersson, MAI**

*Problems on local cohomology – solved and unsolved.*

Abstract: Local cohomology modules are in general not finitely generated. However under certain conditions on the ring, the ideal or the module, they can satisfy other finiteness conditions. In particular I will consider the question of cofiniteness. I have now solved a longstanding problem, namely that the modules cofinite with respect to a one-dimensional ideal in an arbitrary noetherian ring form a full abelian subcategory of the category of modules., i.e. they behave well under exact sequences.

**2 Wednesday 14 December 2011, Benjamin Young, KTH**

*Combinatorics of Donaldson–Thomas and Pandharipande–Thomas invariants*

Abstract: I will discuss a combinatorial problem which comes from algebraic geometry. The problem, in general, is to show that two theories for "counting" curves in a complex three-dimensional space  $X$  (Pandharipande–Thomas theory and reduced Donaldson–Thomas theory) give the same answer. I will prove this in a special case ( $X$  is toric Calabi–Yau), where the difficult geometry reduces to elementary combinatorics. Though this approach is still difficult, it can be handled with a novel use of Eric Kuo's "graphical condensation" technique. There will be many pictures. This is a combinatorics talk, so no algebraic geometry will be used once I explain where the problem is coming from.

# The Mathematical Colloquia held in 2010 at Linköping University

Organized by Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov and Hans Lundmark.

## 2 Wednesday 20 January 2010, Prof. Vladimir Kozlov, MAI

*Complete description of water waves in a canal of finite depth for near-critical values of Bernoulli's constant*

Abstract: I'll talk about our joint work with N. Kuznetsov concerning water waves of arbitrary geometry without any assumptions on their height and slope. I'll give an overview of our previous results and discuss our latest achievements in this direction. The main result here is the fact that only two types of waves are possible for near-critical values of Bernoulli's constant, namely, Stokes waves and solitary waves. Moreover, there is only one (up to translations) solitary wave and Stokes waves are uniquely determined by their height which can take an arbitrary value from supercritical depth of uniform stream to the height of the solitary wave.

## 2 Wednesday 27 January 2010, Prof. Åke Björck, MAI

*LITH – från plan till verklighet*

Abstract: En prognos gjord av ecklesiastikdepartementet 1962 visade att behovet av civilingenjörer i Sverige skulle komma att öka kraftigt det kommande decenniet. Behovet kunde inte tillgodoses enbart genom utbyggnad vid befintliga tekniska högskolor. En subkommitte under universitetsutredningen U63 fick uppgiften att planera en ny teknisk högskola diplomatiskt kallad XTH. Mitt föredrag belyser närmare den planering och de övervägningar som ledde fram till starten av LITH den 1/7 1969. Verksamheten de första åren samt utbyggnaden av storinstitutionen för tillämpad matematik belyses särskilt.

## 2 Wednesday 3 February 2010, Prof. Dietrich von Rosen, SLU and MAI

*Multivariate linear models and matrices*

Abstract: We will give a brief review of the Growth Curve model, including results on high-dimensional analysis and spatio-temporal relationship. The focus will be on matrix problems connected to multivariate linear models. Moreover, a new matrix structure with integer spectra will be presented.

## 2 Wednesday 10 February 2010, Leif Melkersson, MAI

*Affine algebraic geometry*

Abstract: I will give a survey of important problems about polynomial mappings. The most famous one is the Jacobian conjecture: If  $F$  is a polynomial mapping of  $C^n$  into itself such that the determinant of the Jacobian is constant different from zero, then  $F$  is globally invertible. Even the case  $n=2$  is unknown.

## 2 Wednesday 17 February 2010, Christer Bennewitz, Lunds universitet

*New methods for uniqueness proofs in inverse spectral and scattering theory*

Abstract: A new technique, based on Paley-Wiener type theorems, is presented that yields uniqueness theorems in inverse spectral and scattering theory for Sturm-Liouville equations. We shall concentrate on the case of a so called left-definite equation, which is of recent interest since the spectral problem associated with the Camassa-Holm water wave equation is of this type. This is partly joint work with B. M. Brown in Cardiff and R. Weikard in Birmingham, AL.

## 2 Wednesday 24 February 2010, Prof. Sergey Nazarov, Institute of Mechanical Engineering Problems, Saint Petersburg

*The continuous spectrum of peak-shaped elastic bodies ("Vibrating Black Holes")*

Abstract: It will be shown that the continuous spectrum of a peak-shaped elastic body can be non-empty that provokes for the wave propagation phenomenon in a finite volume (note that the spectrum of a finite body with a Lipschitz boundary is always discrete). This fact is used for creating so-called Vibrating Black Holes which have become a tool for the effective absorption of sound and elastic vibrations. Open questions in the spectral theory of elastic solids will be formulated.

## 2 Friday 5 March 2010, Margaret Beck, Boston University

*Understanding metastability using invariant manifolds*

Abstract: Metastability refers to transient dynamics that persist for long times. More precisely, suppose a PDE has a globally attracting state, meaning that, for any initial condition, the solution will asymptotically approach that state. It can happen that, on its way to the state, the solution spends a long period of time near another, possibly unstable, state. This happens, for example, in the Navier-Stokes equation in two spatial dimensions and Burgers equation in one spatial dimension, both with small viscosity. I will explain how, in the context of Burgers equation, this behavior can be understood using certain global invariant manifolds in the phase space of the PDE.

## 2 Wednesday 10 March 2010, Prof. Sergey Nazarov, Institute of Mechanical Engineering Problems, Saint Petersburg

*Spectral peculiarities of T-shaped waveguides and the Dirichlet ladder*

Abstract: It will be shown that the discrete spectrum of a 2d T-shaped waveguide with soft walls, composed from a strip of unit width and a semi-strip of width  $H > 0$ , is empty in the case  $H \geq H_*$ , but consists of the only eigenvalue in the case  $H \in (0, H_*)$ , where  $H_* \in (1, 2)$  is the critical width (its explicit value is still unknown). The effect of opening gaps in the essential spectrum of the infinite 1-periodic Dirichlet ladder, composed from two parallel strips of the small width  $h$  and cleats (crossbeams) with length  $l$  and thickness  $Hh$ , is caused by the above-mentioned eigenvalue. Several unsolved homogenization problems will be formulated.

## 2 Thursday 11 March 2010, Prof. Bharath Sriraman, The University of Montana

*On identities and theories of mathematics education: A critical view of the field*

Abstract: In this talk a critical view of mathematics education is presented by examining its connections to psychology, social sciences, the history and philosophy of mathematics/science and design sciences. The identity of "our" field is questioned even though it offers multitudes of paradigms, methodologies and the possibilities for innovative research

## 2 Wednesday 17 March 2010, Prof. Viviane Baladi, ENS, Paris

*Linear response for generic smooth unimodal maps*

Abstract: Many interesting dynamical systems admit a "natural" or "physical" measure, which describes the asymptotic time averages of a positive Lebesgue measure set of initial conditions. This measure is called the SRB measure, for Sinai-Ruelle-Bowen. When a smooth one-parameter family  $f_t$  of dynamical systems admits for all (or many) small parameters  $t$  a unique SRB measure  $\mu_t$ , it is natural to ask if the map  $t \rightarrow \mu_t$  is also smooth (possibly in the sense of Whitney). In 1997, David Ruelle solved the case when the  $f_t$  are smooth and uniformly hyperbolic, obtaining a formula for the derivative: the linear response formula. Ten years later, Daniel Smania and I discovered by studying a "toy model" (piecewise expanding maps) that the presence of critical points (which destroy structural stability) may cause obstructions to linear response. We proved that the condition of "tangency" of the family to the topological class was necessary and sufficient for linear response to hold. For smooth nonuniformly hyperbolic unimodal maps, the situation is much more difficult, and for the moment one only considers families  $f_t$  which remain in the topological class of  $f_0$ . Up to 2009, only (nongeneric) situations very close to hyperbolic had been tackled (Ruelle, B-Smania). We hope to finish this talk by stating a new joint result with Daniel Smania, which holds under a generic recurrence condition. (Joint with Daniel Smania)

## 2 Wednesday 24 March 2010, Prof. Mikael Patriksson, Chalmers, Göteborg, and MAI

*Current research topics and trends in the Optimization group at Mathematics Sciences, Chalmers*

Abstract: The optimization group at Chalmers/Mathematical Sciences perform both basic and applied research; often the two inspires each other. The presentation covers especially two such research topics, one being the price-winning research on combinatorial optimization in maintenance planning, the other being investigations into a stochastic hierarchical decision model that has immediate applications both in traffic control and in the cure of cancer. We might also comment on how the group has responded to the recent emergence of the "Areas of advance" ("Styrkeområden"), in particular in transportation and energy, following the recent substantial strategic grants given by the Swedish government.

## 2 Wednesday 14 April 2010, Prof. Warwick Tucker, Uppsala Universitet

*Validated Numerics – a short introduction to rigorous computations*

Abstract: We will present an efficient means of performing numerical computations with rigorous error bounds. The basic idea is to use set-valued mathematics as the underlying framework. This enables us to change focus from approximating the solution to enclosing the same. These ideas have been known since the 1950's, but it is only since quite recently that modern programming languages have allowed for the efficient implementation of such frameworks. The applications range from computer-assisted proofs in pure mathematics to more applied areas such as parameter estimation problems.

## 2 Tuesday 20 April 2010, Vitaj Tjatyрко, MAI

*On Vitali sets and their unions*

Abstract: In 1905 G. Vitali presented first examples of non-Lebesgue measurable sets on the real line. They were called Vitali sets. In this talk I will consider some properties of the Vitali sets and their unions.

## 2 Wednesday 28 April 2010, Mikael Olofsson, ISY

*Matrix representations of Finite Extension Fields*

Abstract: A finite extension field can be viewed as a vector space over any of its subfields, with the additional property that there also is a multiplication of the elements. The elements of the field can therefore be represented as vectors over that subfield with respect to some basis. The choice of the basis affects how the arithmetic operations can be done, and thus also the complexity of the operations. Moreover, the elements of the field can be represented as matrices over that subfield with respect to two bases, which can help the implementation of arithmetic operations in the field. In this seminar, we introduce vector and matrix representations of finite extension fields and investigate structural properties of some of those matrix representations.

**2 Tuesday 4 May 2010, Prof. Svante Linusson, KTH**

*Random orientations in graphs and the bunkbed conjecture*

Abstract: I will describe some results from recent investigations (mostly with coauthors Sven Erick Alm and Svante Jansson) on the probabilities for directed paths in randomly oriented graphs. In particular we study correlations, i.e. how the existence of a path between two vertices of a graph influences the probability of the existence of another path. The underlying graph can either be fixed or be a random graph. The talk will be self-contained and should be understandable to all PhD-student. My starting point for all these questions comes from an old conjecture by Kasteleyn (1985) on probabilities of paths in random graphs (a.k.a. percolation) which seems intuitively obvious, but noone has found a proof yet. This conjecture was named the Bunkbed conjecture and presented by Olle Häggström at FPSAC'03 organized by Linköpings universitet, where he also presented proofs for a related problem. The talk is strongly related to the Master thesis of Erik Aas, which is presented earlier the same day.

**2 Wednesday 5 May 2010, Prof. Claire Tomlin, UC Berkeley. Automatic Control–Mathematics Joint Colloquium**

*Verification and Control of Hybrid Systems using Reachability Analysis*

Abstract: This talk will present reachability analysis as a tool for model checking and controller synthesis for hybrid systems. We will consider the problem of guaranteeing reachability to a given desired subset of the state space. We allow for nonlinear dynamics in each discrete mode, and possibly non-convex state constraints. We make use of techniques from hybrid system verification to compute reachable sets, under bounded model disturbances that vary continuously. We also consider the effects of sampling and quantization. The resulting control policy is an explicit feedback law involving both a selection of continuous inputs and discrete switching commands at each time instant, based upon measurement of system state. We discuss real time implementations of this, and present several examples from multiple UAV control.

**2 Wednesday 12 May 2010, Reiner Lenz, ITN**

*Don't think twice it's alright*

**2 Wednesday 19 May 2010, Prof. Olof Heden, KTH,**

*On vector space partition problems, a survey of the different types*

Abstract: A vector space partition will here be a collection  $\mathcal{F}$  of subspaces of a finite vector space  $V(n, q)$ , of dimension  $n$  over a finite field with  $q$  elements, with the property that every non zero vector is contained in a unique member of  $\mathcal{F}$ . Vector space partitions relate to finite projective planes, design theory and error correcting codes. After a few historical remarks, I will discuss the relations to the other branches of mathematics mentioned above. The other part of the talk contains a survey of the known results on the type of a vector space partition, more precisely: the theorem of Beutelspacher and Heden on  $T$ -partitions, rather recent results of ElZanati et al. on the different types that appear in the spaces  $V(n, 2)$ , for  $n \leq 8$ , a result of Heden and Lehmann on vector space partitions and maximal partial spreads including a new necessary condition, a theorem of Heden on the length of the tail of a vector space partition, and finally, a result of Akman and Pissokho on the lattice of vector space partitions of a finite vector space.

**2 Monday 24 May 2010, Prof. Peter Leach, University of KwaZulu-Natal, South Africa**

*Complete symmetry groups*

**2 Wednesday 2 June 2010, Prof. Göran Bergqvist, MAI,**

*Tensor rank*

Abstract: We give an elementary introduction to the concept of rank for multi-way arrays or tensors, emphasizing the differences between general arrays and two-way arrays (matrices). While the singular value decomposition solves the problem of low-rank approximations of matrices, the corresponding problem for tensors is much more complicated. We also sketch a proof of how some exact probabilities of so-called typical ranks of tensors can be determined, these are the first such exact values known.

**2 Wednesday 25 August 2010, Mikael Vejdemo-Johansson, Stanford University**

*Politikens topologi*

Abstract: Principalkomponentanalys (PCA) och andra verktyg från modern dataanalys har varit stapelvara inom politikvetenskapen det senaste decenniet. Man kan läsa ut en hel del om ett parlaments struktur och variationer från exempelvis de första par koordinaterna från PCA använt på punktmolnet av parlamentariker i vektorrummet uppspätt med en basvektor för varje plenumomröstning. För att hitta nya strukturer och nya angreppssätt använder vi metoder från topologisk dataanalys på motsvarande data; framför allt använder vi en metod med inspiration från Morse-teorin, *mapper*, som ger en topologisk förenkling av punktmolnsdata utrustad med en täthetsfunktion på datapunkterna. Föredraget kommer att beskriva mapper, och visa på strukturer i svenska, brittiska och amerikanska parlamentariska datamängder både med klassiska och topologiska dataanalysmetoder.

**2 Wednesday 1 September 2010, Mikael Langer, MAI**

*Asymptotics of solutions of a perturbed heat equation*

Abstract: Solutions of a second order parabolic differential equation in a cylinder are studied and their asymptotic properties as the time variable tends to infinity are presented.

**2 Wednesday 8 September 2010, Prof. Nikolay Kuznetsov, Russian Academy of Sciences, Saint Petersburg**

*On the problem of time-harmonic water waves in the presence of a freely floating structure*

Abstract: The two-dimensional problem of time-harmonic water waves in the presence of a freely floating structure (it consists of a finite number of infinitely long surface-piercing cylinders) is considered. The coupled spectral boundary value problem modelling the small-amplitude motion of this mechanical system involves the spectral parameter – the frequency of oscillations – which appears in the boundary conditions as well as in the equations governing the structure's motion. It is proved that any value of the frequency turns out to be an eigenvalue of the problem for a particular structure obtained with the help of the so-called inverse procedure.

**2 Wednesday 15 September 2010, Prof. Milagros Izquierdo, MAI**

*On the (dis)connectedness of the branch locus of moduli spaces of Riemann surfaces*

Abstract: The moduli space  $\mathcal{M}_g$  of compact Riemann surfaces of genus  $g$  has the structure of an orbifold and the set of singular points of such orbifold is the *branch locus*  $\mathcal{B}_g$ . In this talk we study the (dis-)connectivity of  $\mathcal{B}_g$ . More concretely:  $\mathcal{B}_g$  is disconnected for  $g \geq 26$ . Finally we present the known information about this issue for genera  $< 26$ .

**2 Wednesday 6 October 2010, Prof. Jan Nordström, MAI**

*Weak Boundary and Interface Conditions with Multi-Physics Applications*

Abstract: By reusing the main ideas behind the recent development of stable high order finite difference methods (summation-by-parts operators, weak boundary conditions, the energy-method) new coupling procedures have been developed. We will present the theory by analysing simple examples and apply to very complex multi-physics problems.

**2 Wednesday 13 October 2010, Prof. Anders Björner, KTH & Mittag-Leffler Institute**

*A q-analogue of the FKG inequality and some applications*

Abstract: The FKG inequality of Fortuin, Kasteleyn and Ginibre (1971) originated as a correlation inequality in statistical mechanics. It has many applications in discrete probability and extremal combinatorics.

In this talk we present a polynomial coefficient-wise inequality that refines the original FKG inequality. This polynomial FKG inequality has applications to  $f$ -vectors of joins of simplicial complexes, to Betti numbers of intersection of Schubert varieties, and to power series weighted by Young tableaux. The latter case includes a correlation-type inequality for the poissonization of Plancherel measure on symmetric groups, a probability measure on the set of all integer partitions.

The talk will be quite elementary and no previous familiarity with these topics will be assumed.

**2 Wednesday 20 October 2010, Tomasz Adamowicz, MAI**

*On p-Laplacian, variable exponent analysis and image processing*

Abstract: The purpose of the talk is to introduce the fundamental object of nonlinear potential theory the  $p$ -harmonic operator and related  $p(x)$ -harmonic equation, the so-called *variable exponent p-Laplacian*. If  $p = 2$  we retrieve the harmonic case, but in general the geometry of the  $p$ -harmonic world is much more complicated than the harmonic one. We explain the basic properties of the nonlinear Laplacian and  $p(x)$ -Laplacian and show the unexpected and fruitful interplay between planar quasilinear PDEs and the class of mappings of finite distortion. If time permits we will discuss some of the applications of variable exponent PDEs in image processing.

The talk will be accessible to graduate students and a general audience of mathematicians.

**2 Wednesday 27 October 2010, Prof. Tudor Ratiu, EPFL, Lausanne**

*The variational structure of conservative complex fluids*

Abstract: The goal of the talk is to present the geometry and variational principle that gives rise to Eringen's conservative micropolar liquid crystal equations. To get there, the basic Euler–Poincaré equations and variational principle will be presented as well as the examples of the free rigid body, ideal incompressible homogeneous Euler flow, and the Korteweg–de Vries equation. Then it will be explained how the affine Euler–Poincaré reduction gives rise to the conservative micropolar liquid crystal equations.

**2 Wednesday 3 November 2010, Tomas Sjödin, MAI**

*Integrability properties of positive harmonic functions*

Abstract: Positive harmonic functions on a bounded domain in  $n$ -dimensional Euclidean space have many special properties. For instance on a ball they are precisely those functions representable as a Poisson integral of a positive measure on the boundary of the ball. On some domains, for instance a ball, all of them are integrable with respect to Lebesgue measure and in some cases, for instance on a square in two dimensions, this is not true. The aim of this talk will be to sketch a proof of the fact that on a quadrature domain (where a



ball is an example of such) they are always integrable. The proof builds on blow-up arguments from the theory of free boundaries and Martin boundary techniques.

2 **Wednesday 10 November 2010, No Colloquium. (Seminar on Intercultural Communication with Nigel Musk, IKK.)**

2 **Wednesday 17 November 2010, Tatiana Shaposhnikova, MAI**

*Regularity properties of solutions to the Dirichlet problem for higher order elliptic systems with rough coefficients and the boundary*

Abstract: Given a bounded Lipschitz domain, we consider the Dirichlet problem with boundary data in Besov spaces for divergence form strongly elliptic systems of arbitrary order with bounded complex-valued coefficients. The main result gives a condition on the local mean oscillation of the coefficients of the differential operator and the unit normal to the boundary which guarantee that the solution operator associated with this problem is an isomorphism. This is a joint work with Vladimir Maz'ya and Marius Mitrea. I also describe higher regularity results in a subclass of Lipschitz domains characterized in terms of Sobolev multipliers, following the recent book by Vladimir Maz'ya, Tatyana Shaposhnikova "Theory of Sobolev Multipliers with Applications to Differential and Integral Operators", Springer, Grundlehren der Mathematischen Wissenschaften, 2009. I'll pay much attention to classes of functions used in these results.

2 **Friday 26 November 2010, Prof. Bernt Wennberg, Chalmers Tekniska Högskola**

*Propagation of chaos in many-particle systems*

Abstract: Boltzmann's and Maxwell's kinetic theory aims at explaining the behavior of gases from an atomistic point of view. When Boltzmann's famous paper was published in 1872, the concept of atoms was not generally accepted, and the kinetic theory was much criticized. Poincaré and Zermelo gave examples that pointed at the apparently paradoxical fact that the Boltzmann equation is irreversible, while the microscopic dynamics is reversible. While this paradox is now resolved, there is still no complete, rigorous derivation of the Boltzmann equation. A key concept is "propagation of chaos". In my talk I will explain what this means, and discuss some simplified cases where propagation of chaos can be proven to hold, and hence the Boltzmann equation to be valid.

2 **Wednesday 1 December 2010, Peter Basarab-Horwath, MAI**

*Some aspects of classifying evolution equations*

Abstract: The problem of classifying evolution equations is presented from a systematic symmetry point of view. Using standard techniques from Lie symmetry theory allied with representation theory and the equivalence group of a differential equation (also known as the covariance group), we are able to give a complete classification of third-order evolution equations in terms of symmetry properties.

2 **Wednesday 8 December 2010, Lukáš Malý, MAI**

*Calderón-type theorems for operators with nonstandard endpoint behavior*

Abstract: Calderón's theorem states that every quasilinear operator, which is bounded both from  $L^{p_1,1}$  to  $L^{q_1,\infty}$ , and from  $L^{p_2,1}$  to  $L^{q_2,\infty}$  for properly ordered values of  $p_1, p_2, q_1, q_2$ , is bounded on some rearrangement-invariant space if and only if the so-called Calderón operator is bounded on the corresponding representation space.

We will establish a Calderón-type theorem for nonstandard endpoint behavior, where Lorentz  $\Lambda$  and  $M$  spaces will be the endpoints of an interpolation segment. Two distinctive types of nonstandard behavior are to be discussed; first, we'll explore the operators bounded simultaneously from  $\Lambda(X_1)$  to  $M(Y_1)$ , and from  $M(X_2)$  to  $M(Y_2)$ , next, operators bounded simultaneously from  $\Lambda(X_1)$  to  $\Lambda(Y_1)$ , and from  $\Lambda(X_2)$  to  $M(Y_2)$ . For that purpose, we evaluate Peetre's  $K$ -functional for varied pairs of Lorentz spaces.

# The Mathematical Colloquia held in 2009 at Linköping University

Organized by Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov, Hans Lundmark and Stefan Rauch-Wojciechowski.

## Tuesday 13 January 2009, Jonas Bergman Ärlebäck, MAI (Joint Seminar with Mathematics Education)

Title: *Introducing mathematical modelling using Fermi problems in upper secondary school.*

Abstract: In the documents governing the Swedish upper secondary mathematics education more and more emphasis is put on mathematical modelling. However, research suggests that the explicit teaching of mathematical modelling is not part of the implemented curricula in the classrooms. In this background, I report on and discuss a study aiming to investigate the potential of using Fermi problems to introduce mathematical modelling to Swedish upper secondary school students. The work of three groups of students engaged in solving realistic Fermi problems was analysed using an analytic tool referred to as the MAD framework, providing 'modelling activity diagrams' of the groups' problem solving processes. Using these diagrams, I conclude that the processes involved in a mathematical modelling were richly represented in the problem solving processes of the groups.

## Wednesday 14 January 2009, Magnus Herbertsson, MAI

Title: *On the potential method for calculating radar cross sections*

Abstract: Under some natural conditions, the problem of determining the radar cross section of a body with boundary surface  $S$  results in a vector valued integral equation over  $S$ , which involves only quantities tangential to  $S$ . Using Hodge decomposition theorem, this equation can be formulated in terms of scalar functions. I will describe some recent developments using this technique.

## Wednesday 21 January 2009, Jan-Åke Larsson, MAI

Title: *Linköping University researchers break "unbreakable" crypto*

Abstract: The risk of illegal accessing of information, for example in money transactions, is necessitating more and more advanced cryptographic techniques. A new technology called quantum cryptography is supposed to be absolutely secure, based on properties of quantum-mechanical objects. I will talk about why not even quantum cryptography is 100-percent secure: there is a theoretical possibility that an unauthorized person can extract the key without being discovered, by simultaneously manipulating both the quantum-mechanical and the regular communication needed in quantum cryptography. The attack route is through the authentication, intended to secure that the message arriving is the same as the one that was sent. Even if the authentication itself is cryptographically secure, it does not work as intended in the system as a whole. I will also propose a change that solves the problem and reestablishes the security.

## Friday 30 January 2009, Ewa Kozłowska-Walania, University of Gdansk, Poland

Title: *Pairs of symmetries of Riemann surfaces*

Abstract: As it is known, symmetries of compact Riemann surfaces correspond to the real forms of smooth projective irreducible complex algebraic curves whose number of connected components equals the number of ovals of symmetries. Therefore, one can study real forms of complex algebraic curves by means of Riemann surfaces and their symmetries, using theory of non-euclidean crystallographic groups.

We focus our attention on qualitative studies on doubly symmetric Riemann surfaces of genus  $g \geq 2$ , assuming first that both symmetries have fixed points filling up in this way a picture described ten years ago by Izquierdo and Singerman. We give some bounds for the total number of ovals of two symmetries and show their attainment, as a corollary obtaining the minimal, with one exception in any genus  $g > 2$ , lower bound for  $g$ , which guarantees the commutativity of the symmetries. Furthermore under some additional assumptions, for commuting pairs of symmetries we give necessary and sufficient conditions for such a pair to exist. In the second part we take into account the fixed point free symmetries and we give some new results, being a "comparative analysis" to the previous case.

## Monday 2 February 2009, Presentation of Maple TA

See <http://www.maplesoft.com/products/mapleta>

## Wednesday 4 February 2009, Prof. Thomas Kaijser, MAI

Title: *Convergence in distribution for Markov chains induced by partitions of transition probability matrices.*

Abstract: Let  $S$  be a denumerable state space,  $P$  a transition probability matrix on  $S$  and  $K$  the set of probability vectors on  $S$ . A partition  $\mathcal{M}$  of  $P$  is a denumerable set of nonnegative matrices such that their sum is equal to  $P$ . To every partition  $\mathcal{M}$  of  $P$  we can associate a transition probability function  $\mathbf{P}_{\mathcal{M}}$  on  $K$  defined in such a way that if  $p \in K$  and  $M \in \mathcal{M}$  are such that  $\|pM\| > 0$ , then, with probability  $\|pM\|$ , the vector  $p$  is transferred to the vector  $pM/\|pM\|$ . Here  $\|\cdot\|$  denotes the  $l_1$ -norm.

In this talk I will present conditions that imply convergence in distribution towards a unique limit distribution independent of initial distribution for Markov chains generated by transition probability functions induced by partitions of transition probability matrices.

I will give some proofs, and also some examples for which such a conclusion is not true.

The motivation for this work is that every so called *hidden Markov chain* induces a partition of a transition probability matrix.

## Wednesday 11 February 2009, Kirsti Hemmi, MAI (joint seminar with Mathematics Education)

Title: *Students' encounters with mathematical proof.*

Abstract: There is a renewed emphasis on mathematical proof in the ongoing revisions of school curricula in many countries and proof is also a vital issue in mathematics education research. I will describe the main issues in the didactical research on proof and present some theoretical and empirical results from my study about students' encounters with proof at a mathematics department in Sweden.

## Wednesday 18 February 2009, Prof. Andrzej Szulkin, Stockholms universitet

Title: *"Magnetic" Schrödinger Equation*

Abstract: Consider the Schrödinger equation  $-\Delta_A u + V(x)u = f(x, u)$  in  $\mathbf{R}^N$ . It describes the behaviour of a particle (or a system of particles) under the influence of an electric potential  $V$  and an external magnetic field  $B = \text{curl } A$ . Here  $-\Delta_A u := (-i\nabla + A(x))^2 u$ , where  $A : \mathbf{R}^N \rightarrow \mathbf{R}^N$  is a magnetic potential having its source in  $B$ , and  $f(x, u) = a(x)|u|^{p-2}u$  ( $a > 0$ ) is a model nonlinearity. We discuss gauge equivalence, properties of the functional associated with the problem, existence of solutions other than  $u = 0$  and, if time permits, the so-called Aharonov-Bohm effect.

## Wednesday 25 February 2009, Prof. Torsten Ekedahl, Stockholms universitet

Title: *A generalised Möbius inversion formula.*

Abstract: The inclusion/exclusion principle (or more generally the Möbius inversion formula) can be used very efficiently to count the number of points of a finite set covered by subsets (or more generally computing the Euler characteristic of a topological space covered by open subsets). There are situations however which are not covered by the principle. Examples are when we have a group acting on the set permuting the elements of the cover and one is interested in computing not just the cardinality but the permutation character. I will give a general abstract setup which allows us to get inversion formulas covering these more general situations.

## Wednesday 4 March 2009, Prof. Milagros Izquierdo, MAI

Title: *On the connectedness of the branch locus of the moduli space of Riemann Surfaces.*

Abstract: We use the equiparametric stratification described by Broughton (1990) to study the connectedness of the branch locus of the moduli space of Riemann surfaces of low genus. We also show that for infinite genera the branch locus contains isolated strata of dimension 1, generalizing a result of Kulkarni (1991) for isolated points.

## Wednesday 18 March 2009, Prof. Natan Kruglyak, Växjö

Title: *Image processing, Sobolev embedding theorem and real interpolation*

Abstract: Nowadays interpolation theory is an important and rather abstract branch of functional analysis which has many applications to harmonic analysis and partial differential equations. However, as I plan to explain during the talk, a part of it which is called real interpolation and goes back to classical theorem of Marcinkiewicz, is connected with modern

mathematical methods in image processing. This connection leads to deep mathematical problems. I plan to discuss them and one approach based on Sobolev embedding theorem and local approximations.

**Wednesday 25 March 2009, Prof. S. A. Vakulenko, St. Petersburg Academy of Sciences**

*Title: Genetic networks.*

Abstract: We consider a mathematical model of genetic networks proposed by J. Reintiz , E. Mjølness and D.Sharp. This model is a generalization of the famous Hopfield model of neural networks (1982) which takes into account diffusion, inhibition and activation of genes.

The talk contains a review of results obtained by the author together with D. Grigoriev (Lille), O. Radulescu (Rennes), S. Genieys (Lyon), J. Reintiz (New -York) . They concern with the following problems: pattern formation process; complicated attractors and patterns for genetic networks; stability of patterning under random noise and parameter variation; evolution of networks.

**Wednesday 1 April 2009, Joanna Pres, University College Dublin**

*Title: Positive harmonic functions on Denjoy-type domains.*

Abstract: This talk concerns the study of positive harmonic functions on a particular type of unbounded domain. Let  $E$  be a closed set in the hyperplane  $\mathbb{R}^{N-1} \times \{0\}$  of Euclidean space  $\mathbb{R}^N$  with  $N \geq 2$ . A domain of the form  $\Omega = \mathbb{R}^N \setminus E$  is called a *Denjoy Domain*. Benedicks studied the cone of positive harmonic functions in a Denjoy domain vanishing continuously on the boundary. He gave an integral criterion in terms of harmonic measure which characterizes when there is a positive harmonic function  $h$  on  $\Omega$  satisfying  $h(x) \geq x_N$ . Later, Cranston and Salisbury obtained a corresponding result in the case of the plane where  $E$  lies on  $n$  rays leaving the origin.

Suppose now that  $E$  is a closed subset of the boundary of an infinite cylinder  $U$  in  $\mathbb{R}^N$  ( $N \geq 2$ ). In this talk a Benedicks-type criterion will be provided for the existence of a positive harmonic function  $h$  on  $\Omega$  of exponential growth in  $U$  vanishing continuously on the boundary. Two applications will be given. The first of these concerns minimal harmonic functions associated with an irregular boundary point, and amplifies a recent construction of Gardiner and Hansen. The second concerns the possible non-approximability of positive harmonic functions by integrable positive harmonic functions.

This is joint work with Marius Ghergu.

**Wednesday 15 April 2009, Tomas Johansson, University of Birmingham**

*Title: Inverse acoustic multiple scattering using topological derivatives*

Abstract: We consider an inverse problem where the structure of multiple planar sound-soft obstacles is to be determined, given the direction of one or more incoming fields and knowledge of the corresponding scattered fields on a curve outside the obstacles. A method involving topological derivatives will be presented together with some of its properties. In this method, no a priori assumption is needed on the number of obstacles present. Numerical results will also be presented and discussed. The obtained results are a joint work together with Prof. Ana Carpio (Univ. Complutense de Madrid) and Dr. Maria-Luisa Rapun (Univ. Polit cnica de Madrid).

**Wednesday 15 April 2009, Oliver Riordan, University of Oxford**

*Title: Sparse graphs: metrics and random models.*

Abstract: One of the main applications of graph theory outside pure mathematics is to provide mathematical models for a wide range of real-world networks, both physical and abstract. Very often, random graphs are used, since one cannot hope to produce a model that exactly reproduces a complex real-world network such as a social network.

A key question that is seldom addressed is the following: How good is the fit between the model and the real network? Of course, one can compare the values of various parameters (for example, degree distribution, or network diameter), but often the model can be 'tuned' to match these parameters, which gives no guarantee that the model is accurate in other ways. It would be better to have one standard measure of similarity between graphs, and so be able to say that the model is a good fit if it produces graphs that are 'globally similar' to the real-world networks.

In the dense case, for graphs with  $n$  vertices and order  $n^2$  edges, the work of Borgs, Chayes, Lov sz, S s, Szegedy and Vesztergombi gives a very nice answer to this question, that is closely related to (inhomogeneous) random graphs. For the more realistic sparse case, there are some partial results, but many more open questions.

**Wednesday 22 April 2009, Prof. Aleman Alexandru, Lunds universitet**

*Title: Some applications of near invariance*

Abstract: We consider Hilbert spaces  $H$  which consist of analytic functions in a domain  $\Omega \subset \mathbb{C}$  and have the property that any zero of an element of  $H$  which is not a common zero of the whole space, can be divided out without leaving  $H$ . This property is called near invariance and is related to a number of interesting problems that connect complex analysis and operator theory. The concept probably appeared first in L. de Branges' work on Hilbert spaces of entire functions and played later a decisive role in the description of invariant subspaces of the shift operator on Hardy spaces over multiply connected domains. There are a number of structure theorems for nearly invariant spaces obtained by de Branges, Hitt and Sarason, and more recently by Feldman, Ross and myself, but the emphasis of the talk will be on some applications. We shall have a look at differentiation invariant subspaces of  $C^\infty(\mathbb{R})$ , and invariant subspaces of Volterra operators on spaces of power series on the unit disc. Finally, we use near invariance in the vector-valued case to study kernels of products of Toeplitz operators. More precisely, I will present in more detail the recent solution of the following problem: If a finite product of Toeplitz operators is the zero operator then one of the factors is zero.

**Wednesday 29 April 2009, Ulla Ouchterlony, MAI**

*Title: 5th Term of the IT-Programme at LIU*

Abstract: The fifth semester of the IT-programme is a Project. It is one course lasting for one semester and it includes cooperation between teachers from seven departments and students from three programmes, the IT-Programme, the Programme of Psychology and LIU School of Management.

I have been responsible for this semester since it started in 1997.

The concept is unique and all students are very satisfied with the organisation of the Project and thus it would be a pity not to inform all MAI about the IT-Project before I retire in September. The technical contents of the project will also be completely change next autumn, so it is time for a summary.

I will also inform about the individual examination in Numerical Algorithms. It is homework assignments with an individual oral presentation.

**Wednesday 13 May 2009, Prof. Tobias Ekholm, Uppsala universitet**

*Title: Floer homology and double points of exact Lagrangian spheres and tori*

Abstract: We first describe a basic relation between finite dimensional smooth topology and some infinite dimensional topology arising in symplectic geometry. More precisely, we describe the Morse-Witten complexes of spheres and tori and show how they are related to the Lagrangian Floer homology in the symplectic manifolds which are their respective cotangent bundles. (A cotangent bundle of an  $n$ -manifold equipped with the exterior derivative  $\omega$  of its action form, is a symplectic  $2n$ -manifold. An  $n$ -submanifold of a symplectic manifold is Lagrangian if the restriction of  $\omega$  to its tangent spaces everywhere equals 0.)

Second we discuss how these tools can be used to prove existence of double points of Lagrangian spheres and tori in complex  $n$ -space. (Here we view complex  $n$ -space as the symplectic manifold which is the cotangent bundle of real  $n$ -space). Our discussion leads to the existence of at least one double point. For the  $n$ -sphere this is an optimal lower bound but for the  $n$ -torus the expected lower bound is  $2^{n-1}$ . The latter expected lower bound has been established under additional hypothesis on the torus-immersion but the general case remains open. We end with a discussion of these matters.

**Thursday 14 May 2009, Prof. Faruk G ng r, Istanbul Technical University**

*Title: Kac-Moody-Virasoro Algebras as Symmetries of 2+1-dimensional Nonlinear Evolution Equations*

Abstract: Symmetry proved to be useful for detecting the integrability of nonlinear partial differential equations. In this talk I will survey some physical models such as generalized Kadomtsev-Petviashvili and Davey-Stewartson equations admitting Kac-Moody-Virasoro algebras as their symmetries and discuss how their presence can serve as a preliminary test for integrability. As a by-product, I present some results on blow-up in finite time using pseudo-conformal transformations as subgroups of Virasoro group.

**Monday 18 May 2009, Magnus  sterholm, Ume  universitet (Joint Seminar with Mathematics Education)**

*Title: Two projects, one theory: How a theory on comprehension can be utilized when studying beliefs and when studying the solving of mathematical tasks*

**Wednesday 20 May 2009, Anders Karlsson, KTH**

*Titel: Heat kernels, zeta functions, and spanning trees*

Abstrakt: I will define heat kernels on discrete groups and explain their relations to Bessel functions. This will be applied to the problem of determining the number of spanning trees on finer and finer discretizations of tori. These considerations, which are joint work with G. Chinta and J. Jorgenson, are interesting for statistical physics, differential geometry, and number theory.

**Tuesday 26 May 2009, Prof. Johannes S j strand, Universit  de Bourgogne, Dijon**

*Title: Weyl asymptotics for non-self-adjoint differential operators with random perturbations*

Abstract: Due to spectral instability the eigenvalues of non-self-adjoint differential operators are often highly unstable under small perturbations. There are now several results stating that when we add a small random perturbation, we get Weyl asymptotic distribution of eigenvalues, with probability close to 1 in the semi-classical limit, and almost surely in the limit of large eigenvalues. Moreover the bounds on the resolvent tend to improve under the action of such perturbations. We describe some of these results, due to M. Hager, W. Bordeaux-Montrieux, and the speaker, as well some underlying ideas and proofs.

**Wednesday 27 May 2009, Prof. Natan Kruglyak, Växjö**

*Title: Linear and Non-Linear Ill-Posed Problems and Real Interpolation.*

Abstract: I plan to discuss:

- a) how iterative procedure combined with real interpolation leads to solution of non-linear ill-posed problems;
- b) real interpolation hidden in regularization theory for linear operators;
- c) extension of regularization theory to Banach spaces.

**Monday 15 June 2009, Andreas Axelsson, LiU och Stockholms universitet**

*Title: Hilbert transforms and the Cauchy integral in euclidean spaces.*

Abstract: The Hilbert transform for a domain in the complex plane is the operator which maps the boundary values of a harmonic function in the domain to the boundary values of its harmonic conjugate function. This operator, as well as the Cauchy integral operator on the boundary, are important examples of singular integral operators and have been much studied in harmonic analysis, for domains with non smooth boundaries. A way to calculate the Hilbert transform is to use that it factors into a product of the Cauchy integral and the inverse of a double layer potential operator. In this talk I will discuss extensions of this method for calculating Hilbert transforms / harmonic conjugate functions to Lipschitz domains in higher dimensional euclidean spaces. This makes use of the Cauchy reproducing integral formula for the Hodge-Dirac system  $(d + \delta)f = 0$ , which generalizes the Cauchy-Riemann system in the plane.

**Wednesday 17 June 2009, Prof. Sergei A. Avdonin, University of Alaska**

*Title: Boundary Control Approach to Inverse Spectral Problems*

Abstract: We establish connections between several approaches to inverse spectral problems: the classical Gelfand-Levitan theory, the Krein approach, the Simon theory, the approach proposed by Remling, and the Boundary Control method. We show that the Boundary Control approach provides simple and physically motivated proofs of the central results of other theories. We demonstrate also the connection between the dynamical and spectral data and derive the local version of the classical Gelfand-Levitan equation.

**10-14 August 2009 Nonlinear problems for  $\Delta_p$  and  $\Delta$**

See <http://www.mai.liu.se/TM/conf09/>

**Monday 17 August 2009, Maciej Błaszak, Adam Mickiewicz University, Poznań**

*Title: Bi-presymplectic representation of Liouville integrable systems and related separability theory*

Abstract: Bi-presymplectic chains of one-forms of arbitrary co-rank are considered. The conditions in which such chains represent some Liouville integrable systems and the conditions in which there exist related bi-Hamiltonian chains of vector fields are presented. In order to derive the construction of bi-presymplectic chains, the notions of dual Poisson-presymplectic pair, d-compatibility of presymplectic forms and d-compatibility of Poisson bivectors are used. The completely algorithmic construction of separation coordinates is demonstrated. It is also proved that Stäckel separable systems have bi-inverse-Hamiltonian representation, i.e. are represented by bi-presymplectic chains of closed one-forms. The co-rank of related structures depends on the explicit form of separation relations.

**Wednesday 19 August 2009, Visa Latvala, University of Joensuu**

*Title: Regularity theory of variable exponent p-Laplace equation*

Abstract: We review the basic regularity theory of the variable exponent p-Laplacian. We also discuss the recent development concerning the borderline cases which allow the value one (or the value infinity) in part of the domain.

**Wednesday 2 September 2009, Prof. Lars Erik Andersson, MAI**

*Title: Existence results for quasistatic frictional contact problems and frictional wedging problems*

Abstract: A frictional contact problems in elasticity consists of finding, for a given load history and an initial state, the time evolution of displacements and reaction forces. The problems may be dynamic, quasistatic or static (incremental). It is well known that for *large* coefficients of friction there are no general results of existence and/or uniqueness for these problems, even for very simple systems. Here some existence and uniqueness for frictional systems with finitely many spatial degrees of freedom, will be summarized. Next we will formulate a so called wedging problem, i.e. whether the elastic system has some non-trivial state of equilibrium in the absence of exterior forces. The main focus of this work is the relation between the wedging problem and the problem of uniqueness for quasistatic evolution problems. In particular we will investigate the relation between critical friction bounds for wedging and for nonuniqueness of rate problems respectively.

**Wednesday 9 September 2009, Prof. Magnus Borga, IMT**

*Title: Medical Imaging: Segmentation of Blood Vessels*

Abstract: The segmentation of blood vessels is a common problem in medical imaging and various applications are found in diagnostics, surgical planning, training and more. Among many different techniques, the use of multiple scales and line detectors is a popular approach. However, the typical line filters used are sensitive to intensity variations and do not target the detection of vessel walls explicitly. In our work, we combine both line and edge detection using quadrature filters across multiple scales. The filter result gives well defined vessels as linear structures, while distinct edges facilitate a robust segmentation. We apply the filter output to energy optimization techniques for segmentation based on Level Set front propagation.

**Wednesday 23 September 2009, Prof. Anna Talarczyk, Warsaw University**

*Occupation time fluctuations of branching particle systems*

Abstract: We consider  $(d, \alpha, \beta, \gamma)$  branching particle system, which consists of particles moving in  $\mathbb{R}^d$  according to a symmetric  $\alpha$ -stable Lévy process and branching with a critical  $1 + \beta$  branching law with probability generating function

$$G(s) = s + \frac{(1-s)^{1+\beta}}{1+\beta}, \quad 0 \leq \beta \leq 1.$$

The initial positions of the particles are given by a Poisson random measure with intensity measure  $\mu_\gamma(dx) = dx/(1+|x|^\gamma)$ ,  $\gamma \geq 0$ . The system is described by its empirical process  $N_t$ , where  $N_t(A)$  is the number of particles in set  $A$  at time  $t$ . We investigate the occupation time fluctuations of the system as the time is accelerated, i.e. we are interested in the limit of the processes

$$X_T(t) = \frac{1}{F_T} \left( \int_0^{Tt} N_s ds - E \int_0^{Tt} N_s ds \right), \quad t \geq 0.$$

as  $T \rightarrow \infty$ , where  $F_T$  is a proper norming. In some cases also the density of the system is increased. Depending on the interplay between the parameters  $d, \alpha, \beta, \gamma$  of the system, we obtain several interesting types of limits of normalized occupation time fluctuations. In particular, in low dimensions" the limits have simple spatial structure (Lebesgue measure) and complicated temporal structure (dependent increments), in large dimensions" the temporal structure is simple (independent increments) but the spatial structure is more complicated. We also discuss some properties of the limit processes, in particular long range dependence. Based on joint work with T. Bojdecki and L. G. Gorostiza.

**Wednesday 30 September 2009, Jörg-Uwe Löbus, MAI**

*Absolute continuity under time shift of trajectories*

Abstract: The talk is divided into two parts. The first part is dedicated to introduce basic objects of the infinite dimensional stochastic calculus. In particular the central role of the Cameron-Martin formula will be explained. The second part is devoted to a class of processes  $(X, P_\nu)$  of the form  $X = W + A$ . Here  $W$  is a two-sided Brownian motion with random initial datum  $W_0$  that follows a distribution  $\nu = m dx$  and  $A = A(W)$  with  $A_0 = 0$  is a certain function of  $W$ . Crucial for absolute continuity under time shift is *temporal homogeneity* in the sense that

$$X(W_{\cdot+v} + A_v 1) = X_{\cdot+v}(W), \quad v \in \mathbb{R},$$

where  $A_0$  is the trajectory taking the constant value  $A_0(W)$ . The density relative to time shift of trajectories of  $X$  is of the form

$$\frac{P_\nu(dX_{-t})}{P_\nu(dX)} = \frac{m(X_{-t})}{m(X_0)} \cdot \exp\left\{\langle e, \nabla_{W_0} A_{-t} \rangle\right\}.$$

The basic idea of its proof will be explained. Applications of this formula will be partial integration and a measure valued counterpart to a class of stochastic equations with delay or anticipation.

**Wednesday 7 October 2009, Sandra di Rocco, KTH**

*Toric geometry and certain convex polytope*

**Abstract:** Toric embeddings are associated to convex lattice polytopes. A lot of the geometry of the embedded variety is detected via combinatorial invariants of the polytope and vice versa. The interplay between toric algebraic geometry and convex geometry will be recalled at the beginning of the talk. The aim of this seminar is to report on some (recent and less recent) results for non singular toric varieties, which translate to unexpected combinatorial properties of convex polytopes. More precisely (as long as time permits) generation of jets, adjoint properties and dual properties of toric embeddings will be presented.

**Wednesday 14 October 2009,**

**Matematik och astronomi – en lång sällskapsresa**

**Wednesday 21 October 2009, Prof. Carel Faber, KTH**

*On the moduli space of curves and its intersection theory*

**Abstract:** The moduli space  $\bar{M}_g$  of curves of genus  $g$  is a variety whose points correspond in a natural way to the isomorphism classes of curves of genus  $g$ . Geometric properties of the moduli space have consequences for arbitrary families of curves. After reviewing some basic facts about the moduli space, I will focus on its intersection theory. Several concrete results will be presented.

**Wednesday 28 October 2009, Uffe Jankvist, Roskilde universitet**

*Using history as a 'goal' in mathematics education*

**Abstract:** Based on a brief, but general, discussion of the use of history of mathematics in mathematics education, I shall discuss a concrete experiment involving two historical teaching modules implemented in a Danish upper secondary class. An often occurring problem when trying to integrate the history of mathematics in mathematics education is that it becomes quite anecdotal and detached from the actual mathematics of the historical case(s). One of the foci of the study carried out in the Danish upper secondary school was to investigate how this may be avoided, i.e. how the students' treatment and discussions of the historical cases could be anchored in mathematics of the modules.

**Wednesday 4 November 2009, Prof. Alexander Nazarov, Saint-Petersburg State University**

*The Dirichlet problem for non-divergence parabolic equations with discontinuous in time coefficients*

**Abstract:** In 2001 N. Krylov observed that for non-divergence parabolic equations coercive estimates for solutions can be proved even when the leading coefficients are only measurable functions with respect to  $t$ . In this lecture I give an overview of results obtained in this direction and present new ones obtained together with Vladimir Kozlov. We consider the Dirichlet problem for non-divergence parabolic equation with discontinuous in  $t$  coefficients in a half space. The main result is weighted coercive estimates of solutions in anisotropic Sobolev spaces. We give an application of this result to linear and quasi-linear parabolic equations in a bounded domain. In particular, if the boundary is of class  $C^{1,\delta}$ ,  $\delta \in [0, 1]$ , then we present a coercive estimate of solutions in weighted anisotropic Sobolev spaces, where the weight is a power of the distance to the boundary.

**Wednesday 11 November 2009, Dr. Riikka Korte, University of Helsinki,**

*An obstacle problem related to minimal surfaces in metric spaces*

**Abstract:** We discuss the existence of a set with minimal perimeter that separates two disjoint sets in a metric measure space equipped with a doubling measure and supporting a Poincaré inequality. A measure constructed by De Giorgi is used to state a relaxed problem, whose solution coincides with the solution to the original problem for measure theoretically thick sets. Moreover, we show that the De Giorgi measure is comparable to the Hausdorff measure of codimension one.

The theory of functions of bounded variation in metric spaces is used extensively to obtain these results. Therefore the talk starts with an introduction to this subject.

These results are based on joint work with J. Kinnunen, N. Shanmugalingam and H. Tuominen.

**Monday 16 November 2009, Dr. Johanna Pejläre, MAI**

*Visualiseringar och åskådning i matematik (Visualisation in Mathematics)*

**Tuesday 24 November 2009, Prof. Antonio F. Costa, UNED, Madrid**

*Representing automorphisms of Riemann and Klein surfaces as restriction of rigid motions in the Euclidean space.*

**Abstract:** Given a surface embedded in the Euclidean space, such surface has a natural structure of Riemann surface given by the conformal structure produced by the Euclidean metric. If the surface is invariant by a rigid motion, then the restriction of such rigid motion to the surface gives an automorphism of the Riemann surface. In this talk the automorphisms of Riemann and Klein surfaces that can be represented in this way will be presented. As a consequence we shall present some representations in dimension 4 of some classical Riemann surfaces as the Klein quartic or the Bolza surface.

**See the talk here.**

**Wednesday 2 December 2009, Prof. Kurt Johansson, KTH**

*Universality in random matrix theory*

**Abstract:** A central problem in the study of spectra of large random matrices is the question of universality. Here universality refers to the fact that for many choices of probability measures on spaces of matrices, e.g., Hermitian matrices, the local statistics of the eigenvalues is independent of the details of the probability measures. I will give some background on the problem of proving universality and give an overview of some results. Recently spectacular progress for the case of random Hermitian matrices with independent elements was made by Tao and Vu and also by Erdős, Schlein and H.T.-Yau.

**Tuesday 15 December 2009, Prof. Faina I. Solov'eva, Novosibirsk State University Russia**

*Perfect codes and related topics*

**Abstract:** The topic of perfect codes is one of the most important topics in the theory of error-correcting codes. The class of perfect codes is very complicated, large (double exponential) and intensively studied by many researches. The investigation of nontrivial properties of perfect codes is significant both from coding point of view (for the solution of the classification problem for such codes) and for combinatorics, graph theory, group theory, geometry, cryptography. Many constructions and properties, for example, for perfect binary codes can be applied for codes with different parameters (lengths, sizes, distances) or for nonbinary cases. In this talk an introduction to the theory of perfect codes is presented. Some links with related subjects are outlined and some open problems are given.

**Wednesday 16 December 2009, Prof. Jan Malý, Charles University, Prague**

*Sobolev homeomorphisms*

**Abstract:** In this talk we will address the following questions:

What additional condition guarantees that the inverse  $f^{-1}$  of a Sobolev homeomorphism  $f$  is again a Sobolev homeomorphism?

If  $f$  is a Sobolev homeomorphism, can the Jacobian  $Jf$  change its sign?

The presented results have been obtained in collaboration with Stanislav Hencl and Marianna Csörnyei.



# The Mathematical Colloquia held in 2008 at Linköping University

*Organized by Armen Asratian, Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov and Stefan Rauch-Wojciechowski.*

## Wednesday 16 January 2008, Mattias Enstedt, Uppsala universitet

*Title. Hartree-Fock equations with decreasing external magnetic fields*

Abstract. In the presence of a decreasing external magnetic field, we present recent results on existence and non-existence of a ground state within the Hartree-Fock theory of atoms and molecules. The ground state exists provided the magnetic field is decreasing and the total charge  $Z$  of  $K$  nuclei exceeds  $N - 1$ , where  $N$  is the total number of electrons. In the opposite direction, no ground state exists when  $N \geq 2Z + K$

## Wednesday 23 January 2008, Andreas Strömbergsson, Uppsala universitet

*Title. The Boltzmann-Grad limit of the periodic Lorentz gas and the distribution of visible lattice points.*

Abstract. The periodic Lorentz gas describes a particle moving in a periodic array of spherical scatterers, and is one of the fundamental mathematical models for chaotic diffusion in a periodic set-up. In my lecture I will describe the recent solution of a problem posed by Y. Sinai in the early 1980s, on the nature of the diffusion when the scatterers are very small. The problem is closely related to some basic questions in number theory, in particular the distribution of lattice points visible from a given position. The main tool in our approach is measure rigidity, a part of ergodic theory which has recently found important applications in several other problems in number theory and mathematical physics, such as the value distribution of quadratic forms at integers, quantum unique ergodicity and questions of diophantine approximation. (This lecture is based on joint work with Jens Marklof, Bristol.)

## Wednesday 30 January 2008, Prof. Stefan Rauch-Wojciechowski, MAI

*Title. What means to explain the motion of the Tippe Top?*

Abstract. The Tippe Top has a shape of a truncated sphere with a peg attached to the flat surface. When spun sufficiently fast on its spherical bottom the tippe top turns up and continues motion on the peg. Research on the Tippe Top has long history since 19-th century and it is presently understood that the gliding friction is responsible for this phenomenon and that it takes place for the values of parameters where measures the eccentricity of the centre of mass.

I shall present results of our work on the phase space picture of TT. It appears that under mild assumptions about the friction force the asymptotic frictionless solutions play a special role, they are periodic and they are global attractors. All solutions tend (in the sense of the LaSalle's theorem) to one of the asymptotic solutions. We have discussed conditions of their stability and have described what happens to the TT in large for all values the parameters and all initial conditions. But detailed dynamics of the Tippe Top, that is description of how a TT is rising to the inverted spinning state remained unexplained. I shall present my recent results that provide tools to capture mathematically the whole dynamics of inversion. I shall demonstrate the motion of the Tippe Top and other rigid bodies.

## Wednesday 6 February 2008, Prof. Joaquim Ortega-Cerdá, Universitat de Barcelona

*Title. The univalent Bloch-Landau constant.*

Abstract. Landau in the 30's estimated the univalent Bloch-Landau constant  $U$ , i.e., the biggest radius  $R$  that such that  $f(D(0, 1))$  always contains a disk of radius  $R$  for any univalent  $f$  normalized with  $|f'(0)| = 1$ . Although the exact value of  $U$  is not known, many authors have provided upper and lower bounds. In a joint work with T. Carroll we have studied fine properties of the extremal functions and shown the connection with other well studied question, the Pólya-Cebotarev problem. This relationship has been exploited to improve (very slightly) the upper bound for the constant.

## Wednesday 13 February 2008, Prof. Juha Kinnunen, Helsinki University of Technology

*Title. On the definition and properties of superparabolic functions.*

Abstract: We discuss potential theoretic aspects of degenerate parabolic partial differential equations of  $p$ -Laplacian type. Solutions form a similar basis for a nonlinear parabolic potential theory as the solutions of the heat equation do in the classical theory. In the parabolic potential theory, the so-called superparabolic functions are essential. For the ordinary heat equation we have supercaloric functions. They are defined as lower semicontinuous functions obeying the comparison principle. The superparabolic functions are of actual interest also because they are viscosity supersolutions of the equation. We discuss their structural, convergence and Sobolev space properties.

## Wednesday 5 March 2008, Prof. John Wermer, Brown University

*Title: The complex Plateau problem: Finding a Riemann surface with prescribed boundary.*

Abstract: Given a simple closed curve  $K$  in a complex manifold  $X$ . When does there exist a Riemann surface in  $X$  having  $K$  as its boundary? We discuss the history of the problem and some recent work on it by Harvey and Lawson, and by the speaker, for the case that  $X$  is complex projective space.

## Wednesday 26 March 2008, Tomas Johansson, University of Birmingham, UK

*Title: An alternating potential based procedure for a Cauchy problem for the Laplace equation in a planar domain with a crack*

Abstract: We consider a Cauchy problem for the Laplace equation in a bounded region containing a crack, where the region is formed by removing a sufficiently smooth arc (the crack) from a bounded simply connected domain  $D$ . The aim is to reconstruct the solution on the crack from values of the solution and its normal derivative on the boundary of the domain  $D$ . In 1989, V.A. Kozlov and V.G. Maz'ya proposed an alternating iterative method for solving Cauchy problems for formally self-adjoint elliptic equations, and we extend their ideas to domains with cracks and present a procedure which involves solving direct mixed problems for the Laplace operator in the same region. These mixed problems have either a Dirichlet or a Neumann boundary condition imposed on the crack and are solved by a potential approach. Each of these mixed problems are reduced to a system of integral equations of the first kind with logarithmic and hypersingular kernels and at most a square root singularity in the densities at the endpoints of the crack. Numerical examples will be presented illustrating the feasibility of the proposed method. I point out that the above is joint work with Roman Chapko from the Ivan Franko National University of Lviv in the Ukraine.

## Wednesday 2 April 2008, Kaj Nyström, Umeå universitet

*Title: Boundary Harnack inequalities for  $p$ -harmonic functions and Regularity of Free Boundaries in Two-phase Problems for the  $p$ -Laplace operator.*

Abstract: In this talk I will discuss joint work with John Lewis on boundary Harnack inequalities for  $p$ -harmonic functions in Lipschitz domains and beyond and their application to the study of the regularity of weak solutions to general two-phase free boundary problems for the  $p$ -Laplace operator.

## Wednesday 9 April 2008, Prof. Antonio Costa, UNED, Spain (Joint Seminar with the Didactics Group)

*Title: E-teaching Differential Geometry at UNED*

Abstract: This year we have started a new experience in the teaching way of differential geometry of curves and surfaces in UNED. It is a method similar to the Problem Based Learning. In this talk we shall present the results of such an experience.

## Monday 14 April 2008, Mikko Parviainen, Helsinki University of Technology

*Title: Global reverse Hölder's inequality for parabolic quasiminimizers*

Abstract: It was recently shown that a parabolic quasiminimizer globally belongs to a higher Sobolev space than assumed a priori. This can be deduced from the fact that the gradient satisfies a reverse Hölder inequality near the boundary. In this talk, we discuss the parabolic quasiminimizers and touch some aspects of the higher integrability proof.

## Wednesday 16 April 2008, Prof. Ragnar Sigurdsson, University of Iceland

*Title: Some aspects of pluripotential theory.*

Abstract: In the lecture I will begin by introducing a few concepts of pluricomplex analysis and pluripotential theory in particular and relate them to the corresponding concepts of classical complex analysis and potential theory in one variable. Then I will discuss disc envelope formulas and give examples of such formulas for pluricomplex Green functions.

**Wednesday 23 April 2008, Prof. Juliusz Brzezinski, Göteborgs universitet**

*Title: Diophantine problems, elliptic curves and computer experiments*

Abstract: Solutions of many diophantine problems like Fermat's Last Theorem or the problem of congruent numbers (natural numbers which are areas of right triangles with rational sides) depend on existence of rational points on particular elliptic curves. We discuss several more or less known situations in which diophantine problems lead to questions concerning existence of rational points on elliptic curves and when solutions can be found by sometimes very extensive computer based computations. We concentrate on two problems: finding (relatively) long non-trivial sequences of integers for which second differences of the squares are constant (e.g. 6, 23, 32, 39) - a question closely related to Hilbert's X-th problem, and on a "teacher's problem" related to elementary Linear Algebra courses.

**Wednesday 7 May 2008, Prof. Alexander Nazarov, University of St. Petersburg**

*Title: The A.D. Aleksandrov maximum principle.*

Abstract: The talk is devoted to an excellent geometrical tool in nonlinear PDEs - the Aleksandrov maximum principle. I shall give a historical review and prove the simplest version of this statement.

**Monday 12 May 2008, Prof John McNamara, Bristol University (joint with Senior Lectures in Biology)**

*The importance of individual differences in conflict and the evolution of cooperation*

Abstract: Animals are often in competition with other members of the same population. They compete over access to resources such as food, mates and breeding sites. Even parents compete with each other over who should provide care for their common young. When there is competition the fitness of one member of the population usually depends on the behavioural strategies adopted by others. In such circumstances the evolutionary endpoints can be characterised using evolutionary game theory. I first present a brief outline of this theory as it is usually formulated and used. However, uses often ignore differences between individuals. Using a series of examples I will demonstrate that such differences are not innocuous noise, but can fundamentally change the nature of a game. Differences promote the need to negotiate. Difference can completely reverse the direction of evolution in a simple prisoner's dilemma game, and can interact with lifespan to determine how cooperative individuals are with each other. Finally, differences in personality promote the need to be socially sensitive; and once individuals are socially sensitive, this can lead to the maintenance of differences.

**Wednesday 21 May 2008, Tomas Sjödin, KTH**

*Title: Selected topics on quadrature domains.*

Abstract: Quadrature domains arise naturally in many contexts where gravitational equivalence between mass distributions and similar situations are being considered. To name a couple of specific well-known examples we have the classical obstacle problem, whose solution is a quadrature domain, as well as the important Hele-Shaw flow from fluid mechanics. In this talk, which aims to be as self-contained as possible, we will give the basic definitions and look at the most natural questions (and their answers where known) from a potential-theoretic point of view. We will also indicate how the methods developed to describe quadrature domains can be used to tackle other problems in potential theory.

**Wednesday 28 May 2008, Prof. Kalle Åström, Lunds universitet**

*Title: Solving systems of polynomial equations and geometric problems in computer vision*

Abstract: In this talk I will give examples of geometric problems in computer vision and present some of our latest results concerning (i) finding globally optimal solutions to structure and motion problems and (ii) new techniques for solving systems of polynomial equations. To exemplify these results I will use two simple examples of two view structure and motion and one dimensional retina vision. One-dimensional cameras have proven useful in several different applications, most prominently for autonomous guided vehicles, but also in ordinary vision for analysing planar motion and the projection of lines. Previous results on one-dimensional vision are limited to classifying and solving minimal cases, bundle adjustment for finding local minima to the structure and motion problem and linear algorithms based on algebraic cost functions. In this talk, we present a method for finding the global minimum to the structure and motion problem using the max norm of reprojection errors. We show how the optimal solution can be computed efficiently using simple linear programming techniques. The algorithms have been tested on a variety of different scenarios, both real and synthetic, with good performance. In the talk I will also discuss some new methods for stable and efficient algorithms for finding solutions to systems of polynomial equations.

**Wednesday 4 June 2008, Prof. Nikolay Kuznetsov, Russian Academy of Sciences, St. Petersburg**

*Title: The Benjamin--Lighthill conjecture for near-critical values of Bernoulli's constant*

Abstract: The so-called Benjamin--Lighthill conjecture will be discussed. The conjecture was formulated in 1954 for the classical nonlinear problem of steady two-dimensional waves on water of finite depth and concerns a possibility to characterise all such waves by means of two non-dimensional parameters located within a certain cusped region and on its boundary. Along with a survey of results obtained since 1954, a new approach (proposed in a joint work with Professor Vladimir Kozlov) to this conjecture will be presented. This approach leads to proving the conjecture for near-critical (that is, close to the cusp point) values of Bernoulli's constant.

**Wednesday 3 September 2008, Anders Björn, MAI**

*Title: Cluster sets for Sobolev and  $p$ -harmonic functions*

Abstract: Consider a Sobolev function  $u$  with boundary values  $f$  (in Sobolev sense) and assume that  $f$  is continuous at a boundary point  $x_0$ . It then turns out that the cluster set of  $u$  at  $x_0$  is connected and contains the boundary value  $f(x_0)$ , under suitable conditions. In this talk we study when this result holds in general metric spaces. We also apply the result to  $p$ -harmonic functions (and the more general quasiminimizers) where we are able to say a bit more.

**Wednesday 10 September 2008, Prof. Erik Larsson, ISY, Communication Systems**

*Title: MIMO Detection*

Abstract: In communications, the receiver often observes a linear superposition of separately transmitted information symbols. This is the case, for example, in so-called MIMO communications, where multiple transmit antennas simultaneously send different data streams. From the receiver's perspective, the problem is then to separate the transmitted symbols. Mathematically, this separation essentially amounts to solving many instances of an integer-constrained least-squares (ICLS) problem. This problem is known to be NP-hard. Therefore in practice, one must resort to approximate algorithms that find the correct solution quickly with high probability, and which are, in addition, suitable for efficient hardware implementation. In this talk we will explain why the problem is so important, and give an overview of current research on the topic.

**Monday 22 September 2008, Prof. Richard Schoen, Stanford University and Institut Mittag-Leffler**

*Title: Riemannian manifolds of positive curvature*

Abstract: In this lecture we will summarize what is known about the structure of Riemannian manifolds of positive sectional curvature, and describe our recent contribution to this problem concerning 1/4-pinched manifolds. The theorem is joint with Simon Brendle and is an application of Hamilton's Ricci flow.

**Wednesday 24 September 2008, Prof. Mikael Passare, Stockholms universitet**

*Titel: Euler, amöbor och tropisk geometri*

Sammanfattning: Jag tänker ge en introduktion till teorin för (matematiska) amöbor och koamöbor, och beskriva deras samband med reell, komplex och tropisk geometri. Som en liten bonus fås ett nytt elementärt bevis för Eulers berömda formel  $\sum 1/n^2 = \pi^2/6$ .

**Wednesday 8 October 2008, Prof. Ralf Fröberg, Stockholms universitet**

*Title: Counting paths in digraphs and Koszul algebras.*

Abstract. I will start with a concrete problem on how to calculate the number of paths of a certain length in a digraph. This will lead to a definition of Koszul algebras, which have been intensively studied in Stockholm. I will review some results about them.

**Wednesday 15 October 2008, Prof. Alan Rendall, Albert-Einstein-Institut, Golm**

*Title: Loss of regularity in solutions of the Einstein-Euler system.*

Abstract. It is well known that classical solutions of the Euler equations often lose regularity after finite time. Physically this corresponds to the formation of shock waves. In this talk I describe work by Fredrik Stahl and myself in which we study this phenomenon for a self-gravitating fluid in general relativity under the assumption of plane symmetry. The strategy is to first obtain control of the geometry and the energy density of the fluid. This shows that the process of breakdown of classical solutions of the Einstein-Euler system is sufficiently similar to the analogous process in flat space to conclude that breakdown must occur.

**Wednesday 29 October 2008, Jana Madjarova, Chalmers-Göteborg Universitet**

*Title: Art and Mathematics*

Abstract: Is mathematics the opposite of art? Or is it an art in itself? We can find traces of mathematics in many works of art but is that all the connection there is? Those who expect deep and difficult mathematics will be disappointed, but, you can look forward to an entertaining galopp along the common history of mathematics and art.

**Wednesday 5 November 2008, Tatyana Shaposhnikova, MAI**

*Title: Theory of Sobolev multipliers and their applications to differential and integral operators*

Abstract: The talk is a survey of the theory of pointwise multipliers in spaces of differentiable functions developed together with Vladimir Maz'ya. The following topics will be discussed: analytic characterization of multipliers; essential norm and compactness of multipliers; traces and extensions of multipliers; maximal subalgebras of multiplier spaces; miscellaneous properties of multipliers (composition and implicit function theorems, etc). I'll present some applications to differential and integral operators.

**Monday 10 November 2008, Prof. Ayse Humeyra Bilge, Istanbul Technical University**

*Title: On the classification of scalar integrable evolution equations in 1 space dimension*

**Wednesday 12 November 2008, Pavel Kurasov, Lunds universitet**

*Title: Triplet extensions of operators: Krein's resolvent formula and models for supersingular interactions*

Abstract: The extension theory for semibounded symmetric operators is generalized by including operators acting in a triplet of Hilbert spaces. We concentrate our attention on the case where the minimal operator is essentially self-adjoint in the basic Hilbert space and construct a family of its self-adjoint extensions inside the triplet. All such extensions can be described by certain boundary conditions and a natural counterpart of Krein's resolvent formula is obtained. This method is applied to Schrödinger operators with interaction given by operators acting in the scale of Sobolev spaces.

**Wednesday 26 November 2008, Danyo Danev, ISY**

*Title: A Family of Ternary Quasi-Perfect Codes*

Abstract: A linear  $q$ -ary  $[n, k]$ -code is a linear subspace of the Hamming space  $H(n, q)$  which is the set of all  $n$ -tuples of elements from the finite field  $F_q$  of cardinality  $q$ . In this talk we shall introduce the notions of minimum distance, packing radius and covering radius of a code. Some special types of codes, such as perfect, quasi-perfect and BCH codes will be defined. As a main result we shall present a novel family of ternary quasi-perfect BCH codes. These codes are of minimum distance 5 and covering radius 3. The first member of this family is the ternary quadratic-residue code of length 13.

**Wednesday 3 December 2008, Vitalij Tjatyрко, MAI**

*Title: On dimensional properties of subsets lying in the long band.*

Abstract: Some examples of closed subsets  $B$  of the product  $[0, \omega_c] \times [0, 1]$  having  $\dim B = 1$  and  $\text{ind } B = \text{Ind } B = 2$  will be presented. These examples answer to questions posed by different authors, in particular, by R. Engelking in the book, Theory of Dimensions Finite and Infinite, 1995.

**Wednesday 17 December 2008, Prof. Johan Tysk, Uppsala universitet**

*Title: Convexity in financial theory*

Abstract. Convexity for prices of financial derivatives is of importance for instance when hedging such instruments. We will consider methods from the theory of parabolic differential equations and stochastic processes to demonstrate such convexity for different types of instruments and various models for the underlying assets.

# The Mathematical Colloquia held in 2007 at Linköpings universitet

*Organized by Armen Asratian, Milagros Izquierdo Barrios, Vladimir Kozlov and Stefan Rauch-Wojciechowski.*

## Wednesday 17 January 2007, Douglas Rogers, University of Hawaii (Joint seminar with the didactic group)

*Title : Bounds Archimedes missed : exercises in geometric extrapolation.*

Abstract : Pi is a topic of abiding fascination that engages the interest of all mathematicians, pure and applied alike. We know, or think we know, that it was Archimedes who early calculated pi to considerable accuracy by bounding a circle inside and out by regular polygons. However, this program, with an explicit argument in the case of inscribed polygons, is already contained in Book XII of Euclid's Elements. Closer examination of the works of Euclid and of Archimedes suggests that everything you can do with inscribed and circumscribed polygons together can be done just as well with inscribed polygons alone. Moreover, it seems that the Chinese mathematician Liu Hui, working over seventeen hundred years ago, was able to improve the lower bound on the area of a circle by interpolation using only inscribed polygons. Perhaps even more surprisingly, whereas the combined work of Euclid and Archimedes shows that the difference between areas of circumscribed and inscribed polygons more than halves on doubling the number of sides of these polygons, an argument that would have been accessible to both of them, as well as to Liu Hui, shows that, in fact, it more than quarters. The talk is presented as an exercise in "mathematics from history", where we take the mathematics from a given period and see what (more) can be extracted by means of it alone. Thus, when we look back on this material from the later perspective of the calculus, we find that these geometric arguments remarkably powerful, giving results akin to Richardson-Romberg integration - the quartering inequality just mentioned is accurate up to the term in the sixth power of the reciprocal of the number of sides of the largest and smallest polygons. It seems that we - not just Archimedes - might have been missing something.

## Wednesday 24 January 2007, Prof. Olle Häggström, Mathematical Statistics, Chalmers (Joint seminar with Mathematical Statistics)

*Title : Problem solving is often a matter of cooking up an appropriate Markov chain.*

Abstract : By means of a series of examples, taken from classic contributions to probability as well as from my own practice, I will try to convince the audience of the claim made in the title of the talk. Along the way, I will have reason to discuss topics such as coupling, correlation inequalities, and percolation.

## Wednesday 31 January 2007, Prof. Armen Asratian, MAI

*Title: On local-global phenomena in graph theory*

Abstract: Interconnection between local and global properties of mathematical objects has always been a subject of investigations in different areas of mathematics. Usually by local properties of a mathematical object we mean its properties in balls with small radii.

In my talk I will give some examples of using balls for investigations of some global properties of graphs. In particular, I will discuss the local nature of some classical theorems on Hamilton cycles.

## Wednesday 7 February 2007, Prof. Stefan Rauch-Wojciechowski, MAI

*Title: The method of separation of variables and an effective criterion of separability.*

Abstract: The method of separation of variables is a very useful tool for finding solutions of differential equations. It consists of finding suitable new variables such that the problem splits into a set of uncoupled ODE's, often of 1-st order, that can be solved by quadratures. I shall discuss the meaning of separability for certain classes of ordinary differential equations and for partial differential equations.

For a given differential equation it is difficult to know if separation variables exist and to find them. For the Schrödinger equation and for the Hamilton-Jacobi equation of natural Hamiltonian systems, that are important in quantum and classical mechanics, we formulated an algorithmic criterion of separability that allows for deciding if the problem is separable and for solving equations by quadratures.

It is a solution of an old problem stated by C.G.J.Jacobi in his book "Vorlesungen über Dynamik"(1866) when he introduced the method of Hamilton-Jacobi equation for finding solutions of the Hamilton equations.

## Wednesday 21 February 2007, Prof. Lars Døvling Andersen, Institute of Mathematical Sciences, Aalborg

## Wednesday 28 February 2007, Prof. Emma Previato, Boston University and Mittag-leffler Institute

*Title: Algebraic curves with automorphisms*

Abstract: Curves with (extra) automorphisms are loci of interest in moduli spaces. We report on ongoing work (joint with T. Shaska) seeking defining equations for such loci in terms of theta-nulls, for small genus. This work uses properties of covers of algebraic curves. We then focus on covers of tori to determine some topological properties of the loci (joint work with R.D.M. Accola) and applications to coding theory (joint with D. Coles).

## Wednesday 7 March 2007, Prof. Antonio F Costa, UNED, Madrid

*Title: "On Hurwitz spaces"*

Abstract: Hurwitz spaces are spaces of pairs  $(S, f)$ , where  $S$  are Riemann surfaces and  $f : S \rightarrow \mathbb{CP}^1$  are meromorphic functions. These spaces appear in algebraic geometry and mathematical physics. We shall present several 1-dimensional examples of Hurwitz spaces.

## Wednesday 14 March 2007, Prof. Nikolay Kuznetsov, Russian Academy of Sciences, St. Petersburg

*Title: How the bottom topography can be recognized from the hovercraft motion*

Abstract: The idea of hovercraft or vehicle supported by air-cushion was proposed in 1716 by Emanuel Swedenborg (a Swedish scientist), but the first functional craft was designed and built only in 1931 by Toivo Kaario (a Finnish engineer). Several figures will illustrate history of this sophisticated vehicle.

The aim of talk is to consider a problem that describes waves on the water surface which are generated by a pressure system in the forward motion (a mathematical model of hovercraft). Some hydrodynamic corollaries that follow from analysis of this problem will be presented. One of them demonstrates that if the bottom topography is sufficiently simple, then information about it can be recovered from the behaviour of resistance (the horizontal component of the water's reaction) to a rapid acceleration of hovercraft.

## Wednesday 21 March 2007, Mats Boij, KTH, Stockholm

*Title: Parameter spaces of graded algebras*

Abstract: In algebraic geometry, geometrical objects like varieties or schemes are often studied in families which are again algebraic varieties or schemes. These families sometimes can be described by universal parameter spaces, as for the Hilbert scheme defined by Grothendieck. Even if the existence of such spaces can be proven abstractly, it is hard to get more detailed geometrical information about them. I will discuss such parameter spaces for graded algebras and will give examples where we can describe the component structure of these parameter spaces. Part of this is joint work with Anthony Iarrobino at Northeastern University in Boston.

## Wednesday 28 March 2007, Prof. Gerard van der Geer, Universiteit van Amsterdam and Mittag-Leffler Institute

*Title: The Schottky Problem*

Abstract: An algebraic curve determines an abelian variety, the Jacobian of the curve. For example, for a Riemann surface the Jacobian is a complex torus associated to the periods of integrals over the Riemann surface. Not every abelian variety is the Jacobian of a curve and the Schottky problem, due to Riemann, asks for a characterization of the Jacobians among all abelian varieties. Various answers have been proposed. We shall discuss the problem, its history and some of the proposed answers to this problem.

## Wednesday 25 April 2007, Olle Axling, MAI

*Titel: Arkimedes palimpsest.*

Sammanfattning: För 100 år sedan upptäckte filologen Heiberg att texten i en boken från 1229 var skriven över en bortskrapad text som bl.a. utgjorde sju av Arkimedes verk. Denna palimpsest var länge försvunnen men dök 1998 upp på Christies auktion i New York och köptes av en IT-företagare för 2 miljoner US dollar. En stor vetenskaplig kraftsamling håller nu på att konservera och tolka de 174 pergamentarkten efter donationen till Walter Arts Museum. Jag ska berätta vad jag vet om detta och om en del nytt man lärt om Arkimedes.

**Wednesday 16 May 2007, Veronica Crispin, KTH**

*Title: Ratliff-Rush Monomial Ideals*

Abstract: Let  $R$  be a Noetherian ring and let an ideal  $I$  in it be regular, that is, let  $I$  contain a nonzerodivisor. Then the ideals  $(I^{l+1} : I^l)$ ,  $l \geq 1$ , increase with  $l$ . The union  $\tilde{I} = \bigcup_{l \geq 1} (I^{l+1} : I^l)$  was first studied by Ratliff and Rush in [1]. They show that  $(\tilde{I})^l = I^l$  for sufficiently large  $l$  and that  $\tilde{I}$  is the largest ideal with this property. Hence,  $\tilde{\tilde{I}} = \tilde{I}$ .

Moreover, they show that  $\tilde{I}^l = I^l$  for sufficiently large  $l$ . We call  $\tilde{I}$  the Ratliff-Rush ideal associated to  $I$ , and an ideal such that  $\tilde{I} = I$  a Ratliff-Rush ideal. The Ratliff-Rush reduction number of  $I$  is defined as  $r(I) = \min \{l \in \mathbb{Z}_{\geq 0} \mid \tilde{I} = (I^{l+1} : I^l)\}$ . The operation  $\sim$  cannot be considered as a closure operation in the usual sense, since  $J \subseteq I$  does not generally imply  $\tilde{J} \subseteq \tilde{I}$ . An example from [2] shows this: let  $J = \langle y^4, xy^3, x^3y, x^4 \rangle \subset I = \langle y^3, x^3 \rangle \subset k[x, y]$ , then  $I$  is Ratliff-Rush but  $x^2y^2 \in \tilde{J} \setminus \tilde{I}$ .

One of the reasons to study Ratliff-Rush ideals is the following. Let  $I$  be a regular  $m$ -primary ideal in a local ring  $(R, m, k)$ . The Hilbert function  $H_I(l) = \dim_k(R/I^l)$  is a polynomial  $P_I(l)$  called the Hilbert polynomial of  $I$  for all large  $l$ . Then  $\tilde{I}$  can be defined as the unique largest ideal containing  $I$  and having the same Hilbert polynomial as  $I$ . Ratliff-Rush ideals associated to monomial ideals are monomial by definition, which makes the computations easier. There is always a positive integer  $L$  such that  $\tilde{I} = I^{L+1} : I^L$ , but it is not clear how big that  $L$  is (see Example~1.8 in [2]). If  $I$  is a monomial

ideal and  $m$  is some monomial, then for all  $l \geq 0$  we have

$$(mI)^{l+1} : (mI)^l = (m^{l+1}I^{l+1}) : (m^lI^l) = m(I^{l+1} : I^l) \quad (1).$$

Principal ideals are trivially Ratliff-Rush. Any non-principal monomial ideal  $J$  in the rings  $k[x, y]$  and  $k[[x, y]]$  can be written as  $J = mI$ , where  $m$  is a monomial and  $I$  is an  $\langle x, y \rangle$ -primary ideal; hence it suffices to consider  $\langle x, y \rangle$ -primary monomial ideals. Moreover, (1) shows that the Ratliff-Rush reduction numbers of  $I$  and  $mI$  are the same.

In this talk we show how to compute the Ratliff-Rush ideal associated to a monomial ideal in a certain class in the rings  $k[x, y]$  and  $k[[x, y]]$  and find an upper bound for the Ratliff-Rush reduction number for such an ideal. We start by giving some results about numerical semigroups that are crucial for our later work. We conclude by discussing several useful examples.

References

[1] L. J. Ratliff, Jr and D. E. Rush, \textit{Two Notes on Reductions of Ideals}, Indiana Univ. Math. J. **27** (1978), no. 6, 929-934.

[2] M. E. Rossi and I. Swanson, \textit{Notes on the Behavior of the Ratliff-Rush Filtration}, Commutative Algebra (Grenoble/Lyon, 2001), 313-328, Contemp. Math., 331, Amer. Math. Soc., Providence, RI, 2003.

**Wednesday 23 May 2007, Elena Kreines, Mittag-Leffler Institute**

*Title: Dessins d'enfants: Solving equations determining Belyi pairs*

Abstract: This paper deals with the Grothendieck dessins d'enfants, that is tamely embedded graphs on surfaces. Detailed introduction to the theory including its relations with other branches of mathematics will be given. We investigate combinatorics of systems of equations determining a Belyi pair corresponding to a dessin, that is a rational function with at most 3 critical values on an algebraic curve, such that the preimage of a segment between two critical values is the dessin under consideration. Several properties of extra, or so-called parasitic, solutions of such systems are described. Some special compactification of the moduli space  $\mathcal{M}_{g,n}$ , related to the dessins, will be discussed.

**Wednesday 30 May 2007, Anna Torstensson, KTH**

*Title: Class numbers of finite groups*

Abstract: In this talk I would like to adress two basic problems concering class numbers (that is the number of conjugacy classes) of finite groups.

1) Given a class number  $k=k(G)$ , which is the largest finite group  $G$  having this class number?

2) Find a (good) upper bound on  $|G|$  in terms of the class number of  $G$ .

In the first part of the talk I will describe some of the work that has been done to answer those questions both generally, and for particular types of groups such as  $p$ -groups and simple groups. In the second part I will describe my own results concering groups of square free order. They include a formula for the class number of any such group, and also an estimate  $|G| \leq k(G)^3$ . Finally I want to describe some ideas I have for future work in this area.

Most of the results I will present can also be found in a preprint available at my home page: <http://www.math.kth.se/~annator/>

**Friday 31 Augusti 2007, Yoshihiro MIZUTA, Hiroshima University**

*Title: Lebesgue point theory for Riesz potentials of Orlicz functions*

Abstract: Lebesgue point theory implies the mean continuity for locally integrable functions. For a locally integrable function  $u$  on  $\mathbf{R}^n$ , a point  $x \in \mathbf{R}^n$  is called a Lebesgue point of  $u$  if

$$\lim_{r \rightarrow 0} \frac{1}{|B(x, r)|} \int_{B(x, r)} |u(y) - u(x)| \, dy = 0.$$

It is well know that this is true for almost every  $x \in \mathbf{R}^n$ . A locally integrable function  $u$  on  $\mathbf{R}^n$  is called a Sobolev function in  $W^{1,p}(\mathbf{R}^n)$  if its first derivatives belong to  $L^p(\mathbf{R}^n)$ . Through integral representations for Sobolev functions, we are concerned with Riesz potentials, which are defined by

$$I_\alpha f(x) = \int_{\mathbf{R}^n} |x - y|^{\alpha-n} f(y) \, dy.$$

Let  $f$  satisfy

$$\int_{\mathbf{R}^n} |f(y)|^p (\log(e + |f(y)|))^{\delta} < \infty$$

for  $p > 1$  and a real number  $\delta$ . Then we know that  $I_\alpha f$  is continuous when  $\alpha p > n$  or when  $\alpha p = n$  and  $\delta > p - 1$ . To extend this, we consider

$$\int_{\mathbf{R}^n} |f(y)|^p \varphi(|f(y)|) \, dy < \infty,$$

where  $\varphi$  is a nonnegative and monotone function on the interval  $[0, \infty)$ . We first show that  $I_\alpha f$  is continuous on  $\mathbf{R}^n$  if

$$\int_0^1 \{r^{n-\alpha p} \varphi(r^{-1})\}^{-1/(p-1)} \frac{dr}{r} < \infty$$

If this condition does not hold, then we discuss Lebesgue point theory for  $I_\alpha f$ . To evaluate the size of exceptional sets, we use the notion of capacity.

**Wednesday 5 September 2007, Gunnar Aronsson, MAI**

*Title: Interpolation of real functions under a gradient bound -- uniqueness aspects.*

Abstract: We consider the interpolation of given boundary values into a bounded domain in euclidean  $n$ -space, under the side condition that the interpolating function should be differentiable a.e. and the gradient must stay below a given pointwise bound. It is well known that this is possible, provided that the pointwise bound satisfies a consistency condition with the boundary values. The solution is in general not unique. The solution may happen to be unique on a subset of the basic domain, where it then has some extra smoothness. The set of uniqueness also has a particular structure. Infimal convolutions and semi-concave functions play important roles in this theory.

**Wednesday 19 September 2007, Petter Brändén, KTH. Joint Colloquium with Mathematical Statistics**

*Title: Negative Dependence and the Geometry of Polynomials.*

Abstract: We develop a theory of negative dependence for the class of *strongly Rayleigh* probability measures. This class is defined by means of geometric properties of the generating polynomials of the measures, and contains uniform random spanning tree measures, determinantal measures (for contractions) and distributions for symmetric exclusion processes. In the process we settle several conjectures of Liggett, Pemantle and Wagner, respectively, and extend Lyons' recent results on determinantal measures.

This is joint work with Julius Borcea (SU) and Thomas M. Liggett (UCLA), ArXiv: 0707.2340.

**Wednesday 26 September 2007, Marta Sanz-Solé, Universitat de Barcelona**

*Title: A class of stochastic partial differential equations driven by a fractal noise.*

Abstract: We will consider a class of non autonomous, parabolic stochastic partial differential equations, defined on a bounded domain of  $\mathbb{R}^d$ , driven by an infinite-dimensional multiplicative fractional noise. We will introduce two types of solutions -weak and mild- and give results on the existence, uniqueness, indistinguishability and Hölder continuity of the solutions.

The presentation aims to be self-contained in order to be understandable to non specialists.



### Wednesday 3 October 2007, Martin Hessler, MAI

*Title : Standard methods in combinatorial optimization and their relation to the PWIT-model*

Abstract: Results in combinatorial optimization have been achieved through a number of different methods based upon ideas from different fields of science. Relating these methods to each other has proven fruitful in furthering our understanding of the field of combinatorial optimization. In this presentation we will predominantly discuss the problem of finding the expected cost of a minimal matching in a graph with edge costs given by exponential random variables. The "Poisson weighted infinite tree"-model translates the discrete structure of an infinite graph into a system of distributional equations. Although the model is based upon ideas from statistical mechanics, one can observe basic ideas and properties from the finite case, some which survive and some which do not survive intact.

The aim of the presentation is to give any listener with basic knowledge of graphs and exponential random variables a feel for the structure of combinatorial optimization. Note that the PWIT-model was briefly covered in the context of statistical mechanics by Johan Wästlund in a previous colloquium.

### Wednesday 10 October 2007 Hans Ringström, KTH

*Title: Future global non-linear stability of cosmological models with accelerated expansion.*

Abstract: The Lorentz manifolds that are currently used by physicists to model the universe are ones in which the expansion is accelerated. Furthermore, the models are usually required to be spatially homogeneous and isotropic, so that the only freedom left is a scale factor. Since our universe is not exactly spatially homogeneous and isotropic, it is of interest to prove that these models are non-linearly stable to the future, and this is the subject of the talk. From a mathematical point of view, the essential problem is that of proving future global existence of solutions to a non-linear hyperbolic PDE.

### Friday 12 October 2007, Genkai Zhang, Chalmers

*Titel: Fundamental solutions for a class of degenerate  $p$ -Laplacian operators on  $H$ -type groups*

Abstract: We introduce a class of vector fields on Heisenberg type groups depending on a parameter  $k$ . For  $k = 2$  they are studied by Greiner for Heisenberg groups; for non-integral  $k$  they do not satisfy the Hörmander condition. We compute the fundamental solution of the degenerate  $p$ -Laplacian.

### Wednesday 17 October 2007, Magnus Herberthson, MAI

*Titel: The potential method for calculations of radar cross sections.*

Abstract: Under some natural conditions, the problem of determining the radar cross section of a body with boundary surface  $S$  results in an vector valued integral equation over  $S$ , which involves only quantities tangential to  $S$ . I will try to describe how this can be exploited.

### Wednesday 24 October 2007, Eulerfest

See <http://www.mai.liu.se/~miizq/Euler/Eulerfest.html>

### Wednesday 7 November 2007, Prof. Sergio Benenti, University of Turin

*Title. Evergreen Topics in Analytical Mechanics*

Abstract. In the last years, many researchers have re-opened, independently and in different sites, a number of classical chapters of Analytical Mechanics. This means that not only old problems have been solved by modern techniques but also that new discoveries have been recognized to have their deep roots in ancient papers of our Masters. An example is that of a paper by young Levi-Civita (1894), dealing with the geodesic equivalence theory, which has been recognized to be the natural background for dealing with a recently discovered class of dynamical systems. In this talk I will briefly illustrate this and other connected cases, concerning the Hamilton-Jacobi separation theory, cofactor and bi-cofactor systems, and non-holonomic systems.

### Wednesday 14 November 2007, Prof. Peter Sjögren, Göteborgs universitet

*Title. Gaussisk harmoniska analys - en översikt*

Abstract. I det euklidiska rummet ersätter man Lebesguemåttet med ett gaussmått. Då finns motsvarigheter till Laplaceoperatoren, värmeledningshalvgruppen mm. Vi skall studera maxalfunktioner och singulara integraler i denna situation.

### Wednesday 21 November 2007, Jens Jonasson, MAI

*Title. The Equation  $\nabla f = M \nabla g$*

Abstract. P. J. Olver & M. Jodeit studied the equation  $\nabla f = M \nabla g$  for any constant matrix  $M$ , and in a paper from 1990 they gave its general analytic solution. This equation is a special case of the class of linear homogeneous systems of PDEs admitting a  $*$ -multiplication of solutions, which is considered in my thesis. The  $*$ -multiplication is a bilinear operation on the solution space that allows for algebraic construction of new solutions. I have proved that the equation  $\nabla f = M \nabla g$  has the remarkable property that any analytic solution can be expressed as a convergent  $*$ -power series of a simple solution, in a similar way as for the well known Cauchy--Riemann equations.

### Wednesday 28 November 2007 No seminar

### Tuesday 4 December 2007, Relativity Day

See <http://www.mai.liu.se/~gober/GRmeetingDec07.html>

### Wednesday 12 December 2007 Johan Thim, MAI

*Title. A Fixed Point Theorem in Locally Convex Spaces*

Abstract. For a locally convex space  $\mathbf{X}$  with the topology given by a family  $\{p(\cdot; \alpha)\}_{\alpha \in \Omega}$  of seminorms, we consider the existence and uniqueness of a fixed point for a mapping  $\mathbf{K}:\mathbf{D_K} \rightarrow \mathbf{D_K}$  on some set  $\mathbf{D_K} \subset \mathbf{X}$ . We require that there exists a linear and positive operator  $-K$ , acting on functions defined on the index set  $\Omega$ , such that

$$p(\mathbf{K}(u) - \mathbf{K}(v); \alpha) \leq K(p(u - v; \cdot))(\alpha), \quad \alpha \in \Omega.$$

Under some additional assumptions, one of which is the existence of a fixed point for the operator  $K + p(K(0); \cdot)$ , we prove that there exists a fixed point of  $\mathbf{K}$ . For the class of functions such that  $K^n(p(u; \cdot))(\alpha) \rightarrow 0$  as  $n \rightarrow \infty$ , we show that fixed points are unique. These theorems are a generalisation of Banach's contraction principle in Banach spaces. We consider several applications by proving the existence and uniqueness of solutions to first and second order perturbed differential equations in Banach spaces.

### Monday 17 December 2007, Prof. Grzegorz Gromadzki, University of Gdansk

*Title. Fixed points on Riemann surfaces*

Abstract: (1) By a symmetry of a Riemann surface  $X$  we understand an antiholomorphic involutions of  $X$ . The classical Harnack theorem asserts that the set of fixed points of a single symmetry of a Riemann surface of genus  $g$  consist in a disjoint sum of at most  $g + 1$  subsets each of which is homeomorphic to a circle and is called oval. Here we deal with the estimates for the total number of ovals of  $k$  nonconjugate symmetries of a Riemann surface of genus  $g$ .

(2) We show that the set  $\text{Fix}(X)$  of fixed points of a Riemann surface  $X$  of genus  $g$  has at most  $82(g - 1)$  elements. This set is obviously invariant with respect to the action of the group  $\text{Aut}(X)$  of holomorphic automorphisms of  $X$  and we study the corresponding representation of  $\text{Aut}(X)$  in the symmetric group. Later we deal with the bounds for the total number of fixed points of  $k$  holomorphic involutions paying special attention to  $k = 2$ .

(3) An automorphism  $\varphi$  of order  $p$  of a Riemann surface  $X$  of genus  $g$  is said to be  $\{\text{it cyclic } p\text{-gonal}\}$  if its orbit space  $X/\varphi$  is the Riemann sphere. It can be characterized as an automorphism of order  $p$  having the maximal possible number of fixed points. The classical Castelnuovo-Severi Theorem implies that for  $g > (p - 1)^2$  the group generated by  $\varphi$  is unique and here we deal with the case  $g \leq (p - 1)^2$ .

(4) By a  $\{\text{it nonorientable Riemann surface}\}$  we mean a nonorientable unbordered topological surface equipped with a dianalytic structure which roughly speaking differs from the classical analytic structure by the fact that reflections  $x + iy \mapsto x - iy$  for the transition functions of charts are allowed. Such surfaces correspond to so called purely imaginary real algebraic curves and we give a formula for the topological type of the set of fixed points of a given automorphism of such surface.

### Tuesday 18 December 2007, Prof. Per Enflo, Kent State University

*Title. Problems in Operator Theory*

**Wednesday 19 December 2007, Prof. Johan Håstad, KTH**

*Title. Verifying proofs by reading only 3 bits.*

Abstract: Probabilistically Checkable Proofs or more succinctly PCPs have played a significant role in complexity theory in the last decade. A PCP is a written proof that is verified by a probabilistic verifier that reads a very small portion of the proof. Not only are PCPs interesting in their own right but they also lead to strong inapproximability results for interesting optimization problems.

As a concrete example take satisfiability of Boolean formulas. A classical NP-proof that a formula is satisfiable is given by an assignment that satisfies the formula and this is verified by reading the entire proof and checking that indeed the assignment satisfies the formula.

The PCP-theorem says that for satisfiability and hence for any NP-statement, there is a PCP that allows proofs of polynomial size and such that the verifier reads a constant number of bits, always accepts a correct proof and rejects any proof of a false NP-statement with probability at least  $1/2$ .

In the application to inapproximability it is important to optimize some of the parameters of the PCP and in particular we will be interested in proofs where the verifier only reads three bits.

In the lecture we will explain, but not prove the PCP-theorem and discuss the connection to inapproximability.

# The Mathematical Colloquia held in 2006 at Linköpings universitet

Organized by Armen Asratian, Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov Svante Linusson and Stefan Rauch-Wojciechowski.

## 2 Wednesday 11 Januari 2006 Natan Krugljak, Luleå Universitet

*Open Property of Invertibility of Operators on Interpolation Scales*

Abstract:

A remarkable theorem of Shneiberg extended to complex interpolation scalee the well-known fact that the set of bounded invertible operators acting on a Banach space is open in the space of all operators. After that this property has been intensively studied in the literature. Recently in joint paper with M.Milman we derived a unified approach to Shneiberg's theorem that works for complex, real and many others methods of interpolation. I plan to discuss known results and their proofs, give some examples and formulate several problems and conjectures.

## 2 Wednesday 18 Januari 2006. Sten Kaijser, Uppsala universitet

*An orthogonal polynomial basis for a Hilbert space of analytic functions in a strip*

Abstract.

A few years ago I amused myself by calculating the orthogonal polynomials with respect to the weight  $\frac{1}{2 \cosh \frac{\pi x}{2}}$ . In my calculations I used some simple operators, and these operators more or less forced me to investigate another system of polynomials that turned out to be orthogonal with respect to the Poisson kernel for a strip. Both these sets turned out to have very simple generating functions and also other interesting properties, in particular they were well adapted to the so called *umbral calculus*, developed by Gian-Carlo Rota and his students. This connection was found by my graduate student Tsehay K. Araaya and was developed in his thesis which was presented in June 2003. I will also present some new results concerning these polynomials.

## 2 Wednesday 25 Januari 2006. Hans Lundmark, MAI

*Peakons and shock waves in the Degasperis-Procesi equation*

Abstract:

The Degasperis-Procesi (DP) equation is a nonlinear wave equation which was discovered by Degasperis and Procesi in 1998 when searching for completely integrable cases in a family of equations similar in form to the famous Camassa-Holm (CH) equation derived as a model for shallow water waves in 1993. Both these equations admit a special kind of solution consisting of a superposition of so-called peakons (peaked solitons) interacting in a nonlinear but explicitly describable fashion. A peakon is a wave shaped like  $\exp(-|x|)$ , so that its derivative has a jump discontinuity at the crest, and consequently these solutions have to be interpreted in a weak sense. The CH equation has attracted a lot of attention because of these peaked waves, which are not possible in the KdV shallow water equation, for example. Now it turns out that the DP equation admits even weaker solutions, where the wave profile itself is discontinuous. Such shock waves are usually studied in the context of hyperbolic conservation laws, where it is well known that additional so-called entropy conditions must be imposed in order for the weak solution to be unique. How this works for the DP equation has recently been sorted out by Coclite and Karlsen. In this talk I will explain the explicit formulas which describe the peakon interactions (and how to derive them), why shock waves are formed at peakon-antipeakon collisions in the DP case (but not in the CH case), and how to construct concrete examples of entropy weak solutions by superposition of something I call shock-peakons. No particular prerequisites will be assumed.

## 2 Tuesday 31 January 2006. Linus Carlsson, Umeå Universitet

*Holomorfiområden i flera komplexa variabler*

Abstract:

Holomorfiområden i flera komplexa variabler skiljer sig avsevärt från holomorfiområden i en komplex variabel eftersom dessa alltid är triviala om grundmängden är öppen. Vi kommer att påvisa skillnaden mellan dessa. Vi kommer även att prata om holomorfiområden för begränsade holomorfa funktioner och avsluta med att visa ett tillräckligt villkor för att ett område ska vara ett holomorfiområde för begränsade funktioner.

## 2 Wednesday 1 February 2006. John Noble, MAI

*The Directed Polymer in a Random Environment*

Abstract: Consider a random walk in  $Z$ , where each step is taken to the left or right, each with probability  $1/2$ , independently of the other steps. After  $t$  steps, the mean squared displacement is  $t$ . Now suppose that each site in  $Z_+ \times Z$ , there corresponds a random weight  $V(i, x)$ . These weights are independent, identically distributed. We associate a random energy to each path as the sum of the energies of the sites visited. This corresponds to the energy of each bond in the polymer chain. Now weight the probabilities corresponding to each path according to the energy associated with the path. Numerical simulations show that, under this change of measure, the mean squared displacement is no longer  $t$ , but is instead superdiffusive, growing according to  $t^{4/3}$ . We discuss a continuous space time analogue of this problem, where the superdiffusive exponent of  $4/3$  can be shown explicitly. The proof relates the mean squared displacement problem to the two traveller problem; if two travellers take independent random walks on the same environment, how quickly do they separate? Unfortunately, while the continuous model under discussion preserves the mean squared displacement, the low temperature phase (another very important aspect of the problem) has been eliminated.

## 2 Wednesday 8 February 2006, Lars Ingelstam, LiU

*Between pure art and pragmatic use: on the place of mathematics in the Systems society.*

Abstract:

The first step will be to identify mathematics as one of several "knowledge cultures" with reference to aspects such as usefulness, scientific principles and self-image. This is particularly important in relation to the teaching of mathematics in schools as well as in technical universities. As a second step I will discuss where the center of gravity in mathematics teaching should lie. Alternatives advocated in the debate are analysis and computations. Another point of departure could be systems mathematics. This will be discussed at some length in the seminar. In the seminar I will use and elaborate ideas from my two recent books:

System - att tänka över samhälle och teknik. Energimyndighetens förlag, Eskilstuna 2002 (finns även som CD-rom)

Kampen om kunskapen. Lärarförbundets förlag, Stockholm 2004.

Short biography of the speaker:

Lars Ingelstam was professor of Technology and social change in the Tema Institute, Linköping University, 1980-2002, and is now author and researcher based in Bromma. 1973-1980 he was Director of the Secretariat for Futures Studies in Stockholm. He has a Dr Tech in mathematics (1964) and was docent and Associate Professor of Mathematics at KTH from 1964 to 1973.

## 2 Wednesday 15 February 2006, F. Calogero, University of Rome "La Sapienza", Italy

*The transition from regular to irregular motions, explained as travel on Riemann surfaces.*

Abstract:

We introduce and discuss a simple Hamiltonian dynamical system, interpretable as a 3-body problem in the (complex) plane and providing the prototype of a mechanism explaining the transition from regular to irregular motions as travel on Riemann surfaces. The interest of this phenomenology -- illustrating a mechanism for the onset in a deterministic context

of irregular motions -- is underlined by its generality, suggesting its eventual relevance to understand natural phenomena and experimental investigations. This is joint work with David Gomez-Ullate, Paolo Santini and Matteo Sommacal.

**2 Wednesday 8 March 2006, Milagros Izquierdo, MAI**

*Art and Mathematics: The Möbius Band*

**Abstract**

A Möbius band is formed by identifying the short sides of a rectangle after giving a half turn to one of them. Or equivalently turning upside down a partiture in music.

The Möbius band has inspired many artists during the XXth century. We can use the Möbius band to illustrate how mathematical concepts help us to "see" art and how art helps us to illustrate mathematical concepts.

**2 Wednesday 15 March 2006, Jan Snellman, MAI**

*Walks in directed graphs*

**Abstract**

Many problems in combinatorics can be formulated as follows: determine the number of walks (of a certain length) in a certain digraph (directed graph). Furthermore, since such walks can be encoded as non-commutative monomials, there is also a close connection to the theory of Hilbert series of non-commutative graded rings.

I will describe the well-known "transfer matrix method" for finding the (rational) length generating function of walks in the digraph. I will also discuss a formula by Fröberg, Bruns-Vetter, and others, relating the generating function for a graph  $G$  with the g.f. of its complement  $\bar{G}$ . Together with a theorem of Backelin, classifying graphs with a fixed number of edges having a maximal number of walks of length 2, these results will enable us to answer the following question: what is the maximal spectral radius (i.e., the pole closest to the origin of the rational g.f.) of a digraph with  $n^2$ -s edges?

**2 Wednesday 22 March 2006, Nils Dencker, Lunds universitet**

*Solvability and the Nirenberg-Treves Conjecture*

**Abstract**

In the 50's, Ehrenpreis and Malgrange proved that all constant coefficient linear partial differential equations are solvable. The consensus at that time was that at least all linear PDE's were solvable. Therefore, it came as a surprise when Hans Lewy in 1957 constructed a non-solvable complex vector field, whose image is a set of the first category. The vector field is a natural one; it is the Cauchy-Riemann operator on the boundary of a strictly pseudo-convex domain.

A rapid development in the 60's lead to the conjecture by Nirenberg and Treves in 1969: that condition (y) is necessary and sufficient for solvability of (pseudo-)differential operators of principal type. This is a condition only on sign changes of the imaginary part of the principal symbol along the bicharacteristics of the real part. Thus, it only depends on the highest order term of the operator.

The Nirenberg-Treves conjecture has recently been resolved, and we shall present the background and the ideas of the proof, which will appear in Annals of Mathematics, 163:2, 2006.

**2 Wednesday 29 March 2006, Jana Björn, MAI**

*Maximal functions,  $A_p$ -conditions and Poincare inequalities*

**Abstract**

I will show how these three notions are connected and discuss relations between them. The talk will be rather elementary and does not require any preliminary knowledge of the topic.

**2 Wednesday 5 April 2006, Carles Broto, Universitat Autònoma de Barcelona**

*Local properties of finite groups and classifying spaces*

**Abstract**

I will explain the group theorist point of view of local properties of finite groups, the parallel development of localization and completion of classifying spaces in homotopy theory and how recent developments of algebraic topology led to the definition of new objects, called  $p$ -local finite groups, that unify both points of view.

**2 Wednesday 26 April 2006, Kathryn Hess, EPF Lausanne**

*A gentle introduction to a powerful tool: operads and their bimodules*

**Abstract**

Operads were developed in the 1970's as a powerful tool for describing operations and the identities they must satisfy in different types of algebras, such as associative algebras, commutative algebras, Lie algebras, Poisson algebras, etc. Recently, Paul-Eugène Parent, Jonathan Scott and I realized that bimodules over operads play a similar role in describing highly structured homomorphisms between different types of algebras.

In this talk I will first define operads and explain how they parametrize algebraic operations. I will then outline our new understanding of the role of bimodules over operads.

**2 Wednesday 3 May 2006, Christer Bergsten, MAI (Joint seminar with the didactic group)**

*The role of algebra in reasoning about limits*

**Abstract**

The role of algebra in students' mathematical reasoning about limits of functions is analysed, using data from a video study of six students working in pairs to solve problems on limits. It is argued that algebra is at the same time a key and a lock to reach the limit in these problems. This double effect is related to the mathematical organisation taught, and if the students' sense of authority is internal or external.

**2 Wednesday 10 May 2006, Douglas Rogers, University of Hawaii (Joint seminar with the didactic group)**

*Dissecting the Pythagorean proposition*

**Abstract**

Euclide gives the Pythagorean proposition a double take, in Book I of the Elements in terms of congruent triangles, and in Book VI in terms of Eudoxos' doctrine of proportionality. However, it has been suggested that the prototypical proof was rather by dissection. I will focus on proofs by dissections, in particular after the manner of the ancient Chinese mathematical commentators, notably Lui Hui.

**2 Wednesday 17 May 2006, José M. M. Senovilla, University of the Basque Country**

*Riemannian vs. semiriemannian geometry*

**Abstract**

Semi-Riemannian geometry is a largely unexplored mathematical world, specially comparing with its older sibling branch, Riemannian geometry, much more developed and better understood. In this talk I will present, by means of pertinent examples of mathematical interest, several underdeveloped routes within semi-Riemannian geometry which are surely very appealing to mathematicians and physicist alike and have a direct relevance to gravitational and other physical theories as well as to pure differential geometry.

**2 Tuesday 23 May 2006, Hans Thunberg, KTH (Joint seminar with the didactic group)**

*Matematik från gymnasium till högskola. Gamla problem och pågående reformer.*

**Abstract**

I en undersökning gjord vid KTH Matematik har vi velat studera den välkända övergångsproblematiken i matematik ur ett perspektiv där vi jämför gymnasieskolans mål och ambitioner med den tekniska högskolans förväntningar och förkunskapskrav. En slutsats man kan dra är att problemet till stor del är strukturellt. De särskilda behörighetskraven i matematik till civilingenjörsutbildningarna har sänkts i flera avseenden under de senaste tio åren, i många fall utan motsvarande reformering av högskolans matematikkurser, och gymnasieskolans agenda i matematik har förändrats successivt på ett för vidare matematikstudier inte alltid gynnsamt sätt.

Det finns flera klart definierade stoffområden som högskolan förväntar sig som förkunskaper som antingen inte ingår i gymnasiets kurser överhuvudtaget, eller som behandlas med helt andra förtecken och kunskapsmål än vad högskolan tycks föreställa sig. Man iakttar också en skild syn på vad matematisk kunskap är. Det gäller bl a synen på räknefärdighet och formelkunskap - är detta ytliga svårigheter som hamnar matematisk förståelse och därför bör tonas ner och undanröjas med räknehjälpmiddel och formelsamlingar eller handlar det tvärtom om omistliga komponenter utan vilka ett större och djupare matematiskt kunnande blir omöjligt?

Medvetenheten om dessa problem verkar i dag vara stor. Seminariet avslutas med en diskussion kring aktuella satsningar och reformer.

**Referenser:**

\* Thunberg, Filipsson och Cronhjort. "Gymnasiets mål och högskolans förväntningar". Nämnaren Årgång 33 (2006) nr 2, sid 10 - 15.

## 2 Wednesday 31 May 2006, Peter Rand, MAI

*Asymptotic analysis for large  $t$  of solutions to parabolic systems*

Abstract

Title: Asymptotic analysis for large  $t$  of solutions to parabolic systems

Abstract: We study asymptotics as  $t \rightarrow \infty$  of solutions to the parabolic system of the form  $u_t + Lu = 0$  in  $\Omega \times [0, \infty)$ , where  $\Omega$  is a bounded domain. On  $\partial\Omega$  we prescribe the homogeneous Dirichlet boundary condition. The elliptic part  $L$  stabilizes for large  $t$  in a certain integral sense, i.e. the leading coefficients, which depend on  $t$ , tend to coefficients independent of  $t$  in integral sense (not necessarily in  $L^\infty$ -norm). In particular, we include such situations when leading coefficients may take different values on different parts of  $\Omega$  and the boundaries between them can move with  $t$  but stabilize as  $t \rightarrow \infty$ . Under these conditions we derive an asymptotic representation of solutions for large  $t$ .

## 2 Wednesday 7 June 2006, No seminar

## 2 14-15 June 2006 Geometry and Relativity Meeting.

## 2 Wednesday 13 September 2006, Johan Wästlund, MAI

*Statistical mechanics and combinatorial optimization.*

Abstract

Recently I obtained a proof of the following theorem: If the edges of the complete graph on  $n$  vertices are assigned independent lengths from uniform distribution on  $[0, 1]$ , then the length of the minimum travelling salesman tour converges in probability (as  $n \rightarrow \infty$ ) to

$$\frac{1}{2} \int_0^\infty y(x) dx \approx 2.0415,$$

where  $y > 0$  is defined as a function of  $x$  by the equation

$$\left(1 + \frac{x}{2}\right)e^{-x} + \left(1 + \frac{y}{2}\right)e^{-y} = 1.$$

In the 1980's, using non-rigorous methods of statistical mechanics, the physicists W. Krauth, M. Mézard and G. Parisi had conjectured a limit based on the solution to a certain integral equation. Inspired by my result, Parisi has recently shown that this integral equation has a unique solution, and that the statistical mechanics prediction of the "ground state energy" agrees with my rigorous result. I will describe the statistical mechanics approach and Parisi's recent calculation that leads to the integral above. If time permits, I will also briefly discuss how to make the result rigorous.

## 2 Wednesday 20 September 2006, Nikolay Kuznetsov Russian Academy of Sciences, St. Petersburg

*Two-dimensional steady waves on water of finite depth: modified Bernoulli's equation and its applications*

Abstract:

The nonlinear two-dimensional problem of arbitrary bounded steady waves on water of finite depth is considered, and a new setting of this classical problem is proposed. For this purpose averaging procedure is applied to the velocity potential over vertical cross-sections of the water domain, which leads to modified Bernoulli's equation. The latter involves the difference between the potential and its average along with the free surface elevation. Several applications of new equation are presented. First, necessary conditions for the existence of non-trivial solutions to the general steady-wave problem are obtained. (Earlier, these conditions, that have the form of bounds on the Bernoulli constant and other wave characteristics, were established only for the particular problem concerning simplest periodic waves known as Stokes waves.) Second, the exact upper bound, that must hold only at infinity thus guaranteeing a specific asymptotic behaviour of waves there, is found. Third, a new integral property of arbitrary steady waves is obtained.

The talk is based on the results obtained in the framework of a joint research project with Vladimir Kozlov.

## 2 Wednesday 27 September 2006, Alfonso García-Parrado, MAI

*Title: Causal Structures: a new viewpoint*

Abstract: The notion of causal structure has been used in general relativity most of the times in connection with "conformal structure" or "conformal equivalence". Although these concepts of causal structure have proven themselves quite useful, there are cases in which two spacetimes have similar causal properties but no conformal relation between each other exists. In this work we elaborate on the definition of causal structure arising from the concept of causal relationship which was introduced three years ago. This is a generalization of conformal equivalence and it can be applied to a wide range of examples in a straightforward way. Causal structures can be sorted by means of a partial order and we show in which sense this ordering generalises the classification of spacetimes according to the "standard hierarchy of causality conditions". We also put forward the concept of stability and instability of the causal structure and explain how it can be applied to examples as relevant as Minkowski or de Sitter spacetimes. We also study how our methods enable us to give conditions under which two Mp-waves (generalization of pp-waves) have the same causal structure.

## 2 Wednesday 4 October 2006, Ove Kägesten, ITN (joint seminar with the didactic group)

*Title: Kan studenter lära sig matematik genom att skriva och tala matematik*

Abstract. På seminariet kommer jag att förmedla erfarenheterna av två projekt finansierade av Nylng-projektet respektive Rådet för högre utbildning. I det ena fallet studerade vi hur studenterna påverkas då de tvingas skriva mera förklarande texter etc i samband med tentamen i flervariabelanalys. I det andra studerade vi hur miljön skall utformas för att det skall skapas ett lärande då studenterna genomför muntliga presentationer i matematik.

## 2 Wednesday 11 October 2006, Anders Haraldsson, IDA (joint seminar with the didactic group)

*Title: Inledande programmering och matematik*

Abstract. I nästa alla år (åtminstone 20 år bakåt) har vi i Linköping på dataprogrammen (D, C och IT) samt även Y (lite senare) börjat den inledande programmeringen med den funktionella programmeringsparadigmen med användning av Lisp/Scheme. Parallellt för D och C har alltid den diskreta matematikkursen gått. Med åren har det utvecklats mer och mer kontakter mellan dessa kurser, som jag tänkte berätta om och diskutera med er. En annan punkt som jag gärna talar om är den nya gymnasieskolan där man på Naturvetenskapliga programmet kommer att ha en inriktning Matematik och datavetenskap, där jag tror vi har uppgiften att vidareutveckla lärare för att, som jag vill se det, ämnesmässigt utveckla läran i "datavetenskap" och t ex att koppla matematik till datavetenskapen, dvs skolans datakurser. Tyvärr har man i denna stora reform inte haft någon datavetare med, så ämnet datavetenskap har ej kunnat få ett vettigt form, utan kurserna är fortfarande praktiska kurser att kunna handha system eller installera på datorer. Om vi arbetar bra kan en sådan inriktning, kanske bli bättre och bli mycket intressant och ge underlag för sökande till våra datautbildningar.

## 2 17-18 October 2006, Workshop on Applied Mathematics

## 2 25-26 October 2006, Forskarskolans Jubileumskonferens

## 2 Wednesday 8 November 2006, Francisco Bernal, Universidad Carlos III, Madrid

*Title: A meshless simulation of the isothermal Hele-Shaw flow*

Abstract: Plastic injection molding is a process of industrial relevance whereby molten polymer is driven into a cavity in order to manufacture small plastic parts. If the mould is thin compared to its planar dimensions, the polymer flow can be simulated based on the Hele-Shaw approximation for pressure, temperature, and velocity. If an isothermal power-law for viscosity is assumed, the flow is completely determined by solving a two-dimensional p-harmonic equation at every time step. State-of-the-art solvers rely on finite elements and finite differences together with some tracking technique for front motion, which often involves remeshing around the front.

In this talk, a novel approach is proposed for the isothermal Hele-Shaw flow. Pressure is solved with a meshless method -meaning that the discretisation support is made up of disconnected points scattered across the domain. This is useful to describe complicated geometries and also seems an appealing feature to deal with free boundary problems. More concretely, asymmetric RBF collocation, better known as Kansa's method, is employed, where RBF stands for Radial Basis Function. Moreover, the front is captured rather than tracked by means of the Fast Marching method developed by Sethian. Front-capturing allows for a sharp frontline along which the boundary conditions can be enforced, and also manages front collisions and break-ups in a natural way.

A short introduction to RBFs, Kansa's method and Fast Marching will be presented. We will next focus on its application to Hele-Shaw flow and the advantages and difficulties detected so far in such a formulation.



2 **Wednesday 15 November 2006, Julius Borcea, SU**

*Title: Pólya-Schur problems for hyperbolicity and stability preservers with applications*

Abstract: A linear operator  $T$  on  $\mathbb{C}[z]$  is called hyperbolicity-preserving or an HPO for short if  $T(P)$  is hyperbolic whenever  $P \in \mathbb{C}[z]$  is hyperbolic, i.e., it has all real zeros. One of the main challenges in the theory of univariate complex polynomials is to describe the monoid of all HPOs. This reputedly difficult problem goes back to Pólya-Schur's characterization of multiplier sequences of the first kind, that is, HPOs which are diagonal in the standard monomial basis of  $\mathbb{C}[z]$ . Pólya-Schur's celebrated result (Crelle, 1914) generated a vast literature on this subject and related topics at the interface between analysis, operator theory and algebra but so far only partial results under rather restrictive conditions have been obtained. In this talk I will report on the progress towards complete solutions to both this problem and its analog for stable polynomials as well as their multivariate extensions made in an ongoing series of papers jointly with Petter Brändén and Boris Shapiro. If time permits I will also discuss applications to mixed determinants and Lax-type problems for real stable and (Gårding) hyperbolic polynomials.

2 **Wednesday 22 November 2006, Bengt Josefsson, MAI**

*Title: Projections in Banach spaces.*

Abstract: Let  $E \subset F$  be Banach spaces. A linear, bounded mapping  $P: F \rightarrow E$  is called a projection if  $P$  restricted to  $E$  is the identity. The existence of a projection is equivalent to  $E$  being complemented in  $F$ , i.e. there exists a Banach space  $G$  such that  $E + G = F$ . We shall discuss two types of uncomplementedness for a separable Banach space, one separable and one unseparable, and its similarity (relation?) to other problems.

2 **Wednesday 29 November 2006, Göran Bergqvist, MAI**

*Title: Complex methods for real Lorentzian geometry*

Abstract: We give a simple introduction to how four-dimensional real Lorentzian (the metric is not positive definite) geometry can be described in two complex dimensions. Natural objects in this formalism are so-called spinors and we present some results whose (only or first) proofs were obtained by complex methods. Our examples are related to the Weyl conformal curvature tensor or to purely geometric characterisations of solutions to Einstein's field equations in general relativity.

2 **Monday 4 December 2006, Richard Gill, Mathematical Institute, University of Leiden**

*Title: Optimal Passion at a Distance*

Abstract: I explain quantum nonlocality experiments and discuss how to optimize them. Statistical tools from missing data maximum likelihood are crucial. New results are given on Bell, CGLMP, CH and ladder inequalities. Open problems are also discussed.

It is advantageous not to have any prior understanding of quantum theory or indeed physics. It may be difficult to resist discussion of the philosophical implications of Bell's inequality.

2 **Friday 8 December 2006 Magnus Österholm**

defends his thesis "Kognitiva och metakognitiva perspektiv på läsförståelse inom matematik"

See <http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-7674>

2 **Monday 11 December 2006, Francois Murat, Université Pierre et Marie Curie (Paris VI)**

*Title: Existence and a priori estimate for elliptic problems with subquadratic gradient dependent terms*

Abstract: In this lecture I will consider the nonlinear elliptic model problem

$$u \in H_0^1(\Omega), \quad -\operatorname{div} A(x)Du + \alpha_0 u = \gamma |Du|^q + f(x) \quad \text{in } \mathcal{D}'(\Omega),$$

with  $A$  a coercive matrix with bounded coefficients,  $\alpha_0 \geq 0$ ,  $0 \leq q \leq 2$  and  $f \in L^m(\Omega)$  for some suitable  $m$ . This is a model problem, and there are many possible variants of it.

In the case where  $0 \leq q < 1$ , existence is classical for  $f \in H^{-1}(\Omega)$ . When  $\gamma$  is large, the case where  $q = 1$  and  $f \in H^{-1}(\Omega)$  is difficult but has been solved by G. Bottaro and M.E. Marina in 1973. On the other hand, the case  $q = 2$  has been treated by many authors, including in particular in a series of papers by L. Boccardo, J.-P. Puel and myself. In a more recent paper, V. Ferone and myself proved the existence of a solution  $u$  which further satisfies  $e^{\gamma u} - 1 \in H_0^1(\Omega)$ , and an a priori estimate for such solutions, when  $f \in L^{\frac{N}{2}}(\Omega)$ .

In this lecture I will mainly report about recent joint work with Nathalie Grenon and Alessio Porretta, the announcement of which has been published in C. R. Acad. Sci. Paris, Serie I, 342, (2006), pp. 23-28. When  $1 + \frac{2}{N} \leq q < 2$  and  $f \in L^m(\Omega)$  with  $m = \frac{N(q-1)}{q}$  (we also solved the case where  $1 \leq q < 1 + \frac{2}{N}$ , but I will not discuss it since it uses the notion of renormalized solution), and when either  $\alpha_0 > 0$  or  $f$  is sufficiently small in  $L^m(\Omega)$ , we prove the existence of a solution  $u$  which enjoys the further regularity  $|u|^\sigma \in H_0^1(\Omega)$  with

$\sigma = \frac{(N-2)(q-1)}{2(2-q)}$ , as well as an a priori estimate for any solution which enjoys this further regularity. One of the main interests of our result lies in the a priori estimate, the proof of which is non standard.

2 **Thursday 14 December 2006 Daniel Ying defends his thesis**

Abstract:

Title: On the Moduli Space of Cyclic Trigonal Riemann Surfaces of Genus 4

Abstract: A closed Riemann surface which can be realized as a 3-sheeted covering of the Riemann sphere is called trigonal, and such a covering is called a trigonal morphism. Accola showed that the trigonal morphism is unique for Riemann surfaces of genus  $g \geq 5$ . This thesis characterizes the cyclic trigonal Riemann surfaces of genus 4 with non-unique trigonal morphism using the automorphism groups of the surfaces. The thesis shows that Accola's bound is sharp with the existence of a uniparametric family of cyclic trigonal Riemann surfaces of genus 4 having several trigonal morphisms. The structure of the moduli space of trigonal Riemann surfaces of genus 4 is also characterized.

Finally, by using the same technique as in the case of cyclic trigonal Riemann surfaces of genus 4, we are able to deal with  $p$ -gonal Riemann surfaces and show that Accola's bound is sharp for  $p$ -gonal Riemann surfaces. Furthermore, we study families of  $p$ -gonal Riemann surfaces of genus  $(p-1)^2$  with two  $p$ -gonal morphisms, and describe the structure of their moduli space.

# The Mathematical Colloquia held in 2005 at Linköpings universitet

Organized by Armen Asratian, Anders Björn, Milagros Izquierdo Barrios, Vladimir Kozlov, Svante Linusson and Stefan Rauch-Wojciechowski.

## Friday 14 January 2005.

Meeting of the Swedish Mathematical Society.

## Monday 17 January 2005.

**Professor Per Enflo, Kent State University,**  
*Minimal points and contractive projections*

*Abstract:*

The talk connects to applications in mathematical economy.

## Wednesday 19 January 2005.

**Lars Inge Hedberg,**  
*Spectral synthesis in function spaces; atomic theory*

*Abstract:*

I will discuss the modern definitions of classes of function spaces by means of "smooth atoms", formulate the spectral synthesis problem in this general context, and give some indications of its solution by Netrusov. No prerequisites beyond basic analysis will be assumed. In particular, the talk is independent of my talk in December 2004.

## Wednesday 26 January 2005.

**Anders Björn,**  
*A boundary regularity classification for harmonic and  $p$ -harmonic functions*

*Abstract:*

Let  $G$  be a bounded open set in  $\mathbf{R}^n$  and let  $f$  be a continuous function on the boundary of  $G$ . The *Dirichlet problem* asks for finding a harmonic function  $u$  in  $G$  which has  $f$  as its boundary values.

In general it is not possible to require that  $\lim_{y \rightarrow x} u(y) = f(x)$  holds for all  $x$  in  $G$ . It is however possible to obtain unique solubility in general by saying that the equality above should hold for almost every boundary point (in a sense that will be made precise in the talk).

A boundary point  $z$  is said to be *regular* if  $\lim_{y \rightarrow z} u(y) = f(z)$  for all continuous boundary functions  $f$ . In fact, most boundary points are regular, but there are also irregular boundary points.

The dichotomy between regular and irregular boundary points has been studied extensively. It is less known that one can go one step further and divide the irregular boundary points into two classes, semiregular points and strongly irregular points, with vastly different boundary behaviour. In this talk I will discuss the trichotomy between regular, semiregular and strongly irregular boundary points both for harmonic functions and for their nonlinear generalizations  $p$ -harmonic functions.

## Wednesday 2 February 2005.

**Professor Igor Abrikosov, IFM,**  
*Quantitative quantum description of materials properties: Physics and Mathematics*

*Abstract:*

Quantum mechanics revolutionized physics in the beginning of the last century. In 1927 Sommerfeld applied the quantum description to metals, within the model of free electrons, and obtained a qualitatively correct picture of some basic properties. But due to an enormous complexity associated with quantum mechanical calculations for real materials, accurate quantitative results were rarely obtained. The possibilities to study material properties from the basic principles of quantum mechanics were enormously enhanced when the density functional theory (DFT) and the local spin density approximation (LSDA) were formulated by Kohn and co-workers in the mid-60s. In 1998 this groundbreaking theory was awarded the Nobel Prize. I will give a very brief and informal description of the DFT equations. I will also explain general ideas on how one numerically solve the equations within the DFT. I will start with a variational principle, and show how one derives the so-called secular equation, the main equation to be solved numerically. I will particularly point out unsolved mathematical/numerical problems that we are dealing with in our research.

## Wednesday 9 February 2005.

**Professor Bo Berndtsson, Chalmers,**  
*Prekopa's theorem and its complex versions*

*Abstract:*

A celebrated theorem, due to Prekopa, states that if  $g(x, y)$  is a convex function on  $\mathbf{R}^n_x \times \mathbf{R}^k_y$ , then the function  $h$  defined by

$$e^{-h(x)} = \left[ \int e^{-g(x,y)} dy \right]$$

is also convex. This theorem implies several other results in convex analysis like the Brunn-Minkowski inequality. If we think of the function

$$e^{h(x)}$$

as the density of the operator that assigns to a function  $f(y)$  its weighted mean value with respect to  $e^{-g(x,y)}$  we see that the corresponding object in the complex case is the Bergman kernel. We will discuss a theorem on the subharmonic dependence of the Bergman kernel on a parameter. This theorem implies the theorem of Prekopa and also has several other applications related to interpolation theory and the local structure of plurisubharmonic functions.

## Wednesday 16 February 2005.

**Niko Marola, Helsingfors tekniska högskola,**  
*Moser's argument for minimizers of the  $p$ -Dirichlet integral*

*Abstract:*

The regularity of the nonlinear elliptic equations can be proved using the celebrated Moser iteration technique. It seems that Moser's argument is strongly based on the differential equation, which in our case is the  $p$ -Laplace equation. An alternative approach to consider the  $p$ -Laplace equation is to study its variational formulation, the nonlinear  $p$ -Dirichlet integral. It can be shown that to run Moser's method a differential equation is not needed. In this talk we prove by using Moser's technique that minimizers of the  $p$ -Dirichlet integral

satisfy Harnack's inequality from which the Hölder continuity follows.

2 **Thursday 17 February 2005.**

**Ingemar Eriksson defended his licentiate thesis,**  
*The Chevreton tensor and its trace*

*Abstract:*

In this thesis we investigate the Chevreton tensor in Einstein-Maxwell theory. It was introduced in 1964 as the counterpart, for electromagnetic fields, of the well-known Bel-Robinson tensor of the gravitational field. We prove that, in the absence of electromagnetic sources, this tensor is completely symmetric. We consider currents constructed from the Chevreton tensor with Killing vectors and show that these currents are conserved for some types of spacetimes with a hypersurface orthogonal Killing vector or two commuting Killing vectors that act orthogonally transitive on non-null surfaces. In addition, we show that the trace of the Chevreton tensor is a rank-two, symmetric, trace-free, divergence-free tensor and that it is related to the Bach tensor. This allows us to investigate Einstein-Maxwell spacetimes with a vanishing Bach tensor.

2 **Friday 18 February 2005.**

**Geometry and Relativity Meeting.**

2 **Wednesday 23 February 2005.**

**Professor Ari Laptev, KTH,**  
*Mass transportation approach to sharp functional inequalities*

*Abstract:*

Following the recent articles of C. Villani with his co-authours we shall discuss new proofs of Sobolev and Brezis-Lieb inequalities.

2 **Wednesday 9 March 2005.**

**Professor Anders Melin, Lunds universitet,**  
*Backscattering and multilinear singular integral operators*

*Abstract:*

The mathematical colloquium at Linköping university March 9, 2005. Speaker: Anders Melin, Lund University

**Backscattering and multilinear singular integral operators**

**Abstract:** Let  $H_v = -\Delta + v(x)$  be the Schrödinger operator in  $\mathbf{R}^n$ , where  $n \geq 3$  is odd and the potential  $v$  is real valued and satisfies appropriate decay and smoothness conditions. The wave operators are the strong limits as  $t \rightarrow \pm\infty$  of  $e^{itH_v}e^{-itH_0}$  and the scattering operator  $S = S(v) = W_+^*W_-$  is unitary in  $L^2(\mathbf{R}^n)$ . The anti-diagonal part of the distribution kernel of  $\mathcal{F}S\mathcal{F}^*$ , where  $\mathcal{F}$  is the Fourier transform, forms the backscattering data and the real part of its inverse Fourier transform is after suitable normalization given by the expression

$$B_0v(x) = 2^n \int v(y)W(x-y, x+y)dy,$$

where  $W(x, y)$  is the distribution kernel of  $(W_+ + W_-)/2$  and the integral is interpreted in the distribution sense.

Here  $Bv$  depends on  $v$  in a highly nonlinear way, but it turns out that the restriction of  $Bv$  to an open set of small  $v$  extends to an entire analytic function of  $v$  in suitable Banach spaces of functions containing  $\mathcal{S}(\mathbf{R}^n)$ . We shall denote by  $Bv$  this entire analytic mapping and remark that  $Bv - B_0v$  can be expressed in terms of the bound states of  $H_v$ .

By considering suitable representations for  $Bv$  one finds that  $Bv$  is defined and analytic in  $L_{\text{cpt}}^s(\mathbf{R}^n)$  when  $s > 2n$ . Let

$$Bv = \sum_{N=1}^{\infty} B_Nv$$

be the power series expansion of  $Bv$ , where  $B_Nv$  is  $N$ -linear in  $v$ . The operators  $v \mapsto B_Nv$  are multilinear singular integral operators. Their complexity increase with  $N$  and  $n$ . We have  $B_1v = v$ , and it turns out that if  $v \in L_{\text{cpt}}^s$  with  $s$  as above, then  $Bv - \sum_{N=k}^{\infty} B_Nv \in C^{N_k}$ , where  $N_k \rightarrow \infty$  as  $k \rightarrow \infty$ . Thus  $B_N$  is smoothening when  $N$  is large.

In my lecture I will describe how one derives representations for  $Bv$  and  $B_Nv$  suitable for analysis, and then I will discuss PDE-methods to examine the mapping properties of  $B_N$  in various Sobolev spaces. Because of the invariance of  $Bv$  with respect to the family of transformations  $v \mapsto \rho^2v(\rho x)$ , where  $\rho > 0$ , the Sobolev space  $L^{p,m}$  of functions with  $m$  derivatives in  $L^p$  is a natural candidate for consideration when  $p = 1$  and  $m = n - 2$ . In that case good mapping properties are already established when  $n = 3$ , and they are likely to hold for arbitrary  $n$ . When  $p > 1$  the situation becomes much more complicated because the high degree of singularity of the distribution kernels of the  $B_N$ . We finally notice that if a Banach space  $X$  is invariant under the backscattering transform then it follows that  $v$  is uniquely determined by  $Bv$ , and hence by  $S(v)$  for all small  $v$  in  $X$ , and it would be interesting to develop numerical schemes for computing  $v$  from  $Bv$ .

2 **Wednesday 23 March 2005.**

**Professor Richard Ehrenborg, University of Kentucky, Lexington,**  
*Lifting inequalities for polytopes*

*Abstract:*

The  $f$ -vector enumerates the number of faces of a convex polytope according to dimension. The flag  $f$ -vector is a refinement of the  $f$ -vector since it enumerates face incidences of the polytope. To classify the set of flag  $f$ -vectors of polytopes is an open problem in discrete geometry. This was settled for 3-dimensional polytopes by Steinitz a century ago. However, already in dimension 4 the problem is open.

We will discuss the known linear inequalities for the flag  $f$ -vector of polytopes. These inequalities include the non-negativity of the toric  $g$ -vector, that the simplex minimizes the cd-index, and the Kalai convolution of inequalities.

We will introduce a method of lifting inequalities from lower dimensional polytopes to higher dimensions. As a result we obtain two new inequalities for 6-dimensional polytopes.

The talk will be accessible to a general audience.

Monday 4 April 2005.

**Professor Richard P. Stanley, MIT,**  
*Ordering events in Minkowski space*

*Abstract:*

Suppose that we are given  $k$  points (events) in  $(n+1)$ -dimensional Minkowski space ( $n$  space dimensions and one time dimension). The events need not occur in the same order to observers in different reference frames. What sets of orders are possible, and how many such orders are there? We will show how these questions can be investigated using the theory of hyperplane arrangements. Much of the talk will consist of background information on hyperplane arrangements, focusing on the question of counting the number of regions into which a real vector space is divided by a finite set of hyperplanes.

Wednesday 6 April 2005.

**Peter Basarab-Horvath,**  
*Symmetries and classification of differential equations*

*Abstract:*

Symmetries are an important aspect of differential equations and are a useful tool in finding solutions to nonlinear differential equations as well as providing us with useful information such as conservation laws. Lie algebras first arose in the study of symmetries of differential equations. In the present talk I shall give a survey of some work concerning the classification of evolution equations according to symmetry properties.

Tuesday 12 April 2005.

**Andreas Rietz defended his Ph.D. dissertation,**  
*Existence theorems for noncoercive incremental contact problems with Coulomb friction*

Opponent was Professor Jaroslav Haslinger, Charles University, Prague.

*Abstract:*

Friction is a phenomenon which is present in most mechanical devices and frequently encountered in everyday life. In particular, understanding of this phenomenon is important in the modelling of contact between an elastic object and an obstacle. Noncoercive incremental contact problems with Coulomb friction constitute an important class of such friction problems due to their frequent occurrence in mechanical engineering. They occur for example when modelling an object which is not fixed to a support. The topic of this thesis is to study this class of friction problems.

This thesis considers both discrete and continuous systems. For the continuous systems we consider both problems with a nonlocal friction law where the contact force is mollified and problems with a normal compliance friction law where the body may penetrate the obstacle. For all friction problems we derive a sufficient condition for the existence of a solution. This condition is a compatibility condition on the applied force field, and if it is violated there exists a nontrivial solution to a corresponding dynamical problem.

Wednesday 13 April 2005.

**Professor Jaroslav Haslinger, Charles University, Prague,**  
*Approximation and numerical realization of contact problems with Coulomb friction and a solution-dependent coefficient of friction (static case)*

Wednesday 27 April 2005.

**Milagros Izquierdo Barrios,**  
*Ovals of Riemann surfaces*

*Abstract:*

After Klein a real curve can be represented as a symmetric complex curve. A symmetric complex curve is a Riemann surface together with a symmetry of the surface. A symmetry is an anticonformal involution acting on the surface. The Riemann surface  $X$  and the conjugacy class of the symmetry in the automorphism group give us the real (model of the) curve. The fixed point-set of the symmetry consists of a collection of disjoint Jordan curves: the ovals. Each oval of the symmetry correspond to a connected component of the real curve. In this talk we introduce some combinatorial techniques (Fuchsian and NEC groups) to deal with symmetric Riemann surfaces and we present some results.

Wednesday 4 May 2005.

**Professor Capi Corrales Rodríguez, Universidad Complutense de Madrid, spoke to a wider audience about**  
*From space as container to space as a web in mathematics and painting*

*Abstract:*



Velázquez, *Las Meninas* (1656, Museo del Prado, Madrid)



Picasso, *Las Meninas* (1957, Museo Picasso, Barcelona)

At the end of the XVII century, "space" in mathematics was identified with Physical Space, the space in which natural phenomena take place. And from a corner of this space, conceived as a huge three dimensional container, a box in which objects "float", and using the elements of the geometry of Euclid, mathematicians looked, described and constructed.

By mid XIX century, mathematicians had realised that this identification between Physical Space and mathematical space was just a convention, and a very limiting one. They left the corner of the box and they got closer and closer to the objects, first placing themselves on them, so to speak, with the intrinsic geometry of Gauss, later on touching them with the topology of Poincaré. This allowed them to perceive that any relation between objects, arbitrary objects, can be used to produce a spatial structure. And so, by the beginning of the XX century, a more adequate definition space was reached in mathematics (Hausdorff): any web or net of relations between objects.

The idea of space as a container involves thinking of space as a huge global object given a priori, an external reference we look from. A net space is a space which is not an a priori choice, but a structure constructed for each concrete situation by patching together smaller pieces, local pieces.

From the container space we get a global view. From within the net space we get local views, detailed information of smaller regions. Going back and forth, as contemporary mathematicians do, allows us to coherently patch local pieces of information into global information of whatever it is that we want to look, describe, or construct.

In this talk we will follow the mathematical evolution that takes us from space conceived as a container to space viewed as a web (of relations), and we will do it using as graphical reference the paintings that were being done as the mathematical ideas were being "cooked".

## 2 Wednesday 11 May 2005.

**Professor Nageswari Shanmugalingam, Cincinnati,**

*An introduction to conformal Martin boundary for Euclidean domains*

*Abstract:*

In this talk, we will discuss the construction of singular functions associated with the  $p$ -Laplacian on domains in Euclidean spaces when  $1 < p \leq n$ , where  $n$  is the dimension of the space. We will then use the singular functions to construct the conformal Martin boundary of a Euclidean domain. Such a boundary is a conformally invariant compactification of the domain. We will also explore the behavior of the conformal Martin kernel functions near the boundary of the domain, and use the boundary Harnack principle to discuss the growth behavior of such functions near boundaries of John domains.

## 2 Wednesday 18 May 2005.

**Professor Mireille Bousquet-Mélou, Bordeaux,**

*Algebraic series in enumerative combinatorics*

*Abstract:*

Let  $A(t)$  be the generating function of a class of discrete objects. That is,  $A(t) = \sum a_n t^n$ , where  $a_n$  is the number of objects of size  $n$  in the class. Then  $A(t)$  is said to be algebraic if it satisfies a (non-trivial) polynomial equation  $P(t, A(t)) = 0$ . Algebraic series form a well-behaved family:

- \* It has interesting closure properties (+, \*, /, derivatives, composition, ...).
- \* These series are reasonably easy to handle (elimination, resultants, Gröbner bases, ...).
- \* Their algebraicity can be guessed from sufficiently many of their first coefficients.
- \* The asymptotic behaviour of these coefficients can be obtained in a systematic way.

Most importantly, there is a combinatorial intuition of how a class of objects with an algebraic generating function looks like: VERY roughly speaking, A class of objects has an algebraic generating function if these objects have an algebraic structure: that is, if they admit a recursive description based on the concatenation of smaller objects of the same type."

However, many classes of objects simply refuse to show clearly their algebraicity. The aim of this talk is twofold: I will first present general techniques for proving the algebraicity of a series, and then several challenging problems that still resist these general techniques.

## 2 Friday 20 May 2005.

**Professor Mireille Bousquet-Mélou, Bordeaux, will gave her installation lecture for honorary doctor,**

*What on earth are these mathematicians doing?*

## 2 Wednesday 25 May 2005.

**Professor Alexander M. Khludnev, Novosibirsk,**

*Cracks in solids with possible contact between crack faces*

*Abstract:*

Presence of cracks in solids means that we have to find a solution in domains with cuts (cracks). In the talk we discuss boundary value problems considered in cracked domains. Inequality type boundary conditions given on the crack faces do not allow the crack faces to penetrate each other. New results obtained recently in this field are presented.

## 2 Friday 27 May 2005.

**Jens Jonasson defended his licentiate thesis,**

*The Levi-Civita geodesic equivalence problem and multiplication of cofactor pair systems*

*Abstract:*

When studying equivalence of dynamical systems, in the sense of Levi-Civita, the concept of cofactor pair systems plays an important role. Cofactor pair systems can be constructed through a multiplicative structure of the so called quasi-Cauchy-Riemann equations  $\text{cof } J^{-1} \text{ grad } V = \text{cof } (J')^{-1} \text{ grad } (V')$ , where  $J$  and  $J'$  are special conformal Killing tensors. In this thesis we study this multiplication and its role in the theory of equivalent dynamical systems. We have isolated the properties that are responsible for the multiplication, allowing us to give an elegant characterization of systems that admit multiplication. We describe how the multiplication of cofactor pair systems can be considered as a special case of a more general kind of multiplication. We also investigate algebraic properties of the multiplication and provide several methods for constructing new systems with multiplicative structure.

2 Friday 27 May 2005.

**Professor Sergio Benenti, University of Turin,**

*Dynamical systems with nonholonomic constraints - user friendly approach*

*Abstract:*

The theory of non-holonomic dynamical systems, even in recent times, is treated in a growing number of papers.

Most of them use high-level and refined structures of differential and algebraic geometry, which are not commonly accessible. I think that in using too sophisticated tools for dealing with non-holonomic mechanics, most of its beauty remains hidden. Thus, my aim is to provide a simple and natural approach to this matter, by using the elementary vector calculus in the Euclidean three-space, and the elementary structure of the tangent bundle of a configuration manifold. Starting from the Gauss principle, we shall arrive in a short way to two different (but equivalent) dynamical equations, readily available for any concrete example. These two kinds of dynamical systems are related to the two possible representations of a non-holonomic constraints: by parametric equations or by zero-equations.

In other words, my first aim is to show the essential features of the theory, present in the literature, without redundant and/or inessential notions. The second aim is to propose some special devices realizing non-linear non-holonomic constraints, after that of Appell-Hamel.

2 Wednesday 14 September 2005.

**Johan Wästlund, Linköping**

*The random travelling salesman problem – a challenge to physicists, mathematicians and computer scientists.*

*Abstract:*

Assign independent uniform (0,1) lengths to the edges of the complete graph on  $n$  vertices and let  $L_n$  be the length of the minimum travelling salesman tour. The distribution of  $L_n$  for large  $n$  has

been a challenge to physicists, mathematicians and computer scientists for the last twenty years. According to a long-standing conjecture,  $L_n$  converges in distribution to a certain "universal constant", about 2.0415. I will describe the background as well as some recent progress towards a proof of this conjecture, including a definite integral for the universal constant.

2 Wednesday 21 September 2005.

**Professor Richard Beals, Yale University,**

*The KdV and Camassa-Holm equations: classical and new solitons.*

*Abstract:*

The well-known KdV equation was proposed to describe waves in shallow water. It is now known to have explicit solutions (multi-solitons) with very interesting interactions. The same is true of an equation proposed much more recently: the Camassa-Holm equation. Its special solutions (peakons, antipeakons) have even more interesting interactions. They come from finite-dimensional Hamiltonian systems and can be calculated explicitly, but in a completely different way from KdV.

2 Friday 23 September 2005.

**Professor Natan Kruglyak, Luleå University of Technology,**

*Covering Theorems, Singular Integrals and Applications*

*Abstract.*

The importance of singular integral operators for theoretical and applied mathematics is well-known. For example, they appear naturally in the theory of elliptic and parabolic partial differential equations and in computerized tomography. I plan to discuss two recent results (and some of their applications) obtained on the base of old and new covering theorems.

2 Wednesday 28 September 2005,

**Professor Nikolay Kuzjurin, Russian Academy of Sciences,**

*On-line strip packing and scheduling parallel tasks in Grids*

*Abstract.*

The Grid computing paradigm is originated from a new computing infrastructure for scientific research and cooperation, and is becoming an established technology for large-scale resource sharing and distributed integration. The main problem arises: how to efficiently schedule tasks in such systems. In my talk I am going to present new class of packing problems that related closely to scheduling parallel tasks in grids. Such problems can be formulated as packing of a given set of rectangles (corresponding to tasks) into a set of strips (corresponding to clusters in grid). Strip packing problems have been studied intensively for the case of one strip and some interesting results were obtained. In my talk I'll present some new approximation algorithms for the case of few strips (in the worst and the average cases).

- in the worst case we present an on-line approximation algorithm with constant approximation ratio;
- in the average case we present an on-line strip packing algorithm that achieves  $O(N^{2/3} \log^{1/3} N)$  expected wasted area where  $N$  is the number of rectangles to be packed each with width and height distributed uniformly in  $[0,1]$ .

2 Wednesday 5 Oktober 2005.

**Professor David Sattinger, Yale University,**

*How Deep is the Bay of Bengal?*

*Abstract.*

Russell's velocity formula was at the center of the controversy over the existence of the solitary wave; but today the topic is rarely mentioned. It is an immediate corollary of modern bifurcation theory; and it is fundamental to modelling waves in deep water. A tsunami 60 cm high in an ocean 4 km deep is 377 km long, travels with a velocity of 713 km/hr, and carries a transverse energy density of  $2 \times 10^9$  joules/meter, yet the maximum speed of the surface current is only 2.97 cm/s.

2 Wednesday 12 Oktober 2005, 13.15-14.15, Glashuset.

**Professor Johann Engelbrecht, University of Pretoria**

*Title: Comparison of students' procedural and conceptual understanding in mathematics*

*Abstract.*



When teaching a scientific discipline care should be taken to cultivate procedural and conceptual understanding as opposed to blind application of rules and methods. Procedural understanding enables the student to perceive a process or method in its entirety and not simply as a sequence of steps. Conceptual understanding, on the other hand, requires of the student to grasp the underlying principles in such a way that it can be used in applications, within or outside the particular discipline. The general perception is that high school teaching of mathematics in South Africa tends to be fairly procedural and that students that enter university are better equipped to deal with procedural problems rather than conceptual. In this study we compare the conceptual and procedural skills of first year calculus students in life sciences. We also investigate students' confidence in handling conceptual and procedural problems. The study seems to indicate that these students do not perform better in procedural problems than in conceptual problems. They are also more confident of their ability to handle conceptual problems than to handle procedural problems. Furthermore the study seems to indicate that students do not have more misconceptions about conceptual mathematics than about procedural issues.

📅 **Wednesday 19 Oktober 2005, 13.15-14.15, Glashuset.**  
**Daniel Ying, Linköping**

*Title: A short history about Riemann surfaces*

*Abstract.*

Riemann surfaces have an appealing feature to mathematicians (and hopefully to non-mathematicians

- 📅 as well) in that they appear in a variety of mathematical fields. The point of the introduction of Riemann surfaces made by Riemann, Klein and Weyl (1851-1913), was that Riemann surfaces can be considered as both a one-dimensional complex manifold and an algebraic curve. Another possibility is to study Riemann surfaces as two-dimensional real manifolds, as Gauss (1822) had taken on the problem of taking a piece of a smooth oriented surface in Euclidean space and embedding it conformally into the complex plane. A fourth perspective came from the uniformisation theory of Klein, Poincaré and Koebe (1882-1907), who showed that every Riemann surface (which by definition is a connected surface equipped with a complex analytic structure) also admits a Riemann metric. This is a short survey about the history of Riemann surfaces and the development of such surfaces from Bernard Riemann's doctoral thesis and some of the later results made by Poincaré.

📅 **Wednesday 26 Oktober 2005, 13.00-14.00, Glashuset.**  
**Professor Frank Nijhoff, Leeds University**

*Title: Integrable systems on the lattice and associated partial differential equations*

*Abstract.*

I will review some of the insights that have been obtained in recent years on the integrability of partial difference equations. The intimate interplay between such systems on the space-time lattice and certain parameter-families of partial differential equations will be elucidated.

📅 **Friday 28 Oktober 2005, 13.15-14.15, Glashuset.**  
**Per Enflo,**

*Title: Likformiga homeomorfier i Banachrum*

*Abstract.*

Det är ett gammalt - och ännu icke helt löst - problem, att avgöra i vilken utsträckning följande gäller: Låt  $B$  och  $C$  vara Banachrum. Antag att de är likformigt homeomorfa - dvs. det finns en homeomorfi mellan dem, som är likformigt kontinuerlig i båda riktningarna. Är de då linjärt isomorfa - dvs. kan homeomorfien göras linjär? Jag ska berätta om problemets historia och om nya framsteg som gjorts.

📅 **Wednesday 9 November 2005, 13.15-14.15, Glashuset.**  
**Pertti Mattila, University of Helsinki**

*Title: Rectifiability in Euclidean and metric spaces*

*Abstract.*

Rectifiable 'surfaces' (sets, measures, currents, varifolds) in Euclidean spaces form in many ways an essentially largest possible class of surfaces including smooth surfaces and having their main geometric properties, often interpreted in a generalized sense. They are very central in geometric measure theory and they are useful, for example, in calculus of variations because of strong compactness properties. In Euclidean spaces they can be defined in many natural and equivalent ways. Recently there has been interest for looking at generalizations to metric spaces, both to very general metric spaces and spaces with special non-Euclidean structure such as the Heisenberg groups. Then many new problems arise and it is not even always clear what should be the proper definitions. The talk will be a survey on part of such developments.

📅 **Thursday 10 November 2005, 15.00-16.00, Glashuset.**  
**Professor Martin Aigner, Freie Universität Berlin**

*Title: From Fibonacci to Ramanujan via Lattice Paths*

*Abstract.*

Of the many results due to Ramanujan the Rogers-Ramanujan identities are probably the most famous. They are of interest in number theory, algebra, combinatorics, and statistical mechanics, and they have the touch of the extraordinary because of the mysterious appearance of the number 5. We begin by proving a Fibonacci identity by a double count of two sets of lattice paths, and go on to demonstrate that the Rogers-Ramanujan identities are just a suitably weighted version of it. On the way some of the standard repertoire of combinatorics is encountered, such as inclusion-exclusion, q-binomial numbers, formal power series, and Jacobis's triple product theorem.

📅 **Wednesday 16 November 2005, 13.15-14.15, Glashuset.**  
**Krzysztof Marciniak, ITN**

*Title: Finite-dimensional separable systems give rise to hierarchies of commuting evolutionary PDE's.*

*Abstract.*

Classical dynamical systems that are separable in the sense of the Hamilton-Jacobi theory can be constructed from a set of algebraic equations that are known as separation relations

(or separation curves). In this talk I shall propose a general scheme of constructing sequences of commuting flows of systems of evolutionary PDE's (soliton hierarchies) from separation relations related with dynamical systems separable in the classical (Stäckel) sense. The talk will be on an elementary level with all the necessary notions explained from the basics.

**2 Wednesday 23 November 2005, 13.15-14.15, Glashuset.**

**Emma Previato, Institut Mittag-Leffler and Boston University**

*Title: Commuting Partial Differential Operators.*

*Abstract.*

This talk is an introduction to the question of classification of commutative rings of partial differential operators, especially as regards the geometric aspects, such as their spectral variety and quantum integrable systems. The talk will be centered on examples. Non-trivial examples are provided by algebraic surfaces, Sato's tau function, differential Galois theory.

**2 Tuesday 29 November 2005, 15.15-16.15, Glashuset.**

**Roland Häggkvist, Umeå universitet**

*Title: Paul Seymours lejonklo eller framsteg inom grafteorin.*

*Abstract.*

I detta föredrag som är tänkt att vara populärvetenskapligt, nästan skvallerartat, med bilder illustrerat, och trots detta, även om än icke enbart, intressant för de grafteoretiskt bevandrade tänker jag driva teserna att

a) även stora matematiker är människor

b) även stor matematik kan innehålla delar som går att illustrera så att vi vanliga dödliga har en chans att förstå något.

Den röda tråden utgörs av några frågor so kommit att förknippas med den Oxfordutbildade matematikern Paul Seymour och några av hans medarbetare, främst Neil Robertson, Robin Thomas och nu senast Maria Chudnovsky. Jag tänker beskriva den så kallade perfekta graf-formodan, nedan, formulerad av Claude Berge, vilket ger en alldeles osökt anledning att visa några unika bilder från ett galleri i Paris med en av auditoriet troligen välkänd innehavare. Jag tänker dessutom berätta om några optimeringsegenskaper hos perfekta grafer vilka möjligen indikerar varför perfekta grafer (som visserligen inte är så perfekta som namnet antyder, men ändå) är optimeringstekniska önskeobjekt. Teknisk definition som blir tämligen enkel när den illustreras: En graf är perfekt definitionsmässigt om och endast om hörnfärgningstalet på varje inducerad delgraf är precis så stort som den största kompletta delgrafen (klikstorleken) i den inducerade delgrafen i fråga. Att en 5-cykel inte är perfekt eller att komplementet till en 7-cykel inte heller det är perfekt torde inte komma som någon större överraskning för de som förstår termerna, färgningstalet för en 5-cykel är ju 3 men största klickstorleken 2 exempelvis. Berge's förmodan, bevisad av Chudnovski, Seymour, och Thomas 2002, var att en graf är perfekt om den och dess komplement saknar inducerad udda cykel av längd minst 5. Troligen (?) kommer jag även att gå genom några icke fullt så tekniska varianter av den så kallade Wagners förmodan: Varje oändlig lista av ändliga grafer utan loopar och multipla kanter innehåller två grafer där den ena delvis kan fås ur en delgraf av den andra genom att denna kontraheras och /eller ett antal kanter underdelas. Denna förmodan bevisades under nittitalet av Paul Seymour och Neil Robertson i en serie artiklar om något tusental sidor och även där blev slutresultatet att ett stort antal graffamiljer visade sig ha oväntat bra optimeringsegenskaper. Bli inte alltför förvånade om fyrfärgssatsen nämns heller.

**2 Wednesday 30 November 2005, 13.15-14.15, Glashuset.**

**Chris Eilbeck, Institut Mittag-Leffler and Heriot-Watt University, Edinburgh, UK**

*Title: Breathers in discrete systems.*

*Abstract.*

The concept of a solitary wave or soliton has been around since Scott Russell's discoveries in the 1830's, but the "breather" as a type of wave in nonlinear systems is less well known. Breathers, like solitary waves, are localized, but in addition they have some internal oscillations. They are more prevalent in discrete lattice systems rather than continuum systems. I will concentrate on a simple model for breathers in coupled oscillator lattices, the Discrete Nonlinear Schrödinger equation, and survey briefly both the classical and quantum version of this system.

**2 Wednesday 7 December 2005, 13.15-14.15, Glashuset.**

**Andrew G. Bakan, Nacional Academy of Sciences of Ukraine**

*Title: Representations of measures with simultaneous polynomial denseness in all  $L_p$  spaces.*

*Abstract:*

**Representation of measures with simultaneous polynomial denseness in all  $L_p(\mathbb{R}, d\mu)$ ,  $1 \leq p < \infty$**

Andrew G. Bakan

**SHORT ABSTRACT.** It was found the characterisations of positive finite Borel measures with unbounded support on the real axis so that the algebraic polynomials are dense in all spaces  $L_p(\mathbb{R}, d\mu)$ ,  $1 \leq p < \infty$ . These conditions apply, in particular, to the measures satisfying the classical Carleman conditions.

Dr. Andrew G. Bakan  
Institute of Mathematics  
National Academy of Sciences of Ukraine  
Kyiv  
Ukraine

**COMPLETE ABSTRACT.** For positive Borel measure  $\mu$  on  $\mathbb{R}$  with unbounded support it was proved that algebraic polynomials are dense in all spaces  $L_p(\mathbb{R}, d\mu)$ ,  $1 \leq p < \infty$ , if and only if the measure  $\mu$  can be represented in the following form:  $\mu(A) := \int_A w(x) d\nu(x) \quad \forall A \in \mathcal{B}(\mathbb{R})$ , where  $\nu$  is some finite positive Borel measure on  $\mathbb{R}$  and  $w$  is some upper semicontinuous on  $\mathbb{R}$  function  $w : \mathbb{R} \rightarrow [0, 1]$ ,  $\|x^n\|_w < \infty \quad \forall n \geq 0$ , for which algebraic polynomials are dense in all seminormed spaces  $C_{w^\tau}^0 := \{f \in C(\mathbb{R}) \mid \lim_{|x| \rightarrow \infty} w(x)^\tau f(x) = 0\}, \|\cdot\|_{w^\tau}, 0 < \tau < \infty$ . Here  $\|f\|_w := \sup_{x \in \mathbb{R}} w(x)|f(x)|$  and  $\mathcal{B}(\mathbb{R})$  denotes the family of all Borel subsets of  $\mathbb{R}$ . The more special representations were obtained for the measures satisfying the classical Carleman conditions.

**2 Monday 19 December 2005, 14.15-15.15, Glashuset.**

**Bruce Sagan, Michigan State University**

*Title: Congruences for Combinatorial Sequences*

**Abstract.** We derive congruences for various sequences involving binomial coefficients. In particular, we are able to prove some conjectures of Benoit Cloitre. Surprisingly, the Thue-Morse sequence (from the theory of combinatorics on words) makes an appearance. No prior knowledge of combinatorics will be assumed. (Joint work with Emeric Deutsch.)

# The Mathematical Colloquia held in 2004 at Linköpings universitet

Organized by Anders Björn, Vladimir Kozlov, Svante Linusson and Stefan Rauch-Wojciechowski.

## 2 Wednesday 14 January 2004.

**Professor A. V. Shapovalov, Corresponding Member of Russian Academy of Sciences, Tomsk University,**  
*Commutative and noncommutative separation of variables in the Klein-Gordon equation*

### Abstract:

In the framework of theorem on necessary and sufficient conditions for separation of variables in the scalar 2<sup>nd</sup> order PDE admitting a complete set of 1<sup>st</sup> and 2<sup>nd</sup> order symmetry operators a commutative separation of variables in the Klein-Gordon equation is performed. A method of separation of variables with the use of noncommutative symmetry operators is also explained and compared with the previous one.

## 2 Wednesday 21 January 2004.

**Professor Ernie Kalnins, University of Waikato, New Zealand, Fellow of the Royal Society of New Zealand,**  
*Special functions and group representation theory*

### Abstract:

An explanation of the connection between group theory and the classical special functions of mathematical physics is outlined. Particular attention is paid to the case of Euclidean motions in the plane and the corresponding Hemholtz equation. Properties of special functions such as Mathieu functions, Bessel functions and parabolic cylinder functions are obtained. An addition theorem for Bessel functions is given a group theoretic interpretation. Extension of these ideas to more general cases is briefly discussed.

## 2 Friday 30 January 2004.

**Raffaele Rani, Department of Theoretical Astrophysics, University of Tübingen,**  
*A numerical approach to solving evolution equations for isolated systems in general relativity*

### Abstract:

We present a numerical scheme to evolve isolated systems in General Relativity. Isolated systems serve as models for real systems of physical interest like neutron stars or merging black holes. This particular class of systems is described by asymptotically flat spacetimes which possess the property of admitting a conformal extension. We work directly on the extended conformal manifold and solve numerically the conformal field equations, which assure that Einstein equations hold in the original spacetime. Because of the compactness of the conformal spacetime the whole manifold can be computed on a finite numerical grid. The final goal of this work is to apply these techniques to a self-gravitating, isolated system consisting of a distorted black hole and analyse the outgoing gravitational radiation comparing the results with the quasi normal mode behaviour observed in linear perturbation theory.

## 2 Wednesday 4 February 2004.

**Dr. Rieuwert Blok, Rome University,**  
*Activity on matroids and relations to topology and algebra*

### Abstract:

A matroid can be thought of as a ground set  $E$  together with a family of base subsets. For example, a finite spanning set  $E$  for a vector space, together with the family of linear bases contained in  $E$  forms a matroid. Although initially largely motivated by problems in linear algebra and graph theory, matroid theory provides a unified framework for studying topics from a variety of areas including design theory, combinatorial geometry, lattice theory, hyperplane arrangements, and combinatorial optimization.

After ordering the ground set  $E$  linearly one can introduce the notion of activity. This is best seen in the context of a graphic matroid, but generalizes easily. A graphic matroid is the matroid on the edge set of a connected graph  $G$ , where the base sets are the spanning trees. Given a spanning tree  $T$ , any edge  $e$  not in  $T$  closes off a unique circuit  $C$  in  $T + e$ . If  $e$  happens to be the least edge in  $C$ , then  $e$  is called (externally) active for  $T$ . Note that by inserting  $e$  and removing any other edge from  $C$  one obtains a new spanning tree that is "close" to  $T$ , but cheaper. Activity is used for instance to define search algorithms in graphs. Also, the famous Tutte polynomial is the activity generating function for the base sets of the matroid.

An interesting invariant of a matroid  $M$  is its Orlik-Solomon algebra. For instance, in the context of a complex hyperplane arrangement, it is isomorphic to the cohomology ring of the arrangement's complement. In general however, it is unknown exactly what (combinatorial) aspects of the matroid it captures. It is known though that the independent sets of zero activity yield a (linear) basis for this algebra.

Las Vergnas introduced a partial order on the base sets whose rank function is (external) activity; its atoms correspond to basis elements of the algebra. This poset encapsulates information on the linear dependence of other elements on the basis corresponding to its atoms. A fundamental invariant of this lattice, its Möbius function, was analyzed by Bruce Sagan and the speaker through the topology of its order complex. At this point we present new directions for research, new results, and try to understand their meaning for the Orlik-Solomon algebra and its relation to the matroid.

No prior knowledge of matroid theory, lattice theory, or homology theory is required.

## 2 Wednesday 11 February 2004.

**Armen Asratian,**  
*Localization theorems in Hamiltonian graph theory*

### Abstract:

A *Hamilton cycle* of a graph  $G$  is a walk in  $G$  that starts and finishes at the same vertex and visits each other vertex exactly once. A *Hamilton path* of a graph  $G$  is a path that includes each vertex of  $G$  exactly once. Some problems in algebra and combinatorics can be formulated as problems of the existence of a Hamilton path or cycle in an appropriate graph. It is known that the classical global criteria for the existence of Hamilton cycles and paths only apply to the graphs with large edge density and small diameter.

In 1984-1990 A. Asratian and N. Khachatryan developed some local criteria for the existence of Hamilton cycles in a connected graph, which are analogues of the global criteria due to Dirac, Ore and others. The idea was to show that the global concept of hamiltonicity can, under rather general conditions, be captured by local phenomena, using the structure of balls of small radii. This local approach gives the possibility to find new classes of graphs with Hamilton cycles which, in particular, also contain infinite subclasses of graphs with small edge density and large diameter.

I will give a review of this topic and present some new results.

## 2 Wednesday 18 February 2004.

**Dr. Alfonso García-Parrado, University of the Basque Country, Bilbao,**  
*Causality, geometry and generalized symmetries*

### Abstract:

In this seminar we present a number of results dealing with Lorentzian manifolds and more general manifolds equipped with a metric of arbitrary signature. On the one hand we provide a new tool to study global causal properties of Lorentzian manifolds paying special attention to its applications to the study of the global causal properties of spacetimes. We

are able to provide a simple mean to decide if two given Lorentzian manifolds look the same from the causal point of view and we can generalize the classical hierarchy of causality conditions used in general relativity among other things. On the other hand we generalize the well known conformal motions to the so called bi-conformal vector fields whose differential conditions involves two complementary orthogonal projectors  $P_{ab}$  and  $P_{\bar{a}\bar{b}}$ . These generalized symmetries can in principle be defined in any manifold endowed with a metric and we argue that they may be used to provide an intrinsic geometric characterization of certain spaces in which the metric tensor decomposes in two pieces (double twisted spaces).

## 2 Wednesday 25 February 2004.

**Andreas Rietz,**

*Noncoercive contact problems with friction*

*Abstract:*

I open this talk by discussing the mathematical properties of contact problems with friction. I then focus on noncoercive contact problems, that occur for example when the body has no prescribed displacements. Some recent results on the existence of solutions are presented and these results are interpreted mechanically in terms of dynamical stability. I also present the main ideas on how to derive these results.

## 2 Wednesday 3 March 2004.

**Dr. Federico Incitti, Rome,**

*Bruhat order on the involutions of classical Weyl groups*

*Abstract:*

It is known that a Coxeter group  $W$ , partially ordered by the Bruhat order, is a graded poset, with rank function given by the length, and that it is EL-shellable, hence Cohen--Macaulay, and Eulerian.

We wish to investigate whether a particular subposet of  $W$ , namely that induced by the set of involutions of  $W$ , which we denote by  $\text{Invol}(W)$ , is endowed with similar properties.

The problem arises from a geometric question. In fact, as well as the Bruhat order on the symmetric group encodes the cell decomposition of Shubert varieties, the Bruhat order on its involutions, considered for the first time by Richardson and Springer in 1990 in a more general context, describes the cell decomposition of some symmetric varieties.

In this talk we prove that if  $W$  is a classical Weyl group, then the poset  $\text{Invol}(W)$  is graded, with rank function given by the average between the length and the absolute length, and that it is EL-shellable, hence Cohen--Macaulay, and Eulerian.

The proofs are combinatorial and use the descriptions of classical Weyl groups in terms of permutation groups: the symmetric group for type  $A_n$ , the hyperoctahedral group for type  $B_n$  and the even-signed permutation group for type  $D_n$ .

In particular we obtain, as new results, a combinatorial description of the absolute length of the involutions in classical Weyl groups, and a combinatorial description of the covering relation in the hyperoctahedral group and in the even-signed permutation group.

It is also conjectured that the result proved for classical Weyl groups actually holds for every Coxeter group.

## 2 Wednesday 10 March 2004.

**Doc. Alexander Stolin, Göteborgs universitet,**

*Deformation quantization, Yang-Baxter equation and algebraic orbit method*

*Abstract:*

The fact that any Poisson bracket on a manifold  $M$  leads to a star-product on  $C^0(M)$  was explained by M. Kontsevich a few years ago. However finding of the explicit formulas in concrete cases is still an open problem. In my talk I will explain relations between quantization of the Kirillov-Kostant bracket on certain orbits of the co-adjoint representation of the simple complex finite dimensional Lie algebra  $g$  and certain representations of  $g$ . Relations with the dynamical Yang-Baxter equation will be also explained.

## 2 Wednesday 17 March 2004.

**Professor Nail Ibragimov, Blekinge tekniska högskola,**

*Invariants of families of differential equations*

*Abstract:*

The present talk is a survey of the basic method and recent results in the theory of invariants of families of differential equations.

The problem of invariants of differential equations can be dated back to Laplace's 1773 work, when young Laplace (he was 24) published his renowned method based on what is known today as the Laplace invariants  $h$  and  $k$ . These invariants (rather semi-invariants) were found earlier by Leonard Euler and published in his "Integral Calculus", 1769/70. In 1960, L. Ovsyannikov found two proper invariants for hyperbolic equations. The problem, Laplace's problem, on determining all invariants for hyperbolic equations remained open until recently.

In classical literature, invariants of families of differential equations were considered for linear equations only (J. Cockle, E. Laguerre, G. Darboux, E. Goursat, G.H. Halphen, A.R. Forsyth, etc.). S. Lie (1895) regretted that these authors did not use advantages provided by his theory of infinite continuous groups, but he himself did not undertake further developments in this direction.

Recently, I considered the possibility hinted by Lie's remark and developed the infinitesimal technique in the theory of invariants of families of equations that was lacking in the old methods. In consequence, a simple unified approach was developed for calculation of invariants of algebraic and differential equations independent on the assumption of linearity of the equations. It was employed recently for solution of Laplace's problem.

## 2 Wednesday 24 March 2004.

**Doc. Sergei Silvestrov, Lund,**

*Introduction to  $q$ -difference equations*

*Abstract:*

In this lecture an introduction to the area of  $q$ -difference equations and  $q$ -analysis will be given, algebraic structures behind  $q$ -difference operators will be described, and  $q$ -deformations of KdV-equation will be also discussed.

## 2 Wednesday 31 March 2004.

**Professor José M. M. Senovilla, University of the Basque Country, Bilbao,**

*Trapped submanifolds in Lorentzian geometry*

*Abstract:*

In Lorentzian geometry, the concept of *trapped* submanifold will be introduced by means of the properties of the mean curvature vector. Trapped submanifolds are generalizations of the standard maximal hypersurfaces and minimal surfaces, of geodesics, and also of the trapped surfaces introduced by Penrose. Examples and selected applications to gravitational theories will be presented.

## 2 Wednesday 7 April 2004.

**Vladimir Kozlov,**

*Zeros of eigenfunctions*

*Abstract:*

It is known, that the  $n$ th eigenfunction,  $n=0,1,\dots$ , to the second order Sturm-Liouville ordinary differential operator on an interval has exactly  $n$  zeros. I intend to discuss what is known about multi-dimensional problems and to present some new results.

■ **Wednesday 14 April 2004.**

**Jan-Åke Larsson,**  
*Separation of operators*

*Abstract:*

Separable Hilbert spaces are familiar to those who have been taught functional analysis, and separation of variables in differential equations is a standard tool being researched here at the Mathematics Department. I will here introduce another notion of separability, relevant for positive operators with trace one (e.g., quantum-mechanical "states"), and some criteria to determine separability. Among other things, the difference between positive maps and completely positive maps will be important in this context.

■ **Wednesday 21 April 2004.**

**Dr. Torbjörn Lundh, Chalmers,**  
*Möbius mushrooms and other complex organisms*

*Abstract:*

One of the more intriguing questions in embryology is how we, from one uniform egg, through successive cell-divisions, become those complex creatures we are. This riddle has engaged people for centuries. Today, when we are faced with overwhelming genetic information, we are mainly intrigued by the question how this DNA-code is realized into a developing organism.

In the colloquium, we are going to look at a few embryonic developments, and a couple of mathematical models which are used today in morphogenesis (a term which was introduced by Goethe when he was studying how biological organisms got their shape). We will also speculate if some other mathematical tools could be useful in the study of morphogenesis, such as Möbius mappings.

■ **Tuesday 27 April 2004.**

**Professor Ari Laptev, KTH,**  
*Folyltons and the removal of eigenvalues for fourth order differential operators*

*Abstract:*

A non-linear functional  $Q[u,v]$  is given that governs the loss, respectively gain, of (doubly degenerate) eigenvalues of fourth order differential operators  $L = \partial^4 + \partial^2 u + \partial^2 v$  on the line. (Here  $\partial$  stands for the curly  $d$ .) Apart from factorizing  $L$  as  $A^*A + E_0$ , providing several explicit examples, and deriving various relations between  $u$ ,  $v$  and eigenfunctions of  $L$ , we find  $u$  and  $v$  such that  $L$  is isospectral to the free operator  $L_0 = \partial^4$  up to one (multiplicity 2) eigenvalue  $E_0 < 0$ . Not unexpectedly, this choice of  $u$  and  $v$  leads to exact solutions of the corresponding time-dependent partial differential equations.

■ **Wednesday 5 May 2004.**

**Gunnar Aronsson,**  
*On the  $p$ -Laplace equation including its limit case for  $p=\infty$ , and related problems*

*Abstract:*

The talk will be a survey of the  $p$ -Laplace equation (also called the  $p$ -harmonic equation) and its limit case, the infinity-Laplace equation, in euclidean  $n$ -space. The point of departure will be the classical Laplace equation. Special attention will be given to geometric aspects, regularity questions, some similarities with complex analysis for the case  $n=2$ , and a brief discussion of various solution concepts.

■ **Wednesday 12 May 2004.**

**Doc. Genkai Zhang, Chalmers,**  
*Segal-Bargmann transforms and their generalizations*

*Abstract:*

The Segal-Bargmann transform is a unitary integral operator from the  $L^2$  space on the real space  $\mathbb{R}^1$  to the Fock space of holomorphic functions on the complex space  $\mathbb{C}^1$ , and it intertwines two different models of the Heisenberg commutation relation. We introduce an analogue of the Segal-Bargmann transform on the unit disk in the complex plane, and a general Riemannian symmetric bounded domain.

■ **Wednesday 19 May 2004.**

**Dr. Krzysztof Marciniak, ITN**  
*Geometric approach to Dirac theory of constrained Hamiltonian systems*

*Abstract:*

Given a foliation  $S$  of a manifold  $M$ , a distribution  $Z$  in  $M$  transversal to  $S$  and a Hamiltonian system on  $M$  we present a geometrical method of reducing this system on the foliation  $S$ . We analyse its relation with the classical ideas of P.A.M. Dirac on constrained Hamiltonian systems (Dirac reduction) and with the more modern theory of J. Marsden and T. Ratiu. Our method is constructive and "constraint-independent" in the sense that it does not depend on the functions that define the foliation  $S$ . As a consequence, in case of a second class constraints (in the terminology of Dirac) one can perform not only usual Dirac reduction of Poisson (Hamiltonian) systems to submanifolds but also other types of reductions.

■ **Tuesday 25 May 2004.**

**Johan Lundvall defended his licentiate thesis,**  
*Reconstruction of velocity data using adjoint optimization*

*Abstract:*

In many application areas there is a growing interest in data assimilation or data reconstruction. Data assimilation is a process for integrating observed or measured data into a physical model. The problem originates from a vast array of different topics: traditionally in meteorological and oceanographic modelling, and recently from non-invasive medical measurement devices such as magnetic resonance imaging. The measured data may contain inaccuracies and random noise, given with low spatial and/or temporal resolution.

This thesis presents a method for solving reconstruction problems in fluid dynamics using optimal control theory. The problem considered here includes a known partial differential equation and some spatially and temporarily sparsely distributed data with an unknown initial state. From a given velocity field  $u^*$ , a flow field  $u$  is determined which satisfies a given system of partial differential equations and minimizes  $\|u - u^*\|_{L^2}$ . The function  $u(x,t)$  is known at the boundary and the initial condition  $u_0(x)$  is used as design variable. The optimization problem is solved using adjoint formulation.

■ **Tuesday 1 June 2004.**

**Professor Sergei Avdonin, University of Alaska, Fairbanks,**  
*Boundary control method in inverse problems of mathematical physics*

*Abstract:*

The boundary control (BC) method reveals that the two central problems of the theory of inverse problems and control theory have a direct connection with each other. The first one, together with the recovery of the coefficients, consists in the construction of a map extending the solution inside the domain, from the data on their behavior on the boundary. The second one is the controllability of the corresponding initial boundary value problem. Roughly speaking, the BC method gives the realization for distributed systems of R. Kalman's idea that the controllable (or observable) part of a system can be identified. One of the important results of the BC method is obtaining multidimensional analogs of the Gelfand-Levitan-Krein-Marchenko equations. It is interesting to note that these equations have a clear control-theoretic meaning which makes their derivation much more simple than the original derivation of one-dimensional versions.

The BC method was first proposed for the multidimensional wave equation (Belishev, 1987) and was extended to non-self-adjoint inverse problems, to the heat equation and several

other types of PDEs. In the first part of this talk we shall describe the main ideas of this method on a classical example of the (1d) string equation. Then we shall discuss new controllability and identification results for the Schrödinger equation and for the wave equation on graphs.

**2 Wednesday 9 June 2004.**

**Professor Roland Häggkvist, Umeå,**  
*On the cycle double conjecture*

*Abstract:*

A cycle double cover of a graph  $G$  is a collection  $C$  of cycles of  $G$  such that each edge of  $G$  belongs to exactly two members of  $C$ . Szekeres (1973) and Seymour (1979), motivated by quite different considerations, conjectured that *every 2-edge-connected graph admits a circuit double cover*. This conjecture is called the *cycle double conjecture*.

I shall survey some of my work on this conjecture.

Recently together with Herbert Fleischner I found a simple proof of his old theorem that the strong cycle double conjecture is true for hypohamiltonian graphs. I shall talk about this proof as well as on my joint work with Klas Markström on the following problem: Assume that the 3-regular graph  $G$  is obtained from a 3-regular graph  $H$ , all of whose components have a proper 3-edge-colouring, where every pair of colour classes form a hamiltonian cycle by the rule that into each component of  $H$  we insert an even number of vertices into the edges and arbitrarily adding a matching incident with the new vertices. Show that  $G$  has a cycle double cover!

**2 Friday 27 August 2004.**

**Markus Sköldstam defended his licentiate thesis,**  
*Analysis of the phase space, asymptotic behaviour and stability for heavy symmetric top and tippe top*

*Abstract:*

In this thesis we analyse the phase space of the heavy symmetric top and the tippe top. These tops are one of the very few examples of rigid bodies for which the structure of the phase space can be completely analysed. The heavy symmetric top is the standard example in all textbooks but the proofs of stability of the vertical rotation that are presented there are logically not correct, they provide only the right value of the threshold angular velocity. We provide a complete proof.

The tippe top is a toy that has the form of a truncated sphere equipped with a little peg. When spun fast on the spherical bottom its centre of mass rises above its geometrical centre and after a few seconds the top is spinning vertically on the peg. We study the tippe top through a sequence of embedded invariant manifolds to unveil the structure of the top's phase space. The last manifold, consisting of the asymptotic trajectories, is analysed completely. We prove that trajectories in this manifold attract solutions that stay in the plane of support at all times. We give (for the first time) a complete description of stability/instability of asymptotic motions for all admissible choices of the model parameters and for all initial conditions.

**2 Wednesday 1 September 2004.**

**Peter Rand,**  
*Asymptotic analysis of a nonlinear partial differential equation in a semicylinder*

*Abstract:*

Small solutions of a nonlinear partial differential equation in a semi-infinite cylinder will be studied. We consider the asymptotic behaviour of these solutions at infinity under Neumann boundary condition as well as Dirichlet boundary condition. In the Neumann case it can be shown that any solution small enough either vanishes at infinity or tends to a nonzero periodic solution of a nonlinear ordinary differential equation. In the Dirichlet case every solution small enough vanishes. Parts of the proofs of these statements will be given.

**2 Monday 6 September 2004, 13.15-14.00.**

**Professor Michael Zarichnyi, Lviv University (Ukraine) and University of Rzeszow (Polen),**  
*Large scale topology*

*Abstract:*

The asymptotic topology deals with the large scale properties of metric spaces. Recently, its results found deep applications in geometric functional analysis, group theory, and topology of manifolds. The aim of the talk is to introduce some fundamental notions of asymptotic topology as well as to formulate some results on absolute extensors in asymptotic categories, embedding theorems and coarse invariants.

**2 Wednesday 8 September 2004.**

**Professor Ljudmila Bordag, Halmstad,**  
*Projective differential geometrical structure of the Painlevé equations*

*Abstract:*

The necessary and sufficient conditions that an equation of the form  $y''=f(x,y,y')$  can be reduced to one of the Painlevé equations under a general point transformation are obtained. A procedure to check these conditions is found. The theory of invariants plays a leading role in this investigation. The reduction of all six Painlevé equations to the form  $y''=f(x,y)$  is obtained. The structure of equivalence classes is investigated for all the Painlevé equations. Following Cartan the space of the normal projective connection which is uniquely associated with any class of equivalent equations is considered. The specific structure of the spaces under investigation allows us to immerse them into  $\mathbf{RP}^3$ . Each immersion generates a triple of two-dimensional manifolds in  $\mathbf{RP}^3$ . The surfaces corresponding to all the Painlevé equations are presented.

**2 Wednesday 15 September 2004.**

**Professor Nikolai Kuzjurin, Russian Academy of Sciences, Moscow,**  
*Probabilistic methods in packing and covering problems*

*Abstract:*

Packing and covering problems form a wide class of combinatorial problems where probabilistic methods play an important role. Using probabilistic techniques some best known bounds were obtained in packing and covering problems including the famous result about the existence of nearly perfect packings and coverings (Rödl, 1985). A natural question is: can one obtain similar results without using probabilistic methods? In several cases it is possible to give a positive answer and to present explicit constructions.

In my talk I will describe the main ideas of explicit constructions of nearly perfect packings and then present a simple probabilistic method for counting the number of nearly perfect packings. In the second part of my talk I am going to present a new class of packing problems that arise in scheduling parallel tasks in networks. Such problems can be reformulated as packing of a given set of rectangles (corresponding to tasks) into a set of strips (corresponding to computers in a network). Some new results about approximation algorithms for this problem will be presented for the worst and the average cases.

**2 Wednesday 22 September 2004.**

**Peter Rand defended his licentiate thesis,**  
*Asymptotic analysis of a nonlinear partial differential equation in a semicylinder*

*Abstract:*

We study small solutions of a nonlinear partial differential equation in a semi-infinite cylinder. The asymptotic behaviour of these solutions at infinity is determined. First, the equation under the Neumann boundary condition is studied. We show that any solution small enough either vanishes at infinity or tends to a nonzero periodic solution of a nonlinear ordinary differential equation. Thereafter, the same equation under the Dirichlet boundary condition is studied, but now the nonlinear term and right-hand side are slightly more general than in the Neumann problem. Here, an estimate of the solution in terms of the right-hand side of the equation is given. If the equation is homogeneous, then every solution small enough tends to zero. Moreover, if the cross-section is star-shaped and the nonlinear term in the equation is subject to some additional constraints, then every bounded solution of the homogeneous Dirichlet problem vanishes at infinity. An estimate for the solution is given.



■ **Wednesday 22 September 2004.**

**Professor Henrik Shahgholian, KTH,**

*The structure of the singular set of a free boundary in potential theory*

*Abstract:*

We characterize the structure of the singular set in the following free boundary problem

$$(\Delta u - f)u = 0, \quad \text{in } B=B(0,1),$$

where  $f$  is Lipschitz, and  $u$  in  $W^{2,p}(B)$ ,  $p > n$ . The free boundary  $\partial\Omega$ , represented by  $\partial\Delta u = f$ , appears in certain problems in geophysics and inverse problems in potential theory.

This is joint work with Luis Caffarelli.

■ **Friday 24 September 2004.**

**Jonas Bergman defended his licentiate thesis,**

*Conformal Einstein spaces and Bach tensor generalizations in  $n$  dimensions*

*Abstract:*

In this thesis we investigate necessary and sufficient conditions for an  $n$ -dimensional space,  $n \geq 4$ , to be locally conformal to an Einstein space. After reviewing the classical results derived in tensors we consider the four-dimensional spinor result of Kozameh, Newman and Tod. The involvement of the four-dimensional Bach tensor (which is divergence-free and conformally well-behaved) in their result motivates a search for an  $n$ -dimensional generalization of the Bach tensor  $B_{ab}$  with the same properties. We strengthen a theorem due to Belfagion and Ja' en and give a basis  $(U_{ab}, V_{ab} \text{ and } W_{ab})$  for all  $n$ -dimensional symmetric, divergence-free 2-index tensors quadratic in the Riemann curvature tensor. We discover the simple relationship  $B_{ab} = (1/2) U_{ab} + (1/6) V_{ab}$  and show that the Bach tensor is the unique tensor with these properties in four dimensions. Unfortunately we have to conclude, in general that there is no direct analogue in higher dimension with all these properties.

Nevertheless, we are able to generalize the four-dimensional results due to Kozameh, Newman and Tod to  $n$  dimensions. We show that a generic space is conformal to an Einstein space if and only if there exists a vector field satisfying two conditions. The explicit use of dimensionally dependent identities (some of which are newly derived in this thesis) is also exploited in order to make the two conditions as simple as possible; explicit examples are given in five and six dimensions using these tensor identities.

For  $n$  dimensions, we define the tensors  $\mathfrak{b}_{abc}$  and  $\mathfrak{B}_{ab}$ , and we show that their vanishing is a conformal invariant property which guarantees that the space with non-degenerate Weyl tensor is a conformal Einstein space.

■ **Wednesday 6 October 2004.**

**Professor Natan Krugljak, Luleå,**

*On one new covering theorem and its applications*

*Abstract:*

Classical covering theorems (Vitali, Whitney, Besicovitch) are not only beautiful but they also have important applications in analysis, harmonic analysis, theory of approximations, ergodic theory and PDEs. Some years ago in connection with real interpolation of Sobolev spaces appeared theorems which have simultaneously features of Whitney and Besicovitch covering theorems. I plan to discuss these new theorems and their applications to interpolation and possible applications to singular integrals.

■ **Thursday-Saturday 14-16 October 2004.**

**Conference,**

*The  $p$ -Laplace equation, the infinity-Laplace equation and related topics.*

■ **Wednesday 20 October 2004.**

**Professor Juan J. Manfredi, University of Pittsburgh,**

*Convexity from the PDE point of view*

*Abstract:*

Convex functions in Euclidean space play an important role in the regularity theory of non-linear elliptic partial differential equations. They can be characterized as universal subsolutions of homogeneous fully nonlinear second order elliptic partial differential equations. In the first part of the talk, we will first show that this PDE definition is equivalent to the usual one. Then we will use well-known estimates for subsolutions of familiar PDEs to derive estimates for convex functions.

Another advantage of the PDE definition of convexity is that it can be considered in the case of Carnot groups. In the second part of the talk we will present the theory of convex functions on Carnot groups. Our approach is based on the viscosity theory of subsolutions for subelliptic equations and the geometric role played by infinity-harmonic functions.

■ **Wednesday 27 October 2004.**

**Jonna Gill defended her licentiate thesis,**

*The  $k$ -assignment polytope and the space of evolutionary trees*

*Abstract:*

This thesis consists of two papers.

The first paper is a study of the structure of the  $k$ -assignment polytope, whose vertices are the  $m \times n$   $\{0,1\}$ -matrices with exactly  $k$  1:s and at most one 1 in each row and each column. This is a natural generalisation of the Birkhoff polytope and many of the known properties of the Birkhoff polytope are generalised. Two equivalent representations of the faces are given, one as  $\{0,1\}$ -matrices and one as ear decompositions of bipartite graphs. These tools are used to describe properties of the polytope, especially a complete description of the cover relation in the face lattice of the polytope and an exact expression for the diameter.

The second paper studies the edge-product space  $E(X)$  for trees on  $X$ . This space is generated by the set of edge-weighted finite trees on  $X$ , and arises by multiplying the weights of edges on paths in trees. These spaces are closely connected to tree-indexed Markov processes in molecular evolutionary biology. It is known that  $E(X)$  has a natural CW-complex structure, and a combinatorial description of the associated face poset exists which is a poset  $S(X)$  of  $X$ -forests. In this paper it is shown that the edge-product space is a regular cell complex. One important part in showing that is to conclude that all intervals  $[\hat{0}, \text{genxf}]$ ,  $\text{genxf} \in S(X)$ , have recursive coatom orderings.

■ **Wednesday 27 October 2004.**

**Professor Anders Björner, KTH,**

*Blockers and vanishing ideals of subspace arrangements*

*Abstract:*

The *blocker* of a set family  $A$  is the collection of inclusionwise minimal sets that intersect all sets in  $A$ . This construction is well-known in combinatorics and combinatorial optimization. The corresponding construction on set partitions (and more generally on geometric lattices) arises in the study of vanishing ideals of arrangements of linear subspaces in a vector space.

I will survey examples and properties of blockers and a combinatorial duality that they satisfy, beginning with some new results on blocker duality in general posets. I will then describe the relevance of this concept for vanishing ideals that are generated by products of linear forms. The results also touch on some Turan-type problems from extremal combinatorics.

The talk is based on joint work with A. Hultman, I. Peeva and J. Sidman.

📅 **Wednesday 3 November 2004.**

**Professor Martina Šimunková, Liberec, Czech Republic,**  
*The Poisson Integral and the Kelvin Transform*

*Abstract:*

The Poisson integral which gives an explicit solution of the Dirichlet problem of the Laplace equation on a ball plays an important role in classical potential theory. There are several possibilities to derive the Poisson integral - some of them are based on the Kelvin transform. The connection between the Poisson integral and the Kelvin transform will be shown. Also Kelvin type transforms of elliptical operators will be described.

📅 **Wednesday 10 November 2004.**

**Professor Antonio F. Costa, UNED, Madrid,**  
*On the connectedness of the locus of real Riemann surfaces in the moduli space*

*Abstract:*

This talk is dedicated to expose some results on the connectedness of the set of special types of real curves in the moduli space of complex algebraic curves. We review some known results where the above set is connected. We present some cases where the set of real  $p$ -gonal algebraic curves is not connected. We also show that the set of real elliptic-hyperelliptic curves of even genus  $> 5$  is not connected.

📅 **Thursday 11 November 2004, 10.15.**

**Daniel Ying defended his licentiate thesis,**  
*Cyclic trigonal Riemann surfaces of genus 4*

*Abstract:*

A closed Riemann surface which can be realized as a 3-sheeted covering of the Riemann sphere is called trigonal, and such a covering is called a trigonal morphism. Accola showed that the trigonal morphism is unique for Riemann surfaces of genus  $g > 5$ . This thesis will characterize the Riemann surfaces of genus 4 with non-unique trigonal morphism. We will describe the structure of the space of cyclic trigonal Riemann surfaces of genus 4.

📅 **Wednesday 17 November 2004.**

**Svante Linusson,**  
*A survey on trees in mathematics and biology*

*Abstract:*

A mathematical tree is easy to understand and has been studied not only in pure mathematics. Trees have also been important objects in e.g. optimisation and computer science as for instance search trees, decision trees and computational trees. It would not be unreasonable to believe that such a simple object with many applications would be so thoroughly studied that when questions arise in the new biology related mostly to evolutionary trees, the mathematicians could present all the relevant answers at once. This is however not the case. Several new difficult problems arise.

I will give a survey of interesting theorems/conjectures about trees from both pure mathematics and biology. It is my firm belief that mathematicians have a crucial role in transforming biology into an information science. But also that biology can inspire new beautiful and interesting mathematics.

The talk does not require much prerequisites of the audience. It will be understandable to every mathematician at the department, especially every PhD student.

📅 **Wednesday 24 November 2004.**

**Dr. Mattias Jonsson, KTH,**  
*Singularities in complex dynamics*

*Abstract:*

I will discuss how algebro-geometric methods can sometimes be used to study objects of nonalgebraic nature, e.g. certain dynamical systems.

In dynamics one is often interested in asymptotic behavior as time evolves. For instance, given a polynomial map  $F: \mathbf{C}^2 \dashrightarrow \mathbf{C}^2$  one may ask at what speed the orbit  $p, F(p), F(F(p)), \dots, F^n(p), \dots$  approaches infinity, as  $n$  tends to infinity, if the original point  $p$  is chosen generically near infinity. This speed is governed by the behavior of  $\deg F^n$ , the degree of the highest order term in  $F^n$ . For example, if  $F(X, Y) = (Y, XY)$ , then  $\deg F^n$  gives the Fibonacci numbers, so in a suitable sense, the speed above equals the golden mean.

A classical field of algebraic geometry is the study of singularities, such as the curve in  $\mathbf{C}^2$  parameterized by  $t \mapsto (t^2, t^3)$ , which has a cusp at the origin. It is known that singularities typically can be resolved, i.e. viewed as shadows of nonsingular objects; the cusp above is the shadow of the space curve  $t \mapsto (t^2, t^3)$ .

As I will explain, it turns out that a dynamic version of resolution of curve singularities can be used to understand the speed of convergence to infinity of polynomial maps of  $\mathbf{C}^2$ . As a consequence, the speed is always a quadratic integer.

📅 **Wednesday 1 December 2004.**

**Professor Nikolay Kuznetsov, St. Petersburg,**  
*Uniqueness in the water-wave problem for bodies intersecting the free surface at arbitrary angles*

*Abstract:*

The linearized water-wave problem involving a surface-piercing cylinder in water of infinite depth will be considered. A solution to this problem will be shown to be unique for all values of the radian frequency when the cylinder intersecting the free surface at arbitrary angles is subjected to certain geometric arrangements. The previous result by Simon and Ursell (1984) provided the uniqueness only for bodies intersecting the free surface at angles greater or equal to 45 degrees.

📅 **Wednesday 8 December 2004.**

**Lars Inge Hedberg,**  
*Spectral synthesis in function spaces*

*Abstract:*

I will discuss some old and new results in this area, which is related to some important problems for partial differential equations. No prerequisites beyond basic analysis will be assumed. In particular the concepts mentioned in the title will be defined in the talk.

📅 **Thursday 16 December 2004.**

**Magnus Österholm defended his licentiate thesis,**  
*Läsa matematiska texter: Förståelse och lärande i läsprocessen*  
*(Reading mathematical texts: Understanding and learning in the reading process)*

**The thesis can be downloaded from here.**

*Abstract:*

The focus of this thesis is the reading of mathematical texts, especially how and what you can understand and learn from reading. The main interest is the reading process, that is, the

reading itself and what you understand after reading a text. The main purpose is to study the reading of mathematical texts in particular, in order to test and develop an existing general theory of reading comprehension. An essential part is to study how the use of symbols in mathematical texts can affect the reading process. The thesis consists of theoretical discussions about the reading of mathematical texts and an empirical study among students from the Swedish upper secondary school and from the university.

A study of existing literature that deal with properties of mathematical texts constitute a starting point for the theoretical discussions. In particular, the reading of mathematical symbols and algebraic expressions is discussed.

The empirical study, which includes 106 participants, used three different texts: one history text about the Russian revolution and two mathematical texts about group theory. The same information about groups is included in both mathematical texts, but one of the texts uses mathematical symbols in the presentation while the other does not use symbols at all. Each participant read one of the mathematical texts and the history text, and after each text they got to answer questions about the content of the text.

The group of participants who read the mathematical text without symbols has a better result on the questions about the text than the group of participants who read the text with symbols. This seems to be caused by an inability to articulate the symbols and by the fact that the ability to decode a text does not seem to be used in the same way for the text with symbols as it is for the history text and the mathematical texts without symbols. Thus, the reading of mathematical texts with symbols is a rather special activity and there might be a need for learning how to read such texts. On the other hand, there seems to be many similarities between the reading of the mathematical text without symbols and the historical text. Therefore, the main factor of the texts influencing the reading process is not the content of the text but the form of the text, that is, how the content is presented.

In the theoretical discussions, some suggestions are presented on how the reading of mathematical texts with symbols can be included in the general theory of reading comprehension. In general there is no reason to consider the reading of mathematical texts as a special kind of reading process, different from the reading of other types of texts. Thus, the general theory of reading comprehension can function as a theoretical framework also for the comprehension of mathematical texts, possibly with the suggested additions about mathematical symbols.

# The Mathematical Colloquia held in 2003 at Linköpings universitet

Organized by Anders Björn, Vladimir Kozlov, Svante Linusson, Stefan Rauch-Wojciechowski and Claes Waksjö.

## 2 Wednesday 22 January 2003.

**Docentföreläsning by Jana Björn,**

*Wiener criterion and boundary continuity of solutions to the Dirichlet problem*

*Abstract:*

The classical Dirichlet problem for the Laplace equation is the problem of finding a harmonic function in a given domain so that it has prescribed boundary values on the boundary of the domain. For domains with sufficiently smooth boundary and continuous boundary data, solutions of the Dirichlet problem are continuous up to the boundary and attain their boundary data at every boundary point. This is no longer true for more general domains.

By the celebrated Wiener criterion from 1924, solutions of the Dirichlet problem for the Laplace equation with continuous boundary data are continuous at a boundary point if and only if the so called Wiener integral associated with this point diverges. Similar criteria have later been obtained for p-harmonic equations, which are non-linear analogues to the Laplace equation, and for other elliptic equations.

In recent years, the Dirichlet problem for p-harmonic equations has been studied in the context of metric measure spaces without a differentiable structure. This new theory unites and generalizes some of the earlier results, but it also gives new results in the classical setting.

In the talk, I will give a survey of some older results related to the Wiener criterion and discuss some new results concerning boundary continuity of solutions to the Dirichlet problem.

## 2 Wednesday 12 February 2003.

**Docentföreläsning by Anders Björn,**

*Removable singularities for spaces of analytic functions*

*Abstract:*

Small exceptional sets that can be ignored occur in many parts of mathematics. For instance, when calculating the integral of a function one can ignore the values of the function on any set of (Lebesgue) measure zero.

Another example of exceptional sets is removable singularities. Consider an open connected set  $G$  in the complex plane and let  $E$  be a compact subset of  $G$ . We can then consider the class consisting of all bounded analytic functions on the set  $G \setminus E$ . We say that the set  $E$  is a removable singularity (for this class) if all of these functions are analytic not only on  $G \setminus E$  but also on all of  $G$ . (To be precise they should have analytic continuations to all of  $G$ .) This means that the set  $E$  can be ignored in this context. In this example it is well-known from any first course in complex analysis that a single point is a removable singularity. Painlevé showed already in 1888 that if  $E$  has zero generalized length, in particular if  $E$  has (Hausdorff) dimension less than one, then  $E$  is a removable singularity. It is also true that if  $E$  has dimension greater than one then it is never a removable singularity. For one-dimensional sets the story is more complicated.

In this talk I will discuss removable singularities for bounded as well as other classes of analytic functions.

## 2 Wednesday 12 February 2003.

**Erik Ouchterlony,**

*Primes in  $P$*

*Abstract:*

This seminar will be a presentation of a paper by Prof. Manindra Agarwal and two of his students, Nitin Saxena and Neeraj Kayal, who have recently discovered a polynomial time deterministic algorithm to test if an input number is prime or not. Over the centuries lots of people over have been looking for a polynomial time test for primality, and this result is a major breakthrough. The proof is neither very complex or long, the preprint is only nine pages long, and relies on innovative and insightful use of results from number theory.

## 2 Wednesday 19 February 2003.

**Professor Jan-Erik Björk, Stockholms universitet,**

*Classical mechanics: A lecture about the spinning top*

*Abstract:*

The lecture is devoted to dynamical equations describing the motion of a rigid body which rotates around a fixed point while gravity is the only external force. These dynamical equations are derived from laws of classical mechanics. The proof is very instructive, where two systems of coordinates are used - one is attached to the rigid rotating body and the other is fixed in  $\mathbb{R}^3$ .

Two cases will be discussed in more detail. First, the spinning top - Case of Lagrange - which arises when the body is symmetric and the center of mass is placed on the line of symmetry. Second, the gyroscope of Sonja Kovalevsky which arises when the body is symmetric with respect to the fixed point and the center of mass is placed in the plane of symmetry.

## 2 Wednesday 5 March 2003.

**Dr. Andreas Bette, KTH Syd,**

*On spinning objects in (special) relativity*

*Abstract:*

A relativistic (twistor) phase space formulation of the dynamics of a spinning electrically charged massive particle in an external electro-magnetic field will be presented. The function used to generate such a dynamics is chosen so that it corresponds to the (second order) Dirac operator.

If time permits, a relativistic (twistor) action integral will be presented describing a free massive spinning (no Grassman variables are needed) string.

## 2 Wednesday 12 March 2003.

**Armen Asratian,**

*Some results on proper edge colorings of graphs*

*Abstract:*

A proper edge coloring of a graph  $G$  is an assignment of colors to the edges of  $G$  such that no two adjacent edges receive the same color. I will give a review of some results on proper edge colorings of graphs and point out some applications of these results to scheduling problems and matrix theory.

## 2 Wednesday 19 March 2003.

**Lars Inge Hedberg,**

*The Schwarz lemma, the Poincaré metric, and a theorem of Ahlfors.*

*Abstract:*

In 1938 Lars Ahlfors found a beautiful generalization of the classical Schwarz lemma in complex analysis, which gave new insights into the connections between complex analysis and geometry, and had far-reaching consequences, among others simple proofs of the Picard theorems on omitted values. I will present Ahlfors's theorem from scratch, i.e., without assuming any prerequisites beyond elementary complex analysis.

Föreläget är mycket lämpligt för alla deltagare i kursen *Komplex Analys*

## 2 Wednesday 26 March 2003.

**Professor Ingemar Bengtsson, Stockholms universitet,**

*Metrics with a potential*

*Abstract:*

Differentiating a convex function twice, we get a matrix that can be regarded as a metric tensor. (It sounds like Kähler manifolds, but it is not.) I will sketch some magical tricks that

are performed in mathematical statistics using this construction, and then mention other applications including the theory of Frobenius manifolds and black hole thermodynamics.

**2 Wednesday 2 April 2003.**

**Leif Melkersson,**  
*Local cohomology*

*Abstract:*

I will try to give some glimpses into some of the main objects I deal with in my research, namely the local cohomology modules  $H_a^i(M)$ . They were introduced by Grothendieck. They provide a very powerful technical tool and they have found increasing applications in commutative algebra and neighbouring fields. As an example I may mention that if  $A = \mathbb{C}[X_1, \dots, X_n]$  is the polynomial ring, then the local cohomology modules  $H_a^i(A)$  with respect to an arbitrary ideal  $a$  are (holonomic) modules over the Weyl algebra (the ring of differential operators with polynomial coefficients). Thus in this case they satisfy strong finiteness properties. Using techniques from the theory of local cohomology I have recently solved a problem in the homological theory of noetherian commutative rings, generalizing a result of Hartshorne (Inventiones 9 (1970)).

**2 Wednesday 16 April 2003.**

**Svante Linusson and Johan Wästlund,**  
*A proof of Parisi conjecture on the random assignment problem*

*Abstract:*

An assignment problem is the optimization problem of finding, in an  $m$  by  $n$  matrix of nonnegative real numbers,  $k$  entries, no two in the same row or column, such that their sum is minimal. Such an optimization problem is called a random assignment problem if the matrix entries are random variables. We have in a recent article given a formula for the expected value of the optimal  $k$ -assignment in a matrix where some of the entries are zero, and all other entries are independent exponentially distributed random variables with mean 1. Thereby we prove the formula  $1 + 1/4 + 1/9 + \dots + 1/k^2$  conjectured by G. Parisi for the case  $k=m=n$ , and the generalized conjecture of D. Coppersmith and G. B. Sorkin for arbitrary  $k$ ,  $m$  and  $n$ .

We will give a very basic description accessible to everyone of how one may find the expected value of small random assignment problems in general. We will also give an outline of what enabled us to solve the famous conjecture.

**2 Wednesday 7 May 2003.**

**Dr. Hans Lundmark, University of Saskatchewan,**  
*Inverse scattering and multipeakons*

*Abstract:*

The Camassa-Holm equation is an integrable nonlinear PDE modelling waves in shallow water. It admits multisoliton solutions with waves having peaks of the form  $e^{-|x|}$ . These solutions, known as multipeakons, can be explicitly computed using inverse scattering, a well-known method in the theory of integrable systems. In fact, this problem is an unusually nice example for illustrating the technique, since it can be handled by completely elementary means (in contrast, for example, to the rather complicated inverse scattering theory for the KdV equation).

The main tool for deriving the explicit solution formulas is a theorem about continued fractions due to Stieltjes. The classical theory of orthogonal polynomials and the moment problem can be used to analyse properties of the solutions; in particular, to give sharp results on the steepening of the wave at peakon-antipeakon collisions.

I will give an introduction to these ideas, which go back to work by Krein on inverse spectral problems for inhomogeneous strings, and by Moser on the Toda lattice. The application to the Camassa-Holm equation is due to Beals, Sattinger and Szmigielski. I will also describe some new results by Szmigielski and myself on the corresponding problem for the Degasperis-Procesi equation, which is a recently discovered integrable modification of the Camassa-Holm equation.

**2 Wednesday 14 May 2003.**

**Daniel Mondoc,**  
*Kantor Triple Systems*

*Abstract:*

Kantor triple systems (KTS) are generalisations of Jordan triple systems. KTS are connected to 5-graded Lie algebras by the so called Kantor-Koecher-Tits construction. The KTS are called (real) classical or (real) exceptional if the corresponding Lie algebras are (real) classical or (real) exceptional, respectively. The aim of this talk is to give a presentation of the notions mentioned above and to give results on the classification of real simple compact classical and exceptional KTS defined on tensor products of composition algebras.

**2 Wednesday 21 May 2003.**

**Per-Olov Lindberg,**  
*Dynamiska trafikjämvikter*

*Abstract:*

Jag kommer att modellera s.k. dynamiska trafikjämvikter utgående från enkla tids/flödesrelationer för trafiklänkar. Vi kommer att hamna i Nash-jämvikter, som blir till variationsolikheter och sen optimeringsproblem. Jag kommer förmodligen att behöva hjälp från matematiker för att hitta lämpliga funktionsrum att badda in problemet i, och från numeriker för ev. beräkningar.

Vi har alla upplevt trafikstockningar (i synnerhet Stockholmare). Trafiklänkar karakteriseras av att restiden växer med ökande trafikflöden. Samtidigt vill resenärerna välja (t.ex. i tid) kortaste väg från start till mål, men deras vägval påverkar flödena och alltså tiderna. Vid jämvikt har inga resenärer anledning att byta rutter, under de restider som uppstår då de utnyttjar de rutter de valt.

I den statiska versionen av problemet att bestämma jämvikt, antar man att flödet mellan ett antal givna start- och målpunkter är konstant. Detta problem omformulerades till ett (ändligtdimensionellt) optimeringsproblem redan på 50-talet, och genomförbara beräkningsmetoder togs fram i början på 70-talet, och används ännu idag.

I den mer realistiska dynamiska versionen av jämviktsproblemet, som ska efterlikna t.ex. morgonrusningens uppgång och fall, är flödet mellan start och målpunkter inte konstant. Här har man till dags dato inte hittat någon allmänt vedertagen matematisk beskrivning.

Jag kommer att visa, att om man utgår från gängse tids/flödesrelationer för länkarna, så leds man till en modell som är kontinuerlig i tid såväl som i distans längs länkarna. Länkarna beskrivs av en variant av en partiell differentialekvation för inkompressibel strömning. Nash-jämvikten för detta system går att omforma till en variationsolikhet, som i sin tur omformas till ett (oändligtdimensionellt) optimeringsproblem.

**2 Wednesday 28 May 2003.**

**Dr. Staffan Rodhe, Uppsala universitet,**  
*Samuel Klingenskierna - 1700-talets viktigaste svenske matematiker.*

*Abstract:*

Samuel Klingenskierna är en tidig företrädare för den svenska vetenskapliga revolutionen under 1700-talet. Mycket på grund av sin ovilja att publicera sina skrifter har han kommit i skymundan gentemot de mer kända vetenskapsmännen Carl von Linné, Anders Celsius och Torbern Bergman. Emellertid var Klingenskierna även en internationellt mycket välkänd matematiker. Han hade mött och/eller brevväxlat med alla de stora matematikerna från sin samtid som Johan Bernoulli, Clairault, Cramer och Euler. Föredraget kommer att ge en beskrivning av hans liv och peka på flera av hans vetenskapliga resultat. Vidare kommer hans lösning av det utökade brakystokronproblemet, med en kropp som faller i ett resistent medium, att visas. Lösningen till detta problem är Klingenskierna troligen först med att genomföra, några år före Euler.

**2 Wednesday 4 June 2003.**

**Tomas Johansson, ITN,**  
*Rekonstruktion av temperatur från randdata*

*Abstract:*

Vi studerar ett Cauchy-problem för värmeledningsekvationen. Data är givna på en del av randen till ett begränsat område i  $R^n$ . Detta problem är illaställt i J. Hadamards mening. Vi presenterar en iterativ regulariserande metod vilken kräver lösandet av en följd av välställda problem för samma operator. Metoder baserade på denna idé föreslogs först av V. A. Kozlov och V. G. Maz'ya för en klass av problem som inte inkluderar ovanstående ekvation. Regulariserande egenskaper och konvergens hos metoden vi föreslagit kommer att diskuteras i ett viktat  $L^2$  rum. Välställdhet hos problemen som används i metoden kommer även att påvisas i ett viktat Sobolevrum.

**2 Thursday 12 June 2003.**

**Professor Vladimir Varlamov, University of Texas,**  
*On the wave propagation in the presence of moving boundaries*

*Abstract:*

Nonlinear wave propagation became a popular topic after the historical discovery of solitary waves by the famous Scottish physicist John Scott Russell in 1834. In fact solitons represent a delicate balance between nonlinearity and dispersion. In order to observe nonlinear wave propagation in laboratory experiments, one has to generate these waves

effectively. Usually a moving piston- or flap-type wave maker is employed for this purpose. This wavemaker is mounted at one end of a sufficiently long channel. A Korteweg-de Vries-type equation is chosen to model the nonlinear wave propagation on the surface of water. It is examined in the domain  $x > \gamma(t)$ ,  $t > 0$ , where the function  $x = \gamma(t)$  describes the law of movement of the wavemaker. An interesting feature of the problem is the appearance of a forced nonlinear oscillator equation (Emden-Fowler-type equation) relating the motion of the wavemaker to the wave amplitude at the boundary. Local- and global-in-time solvability of the initial-boundary problem is investigated. Asymptotic solution of the boundary equation is constructed.

## 2 Wednesday 25 June 2003.

**Professor Nageswari Shanmugalingam, Cincinnati,**

*Banach space valued Newton-Sobolev spaces on metric measure spaces*

*Abstract:*

Sobolev spaces have been an integral part of the study of PDEs and quasiconformal mappings between Euclidean spaces. Recent surge of interest in the study of quasiconformal mappings between domains in metric measure spaces indicates the desirability of constructing Sobolev type spaces of functions between metric spaces.

Given that general metric spaces need not have a group structure, spaces of functions between two metric spaces in general need not have a vector space structure. We can however compensate for this by embedding the target metric space isometrically into a Banach space, for example, into  $\ell^\infty$ , and the corresponding space of functions inherit the vector space structure.

In this talk, we will construct an analogue of Sobolev spaces, called Newtonian spaces, of functions from a metric measure space into a fixed Banach space. To do so, we will use the notion of upper gradients first developed by Heinonen and Koskela. We will discuss some elementary properties of this function space, and discuss the independence of Poincaré inequalities from the target Banach space. In particular, we will prove that Newtonian functions from the given metric space  $X$  into any Banach space satisfy a Poincaré inequality if and only if real-valued Newtonian functions from  $X$  satisfy a Poincaré inequality.

We will also discuss how to extract the class of Newtonian functions from  $X$  to a given metric space  $Y$  from the Newtonian space of functions from  $X$  to a Banach space into which  $Y$  was embedded.

This talk is based on joint work with Heinonen, Koskela, and Tyson (2001).

## 2 Wednesday 3 September 2003.

**Professor Vadim Kuznetsov, University of Leeds,**

*Separation by combing and plaiting*

*Abstract:*

We introduce a new notion of a *Fac*  $\rightarrow$  *risedSeparation*  $\in$  ' ( or *simply* Faktura' which is a superstructure responsible for separability of a fairly large class of Liouville integrable systems and their quantum analogues. It consists of two strings of mutually inverse transformations, the combing and the plaiting. The former transform combs out separation variables one by one, while the latter binds down an extra separation variable on each step. Examples include Calogero-Moser systems, monomial/Schur symmetric functions and Jack polynomials.

## 2 Wednesday 10 September 2003.

**Professor Yasunao Hattori, Shimane University, Japan,**

*Special metrics appeared in topology*

*Abstract:*

The notion of a metric function (shortly, a metric) is one of the most fundamental concepts in topology. Metric functions appear almost everywhere in topology. For a metrizable space  $X$ , there are many metrics which induce the original topology of  $X$ . Some of them may determine a topological property of  $X$ . On the other hand there exist admissible metrics which determine any topological property. We shall talk about the relationship between metric properties and topological ones.

## 2 Wednesday 17 September 2003.

**Professor Kyril Tintarev, Uppsala University,**

*Invariant energies and their minimizers in spaces with rich symmetries*

*Abstract:*

Given a differentiable manifold and a topological group acting on it transitively, there is an essentially unique (up to a choice of fixed matrix) invariant metric together with a dual Sobolev quadratic form. Existence still holds if transitivity is replaced by a requirement that the manifold is co-compact. Similar argument extends also to the sub-Riemannian/sub-elliptic case. In the resulting metric the group becomes a group of isometries and, via the abstract concentration compactness method, yields existence of Sobolev minimizers in the case when the manifold (and so the correspondent Sobolev imbedding) is not compact. This is a joint work with K.-H. Fieseler.

## 2 Wednesday 24 September 2003.

**Professor Vitaly Fedorchuk, Moscow State University,**

*Probability measures and infinite-dimensional manifolds*

*Abstract:*

In 1931 O. H. Keller proved that every infinite-dimensional compact convex subset of the Hilbert space  $\ell_2$  is homeomorphic to the Hilbert cube  $Q = \ell_2^\omega$ . It implies that for every infinite compact metric space  $X$  the space  $P(X)$  of all probability measures on  $X$  is homeomorphic to  $Q$ . I'll try to explain why the following topological equalities or non-equalities hold:  $P(\ell_2) = \ell_2$ ,  $P(\ell_2^\omega) = \ell_2^\omega$ ,  $P(\ell_2^\omega) \neq \ell_2^\omega$ ,  $P(\ell_2^\omega) \neq \ell_2^\omega$ . As for  $\ell_2^\omega$  the problem, if  $P(\ell_2^\omega)$  is homeomorphic to  $\ell_2^\omega$ , is unsolvable in ZFC. In order to answer these questions one has to know when a mapping  $f: X \rightarrow Y$ , in particular a mapping of type  $P(f)$ , is a trivial bundle. In early 80th H. Toruńczyk and J. West gave a characterization for mapping  $f: X$  to be either a trivial  $Q$ -bundle, or a trivial  $\ell_2$ -bundle. If time permits, I am going to show how their results work.

## 2 Wednesday 1 October 2003.

**Professor Francesco Calogero, University of Rome I "La Sapienza",**

*Isochronous systems are not rare*

*Abstract:*

A dynamical system is here called isochronous if there exists an open set of initial data (having full dimensionality in the phase space of such initial data) such that all motions emerging out of it are completely periodic with the same fixed period (independent of the initial data). It will be shown how, from a quite arbitrary dynamical system, a deformed one can be generated via a simple trick - amounting to a change of variables - such that the new deformed system is indeed isochronous. Many interesting examples will be exhibited: these include Hamiltonian and non Hamiltonian systems, and systems naturally interpretable as many-body problems, possibly with one- and two-body forces only, possibly rotation- and/or translation-invariant in two- or more-dimensional space, possibly appearing as deformations of classical problems such as the many-body gravitational problem in ordinary (three-dimensional) space. The relevance of these results is underscored by the observation that isochronous systems are generally superintegrable. Although the approach is also applicable to infinite dimensional systems (for instance, also to nonlinear evolution PDEs), the talk will mainly focus on the finite-dimensional case.

## 2 Wednesday 8 October 2003.

**Lars Falk, FOI (totalförsvarets forskningsinstitut), Stockholm,**

*The force on Newton's apple*

*Abstract:*

In Principia Newton proved that a spherical shell attracts as if all mass is concentrated at the centre. This formula gives the force on an apple and simplifies the analysis of the Solar system considered as a system of point masses. The proof in Principia is based on geometry, but many people believe that Newton used calculus in the first place. A number of methods discussed in the literature are reviewed and compared with a simple geometrical argument, which may be close to Newton's original derivation.

## 2 Wednesday 15 October 2003.

**Claes Waksjö, MAI,**

*From elliptic to cofactor-elliptic coordinates*

*Abstract:*

In 1839 Jacobi introduced elliptic coordinates as a means to integrate the geodesic equations on an ellipsoid. Since then, several important problems have been solved by using them to separate variables in the Hamilton-Jacobi or Schrödinger equations. In this talk, I will give a review of the fundamental properties of elliptic coordinates, and show how they can be generalized in a natural way within the framework of the recently developed theory of Newton systems of cofactor type. The so found coordinates, which are called cofactor-elliptic, are given by non-orthogonal families of non-confocal quadrics. These coordinates allow new Newton systems to be solved by separation of variables in a somewhat modified Hamilton-Jacobi sense.

## 2 Wednesday 22 October 2003.

**Andreas Axelsson, MAI,**



*Transmission problems for Dirac's and Maxwell's equations with Lipschitz interfaces*

*Abstract:*

A classical method for solving boundary value problems for the Laplace equation is to reduce the problem to solving an integral equation on the boundary of the domain involving the double layer potential operator. In my PhD work at the Australian National University I have developed a corresponding theory of boundary integral equations for solving transmission problems for the Dirac equation  $(d+\delta) F(x) = 0$ . This uses a singular integral operator, the rotation operator, which replaces the double layer operator. I will discuss how the spectrum of the rotation operator depends on the Lipschitz geometry of the interface. The tool for estimating the spectrum is Hodge decompositions of the function spaces. The whole talk will be focused around the main application: electromagnetic scattering of rough surfaces.

**2 Wednesday 29 October 2003.**

**Dr. Niklas Eriksen,**

*Att blanda och ge - en genordningsodysse*

*Abstract:*

Kombinatoriska problem som att sortera en kortlek med hjälp av enkla tekniker, som att vända de översta  $k$  korten, har länge roat många matematiker. Det visar sig nu att sådana problem har viktiga tillämpningar inom bioinformatik, för att avgöra hur nära släkt olika bakteriearter är. Vi kommer att presentera några av de viktigaste resultaten inom detta decenniegamla område.

**2 Friday 7 November.**

**Professor Sergio Benenti, University of Turin,**

*On the connection between the additive separation of the Hamilton-Jacobi equation and the multiplicative separation of the Schrödinger equation. I. The completeness and Robertson conditions*

*Abstract:*

The fundamental elements of the variable separation theory are revisited, including the Eisenhart and Robertson theorems, Kalnins-Miller theory, and the intrinsic characterization of the separation of the Hamilton-Jacobi equation, in a unitary and geometrical perspective. The general notion of complete integrability of first-order normal systems of PDEs leads in a natural way to completeness conditions for separated solutions of the Schrödinger equation and to the Robertson condition. Two general types of multiplicative separation for the Schrödinger equation are defined and analyzed: they are called "free" and "reduced" separation, respectively. In the free separation the coordinates are necessarily orthogonal, while the reduced separation may occur in nonorthogonal coordinates, but only in the presence of symmetries (Killing vectors).

**2 Wednesday 12 November 2003.**

**Professor Issai Kantor, Lunds universitet,**

*An Introduction in Supermathematics*

*Abstract:*

The Supermathematics is a branch of Mathematics which was started recently (approximately 30 years ago) in connection with existence of two types of elementary particles: bosons and fermions. While behavior of bosons could be described in ordinary variables (called in Supermathematics "even" variables) for fermions one need so called "odd" variables. Using both types of variables together one can consider, for example, the Schrödinger equation and the Pauli equation as the one equation in which both types of variables participate together. This situation has influence on the whole of Mathematics which become more and more what people call Supermathematics. In the talk will be given an elementary introduction to Supermathematics mostly in terms of linear algebra. No preliminary knowledge is needed.

**2 Wednesday 19 November 2003.**

**Professor Maciej Blaszkak, Adam Mickiewicz University, Posnan,**

*How to solve integrable systems by separation of variables*

*Abstract:*

The notion of integrable systems is introduced. Then, the concept of separation conditions (and a related separation curve) in the theory of Hamiltonian integrable systems is discussed. It is demonstrated that such conditions contain a lot of information on properties and solutions of finite and infinite dimensional integrable systems. Finally, the main ideas of modern geometric separability theory are sketched.

**2 Wednesday 26 November 2003.**

**Dr. Jörgen Backelin, Stockholms Universitet,**

*Ramsey theory*

*Abstract:*

The most classical problem in Ramsey theory concerns the smallest size  $R(n_1, \dots, n_r)$  of a complete graph, such that for each colouring of its edges in  $r$  colours, there is an  $i$ , such that there is an  $n_i$ -subset of vertices, with all its edges  $i$ -coloured.

More generally, one may ask questions of the type "How large can a graph be, if it allows an edge colouring without any of certain inheritable properties?". Thus, there are close connections to extremal graph theory; and in particular, there are 'Ramsey extremal graphs' of various kinds, with interesting properties.

I plan to give an introduction to the theory, including a proof of the existence of the limits; a summary of some of the results up to now; and an extension to a general setting. In particular, I'll speak about the numbers  $R(3, k+1)$ , and of some related techniques and extremal graph properties.

I shall make an honest effort to cram this into a 1 hour lecture. No prerequisites are required, above the abstract definition of simple undirected graphs.

**2 Wednesday 3 December 2003.**

**Dr. Alexandru Aleman, Lunds universitet,**

*Analytic contractions and boundary behavior of analytic functions*

*Abstract:*

There is a large class of contractive linear operators on a separable Hilbert space that can be modeled by the operator of multiplication by the independent variable on a Hilbert space of analytic functions on the unit disc. This talk will focus on a basic problem concerning this model namely, the relation between the abstract properties of the operator in question and the boundary behavior of the functions in the model space. Problems of this type emphasize the interplay between complex analysis and operator theory and in this particular case, they are closely related to index theory and polynomial approximation.

**2 Wednesday 10 December 2003.**

**Magnus Herberthson, MAI,**

*Gravitationsdipolen, eller: vad är derivatan av ett svart hål?*

*Abstract:*

Inom elektromagnetismen får man en dipol genom att sammaföra en positiv och en lika stor negativ laddning, där laddningen ökas allt eftersom avståndet minskas. Kan man tänka sig motsvarande förhållande med massor? Att man tillfälligtvis laborerar med en negativ massa är inget problem eftersom resultatet är masslöst. Frågan måste först preciseras och därefter besvaras, och det görs förhoppningsvis vid detta seminarium. Den lösning vi finner ingår i en sedan länge känd klass av lösningar, men frågan är alltså vilken av dessa som svarar just mot en gravitationsdipolen. Frågan ställs differentialgeometriskt, men problemets natur leder oss vad det lider åt andra håll.

# The Mathematical Colloquia held in 2002 at Linköpings universitet

Organized by Anders Björn, Vladimir Kozlov, Svante Linusson and Stefan Rauch-Wojciechowski.

## 2 Wednesday 16 January 2002.

**Professor Mikhail Agranovich, Moscow State Institute of Electronics and Mathematics - Technical University,**

*Spectral problems for strongly elliptic second order systems in smooth and nonsmooth domains*

*Abstract:*

We consider the problems indicated in the title with spectral parameter in the system or in the boundary condition. The domain is bounded and smooth or Lipschitz. Any problem has a discrete spectrum. If it is self-adjoint, then our main aim is to find Sobolev spaces  $H^1$  in the domain or on the boundary, in which the eigenfunctions form a basis. Applications in the case of the Schrödinger equation relate to the mathematical justification of the so-called "R-matrix method" in physics.

## 2 Wednesday 23 January 2002.

**Professor Anders Björner, KTH,**

*Scarf complexes*

*Abstract:*

I will give an introduction to a class of simplicial complexes first considered by the economist H. Scarf (Yale). Originally they arose for purposes having to do with integer programming, but recent research has shown their use for some questions in commutative algebra.

## 2 Wednesday 30 January 2002.

**Professor Alexander Vasil'ev, Universidad Técnica Federico Santa María, Valparaíso, Chile,**

*Univalent functions in the planar dynamics of viscous flows*

*Abstract:*

We apply methods of the theory of univalent functions to some problems of fluid mechanics. Our interest centers on free boundary problems. We study the time evolution of the free boundary of a viscous fluid in the zero- and non-zero-surface-tension models for planar flows in Hele-Shaw cells either with the free boundary extending to infinity or with a bounded free boundary. We consider some remarkable special classes of univalent functions which admit a visible geometric interpretation to characterize the shape of the fluid interface.

## 2 Wednesday 6 February 2002.

**Jan Åslund,**

*Asymptotic analysis of a transmission problem*

*Abstract:*

I will present some basic concepts in asymptotic analysis. First, I consider a conduction problem in a thin layer. The objective is to derive a two-dimensional model that yields an approximation to the original three-dimensional problem. Second, I consider a conduction problem in a body. I prescribe Dirichlet conditions on a thin strip on the boundary and Neumann conditions on the remaining part. The objective is to construct an approximation for the solution and describe the behavior of the solution in the vicinity of the strip. Finally, I consider a transmission problem in the union of the two domains.

## 2 Wednesday 13 February 2002.

**Professor Andrzej Szulkin, Stockholms universitet,**

*Newtonian equations of motion: from periodic solutions to chaotic dynamics*

*Abstract:*

We shall consider the system of second order ordinary differential equations  $d^2q/dt^2 + V_q(q,t) = 0$ , where  $q$  belongs to  $\mathbf{R}^N$  and the function  $V: \mathbf{R}^N \times \mathbf{R} \rightarrow \mathbf{R}$  (the potential) is periodic in  $t$ . Periodic and homoclinic solutions of this system correspond to critical points of an appropriate Euler-Lagrange functional. We shall show how critical points other than local maxima and minima can be obtained. We shall also show how homoclinic solutions give rise to a complicated dynamics of the underlying system of differential equations.

## Wednesday 20 February 2002.

**Milagros Izquierdo Barrios,**

*The spaces of tori and other Riemann surfaces.*

*Abstract:*

We revisit the modular group and some of its normal subgroups. We study the classification of tori by the modular group and present the space of moduli of elliptic curves (or tori). The above explanation will then allow us to give some hints on surfaces of higher genera. Special attention is paid to the, so called, real surfaces.

## 2 Wednesday 6 March 2002.

**Dr. Markku Rummukainen, programchef för SWECLIM, SMHI,**

*On climate modelling and climate scenarios*

*Sammanfattning:*

Frågan om klimatet och människans roll som en drivkraft för klimatförändringen är dagsaktuella vetenskapliga och praktiska frågeställningar. Enligt den senaste internationella sammanställningen av klimatforskning (IPCC 2001), beräknas antropogena utsläpp av koldioxid m.m. leda till en global uppvärmning mellan 1,4 och 5,8 grader från år 1990 till år 2100. Ett annat rön som delvis baseras på klimatmodellering är att människan sannolikt står bakom en stor del av den redan inträffade globala uppvärmningen på 0,6 grader under 1900-talet. Dessa förändringar överträffar de naturliga variationer som samhället är van vid och uppskattas leda till omfattande negativa konsekvenser världen över. I denna presentation diskuteras klimatmodellering och klimatscenarier, med svenska sårfrågor i fokus.

Klimatmodeller är viktiga verktyg i studier av hur klimatsystemet fungerar, hur det kan påverkas och hur känsligt det är för påverkan. Trots sina ofullkomligheter är klimatmodeller nödvändiga för att handskas med det omfattande och komplicerade klimatsystemet, inklusive systemets återkopplingar och interna frihetsgrader.

Jordens klimatsystemet består av atmosfären, havet, biosfären, kryosfären men även människan. Klimatsystemet drivs externt av solinstrålningen. Det finns dock andra aspekter som medverkar bakom klimatets tillstånd, variationer och dess förändring, t.ex. atmosfärens sammansättning, fördelningen av land och hav, orografi, vegetation osv. Dessa betingelser, tillsammans med hydromekanikens och termodynamikens principer och deras matematiska representation ingår i numeriska datormodeller. Eftersom klimatsystemets processer spänner över många storleksordningar är det omöjligt att analytiskt räkna på klimatsystemet. Ekvationerna måste förenklas och de processer som sker på finare skalor än datorkraften räcker till parameteriseras (representeras i termer med större skalor). Olika typer av klimatmodeller finns, från tämligen enkla (t.ex. energibalansmodeller (EBM), 1-dimensionella modeller) till 3-dimensionella klimatmodeller, s.k. generella cirkulationsmodeller (GCM) och kopplade modellsystem. Idag forskas om hur t.ex. vegetation och biogeokemi på ett bra sätt kan inkorporeras i kopplade atmosfär-havsmodeller.

När det är framtidens klimat som studeras, blir simuleringarna typiskt långa (flera hundra år), de måste upprepas för olika antaganden av socioekonomisk världsutveckling (utsläppsscenarier) och genomföras med flera modeller (för att uppskatta säkerheten med avseende på klimatsystemets känslighet för påverkan och betydelsen av systemets interna variationer). Beräkningsuppgiftens omfattning leder till att långa simuleringar, med hög upplösning, inte i praktiken går att genomföra ens med superdatorer. Detta är ett dilemma i regional klimatforskning och för regionala åtgärder/beslut som har med klimatfrågan att göra.

Regionala klimatmodeller (RCM) används för att ta fram behövliga detaljer från globala simuleringar. I Sverige bedrivs utveckling av regional klimatmodellering och genomförs regionala klimatsimuleringar. En central aktivitet är det svenska regionala klimatmodelleringsprogrammet SWECLIM och dess Rosby Centre på SMHI. SWECLIM finansieras med medel från MISTRA, SMHI och vissa andra medfinansierare.

## 2 Wednesday 6 March 2002.

**Dr. Pär Kurlberg, Chalmers,**

*Number theory related to quantum chaos.*

*Abstract:*

Quantum chaos is concerned with properties of eigenvalues and eigenfunctions of "quantized Hamiltonians". For instance, can classical chaos be detected by looking at the spacings

between eigenvalues? Another problem is if classical ergodicity forces eigenfunctions to be equidistributed in a certain sense. We will give a short introduction to quantized Hamiltonians, and then show that the study of the above mentioned questions for some simple dynamical systems gives rise to interesting problems in number theory.

## 2 Wednesday 20 March 2002.

**Lars Inge Hedberg,**

*Slicing of cubes and other convex bodies*

*Abstract:*

Keith Ball proved in 1986 that for any  $n$  the  $(n-1)$ -dimensional slice cut out of the  $n$ -dimensional unit cube by a hyperplane has  $(n-1)$ -dimensional area at most  $2^{1/2}$ . Equality takes place only when the hyperplane contains an  $(n-2)$ -dimensional face of the cube. As Ball observed, for  $n \geq 10$  his result also gives a simple negative solution to a famous problem posed in 1956 by H. Busemann and C. M. Petty, asking whether of two convex bodies in  $\mathbb{R}^n$ , symmetric with respect to the origin, the one whose  $(n-1)$ -dimensional central sections have the greater area always has the greater  $n$ -dimensional volume. I will present a simplified proof of Ball's theorem due to F. Nazarov and A. Podkorytov, and if time permits, I will also discuss the complete solution of the Busemann -- Petty problem due to G. Y. Zhang and others.

## 2 Wednesday 27 March 2002.

**Dr. Vladimir Ya. Eiderman, Moscow,**

*Uniqueness theorems for analytic functions*

*Abstract:*

Assume that an analytic function  $f(z)$  in a domain  $D$  in the plane tends to 0 rapidly on a sequence of points  $\{z_n\}$  in  $D$  as  $z_n$  tends to the boundary of  $D$ . Does it follow that  $f=0$  for all  $f$  that are, say, bounded in  $D$ ? We give a survey of theorems of this type including some new results obtained jointly with M. Essén. Some applications of uniqueness theorems will be given.

## 2 Wednesday 10 April 2002.

**Professor Kjell Rosquist, Physics, Stockholm University,**

*A geometric/tensorial approach to integrable systems*

*Abstract:*

A geometric formulation of the Lax pair equation is described. In this picture, the integrable dynamical system is realized as a geodesic flow on a curved space which carries all the dynamical information in its metric, or equivalently, in its geodesic Hamiltonian. The geometric version of the Lax pair equation then appears as a tensorial equation written entirely in terms of configuration space tensors, one of which may be viewed as a generalized third rank Killing tensor. A similar formulation of the classical R-matrix is also given. The specific geometric nature of the classical R-matrix provides a natural explanation of its transformation properties.

## 2 Wednesday 17 April 2002.

**Jesper Thorén,**

*Quantum groups and the Yang-Baxter equation*

*Abstract:*

Quantum groups first arose in the physics literature in the early 1980s from the inverse scattering method which had been developed to construct and solve integrable quantum systems. They have excited great interest in the past few years because of their unexpected connections with such, at first sight, unrelated parts of mathematics as the construction of knot invariants and the representation theory of algebraic groups in characteristic  $p$ . In their original form, quantum groups are associative algebras whose defining relations are expressed in terms of a matrix of constants called a quantum R-matrix, i.e. the matrix is a solution of the Yang-Baxter equation. It was realized independently by V. G. Drinfel'd and M. Jimbo around 1985 that these algebras are Hopf algebras, which, in many cases, are deformations of universal enveloping algebras of Lie algebras.

In this talk, we will introduce the Yang-Baxter equation and show how we can produce solutions of it (i.e. R-matrices) using representations of quantum groups.

## 2 Monday 22 April 2002.

**Professor José M.M. Senovilla, University of the Basque Country, Bilbao,**

*On the definition and the existence of singularities in general relativity*

*Abstract:*

The question of how to define singular spacetimes, and how to attach properties to the singularities in Lorentzian geometry, is carefully analyzed. It turns out that the existence of singularities is intimately related with the question of extendibility of spacetimes, a problem with no unique solution, if any, in general. The existence of singularities under general conditions is also shown via the famous singularity theorems in general relativity, which are critically reviewed. Many explicit illuminating examples will be given.

## 2 Monday 22 April 2002.

**Professor Valeriu Dryuma, Institute of Mathematics and Informatics, Academy of Sciences of Moldova, Kishinev,**

*Geometric problems in the theory of differential equations*

*Abstract:*

Some properties of the 4-dimensional Riemannian spaces with the metrics

$$ds^2 = 2P_{ik} dx^i dx^k + 2 dx^k d\lambda_{ik}$$

connected with nonlinear differential equations are considered. The properties of 3-dimensional Einstein-Weyl spaces connected with the theory of second and third order ordinary differential equations are also discussed.

## 2 Wednesday 24 April 2002.

**Kristina Crona,**

*Hilbert functions and Gröbner bases*

*Abstract:*

Consider the homogeneous coordinate ring  $S/I$  of a projective algebraic set, where  $S$  is a polynomial ring. The Hilbert function  $H(S/I, n)$  measures the dimension of the  $n$ -th homogeneous piece of  $S/I$ . For  $n \gg 0$ , this function coincides with a polynomial - the Hilbert polynomial - which is an important invariant. Macaulay's theorem and theorems by Gotzmann give a very precise knowledge of these Hilbert functions. My presentation will rely on Gröbner bases theory, in particular generic initial ideals and lexsegment ideals. I will also treat Hilbert functions for other gradings that appear naturally, primarily standard bigradings, corresponding to products of projective spaces. Bounds of Macaulay type as well as a persistence result will be given for bigraded Hilbert functions.

## 2 Friday 3 May 2002.

**Göran Bergqvist,**

*Causal tensors and superenergy tensors*

*Abstract:*

That a quantity is positive is often a required property or a needed assumption in the demonstrations of many important results. We will discuss properties of causal tensors in Lorentzian geometry, these are geometric objects which are positive in a certain sense. We present a way of constructing causal tensors, so-called superenergy tensors, from any given tensor. Applications of both algebraic and analytic nature will be discussed.

## 2 Wednesday 8 May 2002.

**Professor Nikolai Makarov, Caltech,**

*Conformal welding*

*Abstract:*

Let  $G_+$  and  $G_-$  be two complementary Jordan domains on the Riemann sphere, and let  $f_+ : \mathbf{D} \rightarrow G_+$  and  $f_- : \mathbf{D} \rightarrow G_-$  be conformal maps defined on the unit disc  $\mathbf{D} = \{z : |z| < 1\}$ . The equation  $f_+ = f_- \circ a$  on the boundary  $S^1 = \{z : |z| = 1\}$  determines a map  $a : S^1 \rightarrow S^1$ . A classical topic of complex analysis is to relate the properties of the curve which is the common boundary of  $G_+$  and  $G_-$  and the welding homeomorphism  $a$ ; in particular, it includes the problems of existence and uniqueness.

The welding operation and its generalizations have important applications in several areas of mathematics. I will discuss an approach which is based on the study of the action of the homeomorphism  $a$  in the Dirichlet space on the circle, and review some results and open problems.

## 2 Tuesday 14 May 2002.

**Professor Andrzej Trautman, Institute of Theoretical Physics, Warsaw University,**

*Two approaches to spinor fields on manifolds*

A survey dedicated to the memory of Marcel Riesz

*Abstract:*

Spinor fields and the Dirac equation on pseudo-Riemannian manifolds of general relativity theory were introduced, by the great masters (Wigner, Fock, Weyl) shortly after the appearance of the 1928 paper by Dirac. From the very beginning, two competing approaches were present: in one, the emphasis was on orthonormal tetrads and the use of constant Dirac matrices. In the second, initiated by Tetrode and Schrödinger, the Dirac matrices were allowed to depend on curvilinear coordinates. In the language of contemporary differential geometry, the first approach is subsumed by the notion of a principal bundle defining a (s)pin structure, whereas in the second, the basic object is a vector bundle of spinors, carrying a representation of a Clifford bundle. This last notion can be traced back to a 1953 paper by Riesz. In the talk, the two approaches will be compared and shown to be essentially equivalent. Hypersurfaces in Euclidean space have a pin structure which is non-trivial, in general, but the associated bundle of spinors is trivial, a fact that leads to a simple formula for the Dirac operator on such hypersurfaces. General considerations will be illustrated by the example of spheres. Spinor connections on low-dimensional spheres can be identified with simple gauge configurations. The triviality of the spinor bundles of spheres easily leads to a determination of the spectra and eigenfunctions of the Dirac operator on these manifolds.

## 2 Wednesday 15 May 2002.

**Professor Andrzej Trautman, Institute of Theoretical Physics, Warsaw University,**

*Elementary approach to the idea of general relativity*

*Abstract:*

The full Einstein theory of general relativity is rather complicated and requires some knowledge of Riemannian geometry. There are, however, several essential aspects of that theory that can be described and understood using only Newtonian gravitation and rudiments of special relativity and quantum mechanics. In the lecture, the Newtonian aspect of general invariance will be derived from the equality of inertial and gravitational masses and applied to Newtonian cosmology. It will be shown how the law of propagation of photons in a gravitational field implies the curvature of space-time.; Uniformly accelerated motion in special relativity leads to the idea of a horizon, an essential aspect of black holes predicted by Einstein's theory. It will be explained why gravitational radiation is so weak and has escaped, so far, all attempts at detection.

## 2 Wednesday 29 May 2002.

**Dr. Danyo Danev, ISY,**

*Superimposed Codes*

*Abstract:*

The concept of superimposed codes was first introduced by Kautz and Singleton in the beginning of the 1960s. They considered one type of such codes which have application to some retrieval problems in databases. By defining the superposition rule we obtain different kinds of superimposed codes, which can be used for solving different practical problems. A scheme for digital fingerprinting of documents can be described in terms of superimposed codes.

We aim to make a survey of the known types of superimposed codes which have been studied by different researchers. A short list of possible applications will be presented.

The emphasis of our talk will be placed on a special type of superimposed codes which have been introduced by Chien and Frazer in 1966. The superposition mechanism for this type of codes is addition modulo two (i.e. binary XOR") of the codewords. The set where these codes are defined consists of all binary sequences of given length. This set is known as the *Hamming space*. A possible application of this scheme is in identification systems.

## 2 Wednesday 29 May 2002.

**Professor Kari Astala, Jyväskylä University,**

*Optimal  $L^p$ -regularity for the gradients of solutions to elliptic PDEs*

*Abstract:*

Consider elliptic differential equations  $\text{Div}(A(x) Du) = 0$ ,  $u$  in  $W^{1,2}_{\text{loc}}$ , with nonsmooth coefficient matrix  $A(x)$ . It is well known that the solutions have improved smoothness properties, such as higher integrability of their derivatives, with bounds depending only on the degree of ellipticity.

In general, finding the optimal smoothness or higher integrability class is a problem of great difficulty. However, in the two-dimensional case the precise bounds are possible to determine; we give an overview of the recent results here, both in the case of isotropic and non-isotropic equations.

## 2 Wednesday 5 June 2002.

**Professor Pekka Koskela, Jyväskylä University,**

*Lipschitz continuity of metric harmonic functions*

*Abstract:*

Metric spaces equipped with a doubling measure that supports a Poincare inequality have recently been of some interest. One can define harmonic functions and basic results such as Holder continuity have been established. We will discuss the Lipschitz continuity of harmonic functions.

## 2 Wednesday 12 June 2002.

**Jakob Jonsson, RSA Laboratories Europe,**

*How to use the RSA algorithm in a secure manner*

*Abstract:*

The RSA algorithm is a cryptographic system that was introduced in the late Seventies by Rivest, Shamir, and Adleman; RSA is based on the presumably hard problem of factoring large composite integers. Today, RSA is widely used as a building block in cryptographic schemes intended to provide information services such as confidentiality and authenticity.

A specific instantiation of the RSA algorithm can be viewed as a permutation. The security of a scheme based on RSA relies on the hardness of inverting such a permutation on a random input: if the inversion problem can be solved, then the scheme can be broken. Ideally, we would like the converse to be true as well: If the inversion problem cannot be solved, then the scheme is secure. Unfortunately, such a result seems very hard, if at all possible, to establish for the RSA-based schemes used in practice today. We outline their shortcomings and discuss a few other schemes that are equipped with rigorous proofs relating the security of the schemes to the hardness of the underlying RSA inversion problem.

## 2 Wednesday 28 August 2002.

**Professor Yuri Safarov, King's College, London,**

*Multidimensional spectra of self-adjoint operators*

## 2 Wednesday 4 September 2002.

**Professor Jens Hoppe, KTH,**

*Membranes and matrix models*

*Abstract:*

The classical, and quantum, description of certain 3-manifolds in  $n+1$  dimensional Minkowski-space  $R(n,1)$ , corresponding to  $SO(n,1)$ -invariant surface motions in  $R(n)$  has given rise to a variety of interesting relations, such as the local equivalence of extremal hypersurfaces with hydrodynamical systems, and, via the approximation of diffeomorphism-algebras by matrix-algebras, finite-dimensional systems with quartic interactions. I will also talk about multi- linear generalisations of Lax-pairs, and some integrable higher-dimensional systems.

## 2 Monday 9 September 2002.

**Workshop on Inverse Problems.**

## 2 Monday 9 September 2002.

**Professor Yaroslav Kurylev, Loughborough University,**

*Gromov convergence for Riemann manifolds and stability in the Gel'fand inverse spectral problem*

(As part of the above workshop on inverse problems.)

## 2 Wednesday 11 September 2002.

**Professor Alexander Sobolev, University of Sussex, Brighton,**

*Periodic operators and the circle problem*

*Abstract:*

Spectra of periodic operators are known to consist of a collection of closed intervals called bands, separated by spectrum-free intervals called gaps. It is an important mathematical and physical issue to find out if the number of the gaps is finite. It has been conjectured by Bethe and Sommerfeld in the 30s that the spectrum of the Schrödinger operator with a periodic electric potential must have only a finite number of gaps. A rigorous proof of this hypothesis was given only in the 80s. The aim of the talk is to present old and new results in this field and, more importantly, to discuss a connection with the famous circle problem, that is the problem of counting lattice points inside a ball of large radius.

## 2 Wednesday 18 September 2002.

**Professor Dimitri Yafaev, Rennes,**

*A particle in a magnetic field of an infinite rectilinear current*

*Abstract:*

We consider the Schrödinger operator  $H = (i \text{grad} + A)^2$  in the space  $L^2(\mathbf{R}^3)$  with a magnetic  $A$  potential created by an infinite rectilinear current. We show that the operator  $H$  is

absolutely continuous, its spectrum has infinite multiplicity and coincides with the positive half-axis. Then we find the large-time behavior of solutions  $\exp(-iHt)$  of the time dependent Schrödinger equation. Our main observation is that a quantum particle has always a preferable (depending on its charge) direction of propagation along the current. Similar result is true in classical mechanics.

📅 **Wednesday 25 September 2002.**

**Professor Nikolay Kuznetsov, St. Petersburg,**

*On sloshing frequencies in the ice-fishing problem*

*Abstract:*

The ice-fishing problem is a boundary value problem for the Laplace equation with a spectral parameter in a boundary condition on a part of the boundary. This problem describes free oscillations of water in a half-space covered by a rigid dock with apertures, where the above mentioned condition is imposed. The aim of talk is twofold:

(1) Assuming that the problem is two-dimensional and that there are two equal gaps in the dock, it will be demonstrated by differentiating the Rayleigh quotient that all eigenvalues are monotonic functions of the spacing between gaps. For the fundamental eigenvalue and the corresponding eigenfunction the asymptotic formulae for large values of spacing will be discussed.

(2) For the three-dimensional problem, it occurs that the fundamental eigenvalue depends in a complicated way on the geometry of apertures. Some examples illustrating this dependence will be given.

📅 **Wednesday 2 October 2002.**

**Professor Adrian Constantin, Lund,**

*The Camassa-Holm model for shallow water waves*

*Abstract:*

A recently derived nonlinear partial differential equation models the unidirectional propagation of waves on shallow water. The rich structure of the equation is the object of our presentation: the equation is an integrable infinite-dimensional Hamiltonian system and a re-expression of geodesic flow on the diffeomorphism group of the circle. Moreover, it models waves of permanent form as well as wave breaking and its solitary waves interact like solitons.

📅 **Wednesday 9 October 2002.**

**Professor Kurt Johansson, KTH,**

*Probability measures from random matrix theory*

*Abstract:*

Probability measures coming from random matrix theory, usually from appropriate limits of the spectrum as the size of the matrix goes to infinity, have been the focus of much interest. They occur in spectral problems (quantum chaos), number theory and also unexpectedly in certain probability problems related to random growth and random tilings. I will give some background and then concentrate on the last topics.

📅 **Wednesday 16 October 2002.**

**Professor Michael Solomyak, Weizmann Institute of Science, Rehovot, Israel,**

*On the spectrum of the Laplacian on metric graphs*

*Abstract:*

A metric graph is a graph whose edges are viewed as line segments of positive length, rather than just pairs of vertices. The Laplacian on such graph is the operator of second derivative on each edge, complemented by the Kirchhoff matching conditions at vertices. The spectrum of the Laplacian can be quite different, reflecting geometry of a given graph. Recent results on this subject will be reported. The most detailed results concern a special case of graphs, namely the so-called regular trees.

📅 **Thursday 17 October 2002.**

The film

*The CMI millenium meeting*

was shown.

📅 **Wednesday 23 October 2002.**

**Jonas Lundgren defended his licentiate thesis,**

*Reconstruction of stresses in plates by incomplete Cauchy data*

📅 **Wednesday 23 October 2002.**

**Dr. Björn Gustafsson, KTH,**

*Some multiplicative potential theory*

*Abstract:*

In ordinary potential theory one gets the potential of a body by adding (or integrating) the contributions from the individual parts of it. It sounds of course a little crazy to instead multiply the contributions with each other, but sometimes this makes sense and leads to interesting mathematics. One simple example is any polynomial in one complex variable, which can be viewed as a multiplicative version of the (logarithmic) potential for finitely many point masses.

I will discuss another instance of multiplicative potential theory, namely the so-called exponential transform, which originally appeared in operator theory but now is studied for its own sake and for applications in other areas. The talk is based on joint work with Mihai Putinar (Santa Barbara).

📅 **Friday 25 October 2002.**

**Autumn meeting of the Swedish Mathematical Society.**

📅 **Wednesday 30 October 2002.**

**Professor Stanislav Smirnov, KTH,**

*Critical lattice models and conformal invariance*

*Abstract:*

For a number of lattice models in the plane (Percolation, Ising Model, Self Avoiding Random Walk,...) physicists were able to predict exact values of various scaling exponents and dimensions. E.g., they reason that Hausdorff dimension of the critical percolation cluster should be  $91/48$  almost surely.

We will review recent mathematical progress in this area.

📅 **Thursday 31 October 2002.**

A film from the lecture by **Professor Timothy Gowers, Cambridge,**

*The importance of mathematics*

The lecture was given at the Clay mathematics institute millennium meeting in Paris, 2000.

📅 **Wednesday 6 November 2002.**

**Stefan Rauch-Wojciechowski,**

*Theory of quasi-potential Newton equations and an effective criterion of separability for the Helmholtz and Hamilton-Jacobi equations. "Two accidents at work".*

*Abstract:*

The method of separating variables for solving the Hamilton-Jacobi equation of a natural Hamiltonian  $H=p^2/2+V(q)$  has been conceived by W. R. Hamilton and by C. G. J. Jacobi around 1830-40. It consists of finding a curvilinear change of variables  $u(q)$  so that the problem is reduced to integration of a system of uncoupled ODEs. It has been the most successful method of integrating mechanical problems that we now find in textbooks. However for a given potential  $V(q)$ , the variables of separation are a priori unknown and usually do not exist. The problem of deciding and finding  $u(q)$  for any given potential  $V(q)$  has been considered unsolvable by Jacobi (Vorlesungen über Dynamic, 1866) as later cited by Arnold in his Mechanics §47, p. 266.

Jacobi's opinion had a profound influence on the further development of separability theory, which focused on characterising separable Hamiltonians in terms of variables of separation (Stäckel, Levi-Civita) and on classifying all separation variables (Eisenhart, Benenti, Kalnins & Miller).

Being unaware of this classical line of development we have together with C. Waksjö formulated and proved an effective, completely algorithmic criterion of separability. It is based on the structure of quadratic integrals of motion for  $H=p^2/2+V(q)$  and on the use of a set of  $n(n-1)/2$  second order linear PDEs characterising separable potentials.

Closely related to this result is the theory (developed together with H. Lundmark and K. Marciniak) of cofactor pair systems  $d^2q/dt^2=-(\text{cof } G)^{-1}\text{grad } k(q)=-(\text{cof } G')^{-1}\text{grad } k'(q)$  which, naturally generalises the classical separability theory, but has its origin in soliton theory.

This theory leads to a new type of nonorthogonal variables of separation and has two equivalent characterisations in terms of certain Poisson pencils or through a set of *fundamental*

*equations*. It appears that the condition  $(\text{cof } G)^{-1} \text{grad } k(q) = (\text{cof } G')^{-1} \text{grad } k'(q)$  can be interpreted as a generalisation of the Cauchy-Riemann (quasi-CR) equations while the related fundamental equations correspond to the Laplace equation.

I shall present the story and the main ideas behind these results.

📅 **Wednesday 13 November 2002.**

**Professor Torkel Glad, ISY,**

*Lyapunov-teori och styrning av flygplan*

*Abstract:*

Lyapunov-teori är den klassiska metoden att undersöka stabilitet hos system som beskrivs av ordinära differentialekvationer. För styrda system, t.ex. flygplan, modifieras problemställningen så att det gäller att hitta en kombination av Lyapunovfunktion och styrprincip som garanterar stabilitet. För många tekniska system finns en struktur som gör det möjligt att generera Lyapunovfunktioner successivt för delsystem av ökande komplexitet. Denna struktur gör det också möjligt att undersöka robustheten gentemot fel i den matematiska modellen.

📅 **Thursday 14 November 2002.**

A film from the lecture by **Professor John Tate, University of Texas, Austin**

*The millennium prize problems*

*Abstract:*

This lecture is about three of the seven **millennium problems**:

- 📅 the Riemann hypothesis;
- 📅 the conjecture of Birch and Swinnerton-Dyer;
- 📅 the P versus NP problem.

The lecture was given at the Clay mathematics institute millennium meeting in Paris, 2000.

📅 **Friday 15 November 2002.**

**Professor Rafael Benguria, Santiago,**

*Variational characterization for nonlinear eigenvalue problems*

*Abstract:*

I will review a variational technique, developed in collaboration with M. C. Depassier, to characterize eigenvalues of several nonlinear problems arising from mathematical physics. In particular, I will consider the variational characterization of the speed of propagation of travelling waves of the nonlinear diffusion equation; nonlinear equations coming from mechanics (like the Euler elastica); and the characterization of the chemical potential for the Thomas-Fermi equation.

📅 **Monday 18 November 2002.**

**Andreas Rietz defended his licentiate thesis,**

*Existence results for noncoercive incremental contact problems with Coulomb friction.*

📅 **Wednesday 20 November 2002.**

**Professor Peter Hislop, Lexington, Kentucky,**

*Spectral and dynamical properties of random Schrödinger operators: an overview*

*Abstract:*

Schrödinger operators are second-order partial differential operators that represent the energy of a quantum mechanical particle. The coefficients of the Schrödinger operator are called scalar and vector potentials and describe the interaction of the particle with other atoms and electrons, and with external magnetic fields, respectively. Since the mid-seventies, there has been much interest in random Schrödinger operators. These are families of operators for which the potentials are given by stochastic processes. These random families of operators have interesting spectral and dynamical properties. For example, many simple random families exhibit Anderson localization, the occurrence of intervals of dense point spectrum with probability one. In this overview talk, I will introduce the simplest models, and I describe their general spectral and dynamical properties.

📅 **Thursday 21 November 2002.**

A film from the lecture by **Professor Timothy Gowers, Cambridge,**

*The importance of mathematics*

The lecture was given at the Clay mathematics institute millennium meeting in Paris, 2000.

*Abstract:*

I föredraget argumenterar Gowers för hur pass sammanhängande matematiken är. Det är något som jag själv har förstått mer och mer ju mer jag lärt mig, och som jag inte alls var lika medveten om som doktorand.

Gowers pekar också på att det som tillämpade matematiker, eller folk inom mer tillämpade ämnen som datavetenskap eller fysik, håller på med ofta inte är mer samhällsnyttigt än vad en ren matematiker sysslar med.

📅 **Wednesday 27 November 2002.**

**Professor Peter B Gilkey, Eugene, Oregon,**

*Heat content asymptotics with 'exotic' boundary conditions*

*Abstract:*

Let  $M$  be a bounded domain in Euclidean space (or more generally a Riemannian manifold) with smooth boundary. Let  $D$  be the Laplacian on  $M$  (or more generally an operator of Laplace type) and let  $B$  be a suitable boundary condition. Let  $f$  be the initial temperature distribution. The subsequent temperature distribution  $u(x;t)$  is described by the equations

$$u_t + Du = 0, \quad u(x;0) = f(x), \quad Bu = 0.$$

Let  $r$  be the specific heat of the manifold. The total heat energy content of the manifold

$$b(t) := (\text{integral over } M) u(x;t) r(x) dx$$

has an asymptotic expansion as  $t \rightarrow 0$  with locally computable coefficients:

$$(\text{sum over } n \geq 0) b_n t^{n/2}.$$

We discuss local formulas for the  $b_n$  with various physically relevant boundary conditions (Dirichlet, Neumann, transmission, transfer, and spectral).

📅 **Thursday 28 November 2002.**

A film from the lecture by **Sir Michael Atiyah, University of Edinburgh,**

*The millennium prize problems*

*Abstract:*

This lecture is about four of the seven **millennium problems**:

- 📅 the Poincaré conjecture;
- 📅 the Hodge conjecture;
- 📅 quantum Yang-Mills existence;
- 📅 the existence and smoothness of solutions to the Navier-Stokes equations.

The lecture was given at the Clay mathematics institute millennium meeting in Paris, 2000.

📅 **Wednesday 4 December 2002.**

**Dr. Edvin Langmann, KTH,**

*Exactly solvable models of interacting fermions*

*Abstract:*

Models for interacting fermions play an important role in theoretical condensed matter physics.

I will give a general introduction to such models: explain what they are from a general mathematical point of view, mention a few standard methods to study them, and then describe some important examples of current interest. In particular I will mention two-dimensional Hubbard-like models and the problem of understanding correlated fermion systems.



I will then concentrate on a particular class of such models which can be solved exactly using group theory. Simple examples are Hartree-type models which are related to Abelian groups. Another well-known example is a BCS-type model for superconductivity which is related to  $SU(2)$ . I then present a few recent examples which describe 2D correlated fermions.

# The Mathematical Colloquia held in 2001 at Linköpings universitet

Organized by Lars-Erik Andersson, Anders Björn, Svante Linusson and Stefan Rauch-Wojciechowski.

## Friday 26 January 2001.

**Stefan Rauch-Wojciechowski,**

*Three theorems on integrable Newton equations*

*Abstract:*

The Newton equation which says that the acceleration is equal to an external force is fundamental for the whole analytical mechanics. The standard theory studies extensively the case when the force  $M(q) = -\text{grad}V(q)$  is generated by the potential  $V(q)$ . The Kepler problem of motion of Earth around Sun:  $V(q) = k/|q|$  is a pivotal example in this theory. A new theory of Newton equation with quasipotential forces  $M(q) = -B(q)\text{grad}[k(q)]$ , developed by us, gives new light on many results of integrable dynamics and leads to new, interesting mathematics. I shall present three important theorems within this theory: on an effective criterion of separability for potential forces, on triangular systems of Newton equations and on cofactor pair systems of Newton equation.

## Friday 2 February 2001.

**Professor Alexander P. Veselov, Loughborough University,**

*The Hadamard problem and generalised quantum Calogero--Moser systems*

*Abstract:*

The Hadamard problem of description of all second order hyperbolic equations satisfying Huygens' principle (in the narrow Hadamard sense) is one of the classical open problems which still remains open. Over the last decade a substantial progress in this direction has been made as a result of the discovered relations between this problem and the problem of classification of all integrable generalisations of the quantum Calogero--Moser system. In the talk these relations due to Yu. Berest, O. Chalykh, M. Feigin and the speaker will be discussed.

## Thursday 8 February 2001.

**Professor Elżbieta Pol, Warsaw University,**

*On hereditarily indecomposable continua*

*Abstract:*

A continuum is said to be indecomposable if it is not the union of two of its proper subcontinua, and hereditarily indecomposable (h.i.) if each of its subcontinua is indecomposable. In 1922 Knaster gave the first example of h.i. one-dimensional continuum. Later one called it a pseudo-arc because it was homeomorphic to each of its nondegenerate subcontinua, like an arc. But unlike the arc it was homogeneous, like a circle. The pseudoarc has appeared in many areas of continuum theory. In 1951 Bing showed that there were h.i.  $n$ -dimensional continua for every integer  $n = 2, 3, \dots$  and  $\infty$ . The existence of h.i.  $n$ -dimensional continua is a strong tool in the construction of unusual spaces such as Henderson's infinite-dimensional compactum all of whose closed finite-dimensional subsets are zero-dimensional (Levin). In this talk some new results in the theory of h.i. continua will be presented.

## Tuesday 13 February 2001.

**Professor Vitaly V. Fedorchuk, Moscow State University,**

*Manifolds and their dimensions*

*Abstract:*

In celebrated papers of Lebesgue, Brouwer, Urysohn, and Menger the main dimensional invariants  $\dim$ ,  $\text{ind}$ ,  $\text{Ind}$  were introduced. These invariants coincide for subsets of Euclidean spaces, in particular, for metrizable manifolds. Recall that a topological  $n$ -manifold is a Hausdorff connected space which is locally homeomorphic to the Euclidean space  $\mathbf{R}^n$ . For an  $n$ -manifold  $M$ , which is a normal space, we have  $n = \text{ind } M \leq \dim M \leq \text{Ind } M$ . For nonmetrizable manifolds these dimensions generally do not coincide. The talk will concern this topic.

## Friday 23 February 2001.

**Dr. Maria Roginskaya, Chalmers,**

*Some criteria for Riesz sets*

*Abstract:*

The F. and M. Riesz theorem says that, as soon as a finite measure on an interval has Fourier transform supported on the positive half line, the measure has no singular part. A Riesz set (on  $\mathbf{Z}^n$  or  $\mathbf{R}^n$ ) is a set such that, as soon as a finite measure has Fourier transform supported on the set, the measure has no singular part. I'm going to show some examples of Riesz and non-Riesz sets, and discuss two types of criteria for a set to be Riesz.

## Friday 2 March 2001.

**Dr. Stephen Buckley, National University of Ireland, Maynooth,**

*Gromov hyperbolicity*

*Abstract:*

We define Gromov hyperbolicity and discuss one area (in Several Complex Variables) where it arises. Although useful, this concept is still somewhat mysterious. For instance, the literature gives no good guide as to how one can determine whether or not the quasihyperbolic metric on a bounded Euclidean domain is hyperbolic. We discuss new results that answer this and related problems, and shed a little more light on hyperbolicity.

## Friday 9 March 2001.

**Magnus Herberthson,**

*Spacelike infinity -- what is it (good for)?*

*Abstract:*

In order to investigate asymptotic properties (e.g. mass, angular momentum, charge) of isolated systems/spacetimes, one wants to take various limits at infinity. It is important to give these limits a precise meaning, and one way is via conformal transformations. These transformations transform the asymptotic questions to local considerations around added boundary points, corresponding to infinity. Unavoidably, the regularity of the completed manifold is low at some of the added points, and this affects the local analysis. I will try to discuss the problems connected to this in a principal, non-technical, manner.

## Monday 12 March 2001.

**Professor Konstantin Kozlov, Moscow State University,**

*Dimension of equivariant compact extensions*

*Abstract:*

Let  $G$  be a topological group. By a  $G$ -space  $X$  we mean a Tychonoff space  $X$  (phase space) with a continuous action of the group  $G$ . If for a  $G$ -space  $X$  there exist a compact  $G$ -space  $bX$  and an equivariant dense embedding of  $X$  into  $bX$  then we call  $bX$  a  $G$ -compactification of  $X$ . If a  $G$ -space has a  $G$ -compactification then it has a maximal  $G$ -compactification  $\beta_G X$ . In the presented talk it will be shown how the maximal  $G$ -compactifications can be obtained and then their dimensional properties will be discussed.

## Friday 16 March 2001.

**Bengt Josefson,**

*Quasicomplements in Banach spaces*

*Abstract:*

Let  $F$  be a closed subspace of the Banach space  $E$ . A closed subspace  $G$  of  $E$  is said to be a quasicomplement of  $F$  in  $E$  if  $F \cap G = \{0\}$  and  $F + G$  is dense in  $E$ . This notion was introduced around 1940 by Murray and generalizes the notion of complement (i.e.  $F + G = E$ ). Known results and open problems will be discussed.

## Friday 23 March 2001.

**Professor Vladimir E. Nazarov, Moscow State University,**

*Noncommutative analysis: theory and applications*

#### Abstract:

Noncommutative analysis, which deals with functions of several noncommuting operators, has numerous applications to algebraic problems, differential equations, asymptotics, etc. A concise survey of basic elements of the theory will be given along with a variety of specific applications.

#### 1. Theory

Basic definitions. Feynman indices. Functional calculi, symbol classes, and the uniqueness theorem. The Daletskii--Krein formula and general formulas for the noncommutative differential. Composite functions. Commutator expansions and noncommutative Taylor and Newton formulas. The main problem: finding the composition law for functions of a given set of operators. How to solve this problem using the ordered representation.

#### 2. Applications

The Campbell--Hausdorff--Dynkin formula. The Jacobi condition and the Poincaré--Birkhoff--Witt theorem. T-exponentials and T-products. An assortment of exact solutions to various equations arising in mathematical physics. Difference-differential equations. Various kinds of asymptotic solutions for differential equations (asymptotics with respect to a large/small parameter, smoothness asymptotics, mixed asymptotics). Degenerate equations.

#### Literature:

V. P. Maslov, Operational Methods, Mir, Moscow, 1976.

V. E. Nazarov, V. E. Shatalov, B. Yu. Sternin, Methods of Noncommutative Analysis, Walter de Gruyter, Berlin, 1996

#### Friday 30 March 2001.

**Jan Snellman,**

*Combinatorial properties of generic initial ideals*

#### Abstract:

So-called *Gröbner bases* are a standard tool in computational (commutative) algebra. They provide a way of associating to each homogeneous ideal (in a polynomial ring, in several variables, over a field) its *initial ideal*, which is a *monomial ideal*. The initial ideal is a simpler object which shares many properties with the original ideal (i.e. Hilbert series).

From the point of view of algebraic geometers, however, this construction is flawed, since it depends on the choice of basis for the polynomial ring. They much prefer the *generic initial ideal*, the initial ideal of the ideal obtained by a generic linear change of coordinates.

Generic initial ideals give deep understanding of the corresponding variety. However, since I'm not a geometer, I am going to concentrate instead on their combinatorial properties: I'll show that, if the ground field has characteristic zero, generic initial ideals correspond to filters in the Young lattice of numerical partitions. Time permitting, I might say something about the finite characteristic case, and about non-commutative generic initial ideals.

#### Thursday 5 April 2001.

**Royal Academy of Sciences Lecturer Professor Vaughan Jones, University of California, Berkeley,**

#### Knots

The talk is intended for a general scientific audience.

In 1990 Jones was awarded the Fields medal, the most prestigious of all mathematical prizes. The ostensible reason was his discovery in 1984 of a new polynomial invariant for knots and links in 3-space. Knot theory is a rather established field, and his discovery came as a complete surprise to knot-topologists, who had been searching for new invariants for the better part of a century. Striking as such a discovery may have been, it was just a spin-off from discovering startling, and hitherto unsuspected, relationships between von Neumann algebras and geometric topology. At the heart of the matter lies Jones Index theorem, with repercussions, not only on the aforementioned knot-theory, but also tying together seemingly diverse fields like representations of Lie algebras, Quantum groups and Statistical mechanics. It is safe to claim that Jones work along with that of Connes (the 2001 Crafoord Prize winner) has been instrumental in revitalizing the subject of von Neumann algebras giving it a much vaster scope, and making it a central part of Modern Mathematics.

In addition to the Fields medal he has enjoyed a variety of awards and distinctions, the complete list of which may be too tedious to list here. Suffices it to single out, among his fellowships in various prestigious societies like the Royal Society, the AASA and the National Academy of Sciences, the award of the Honorary vice Presidency for life of the International Guild of Knot Tyers, a position he has enjoyed since 1992.

#### Friday 20 April 2001.

**Fredrik Andersson,**

*The Lanczos potential in 3+1 dimensions*

#### Abstract:

This talk concerns the Lanczos potential of the Weyl tensor on Lorentz manifolds. I will start by considering Poincaré's lemma which, when applied to electromagnetic theory, proves the existence of a first order potential of the electromagnetic field tensor. A natural question is then: Does the 'gravitational field' in general relativity also have a first order potential? Two natural interpretations of the 'gravitational field' are the Riemann and Weyl curvature tensors. As it turns out only the Weyl tensor has a first order potential, called the Lanczos potential. I will describe some of the properties of the Lanczos potential, as well as its history. In particular it can be shown to satisfy a linear wave equation, unlike the Weyl tensor itself whose wave equation is non-linear. If time permits I will also briefly describe some applications of the Lanczos potential to other mathematical and physical problems. This talk is intended for a general mathematical audience and requires only some elementary knowledge of differential geometry.

#### Friday 27 April 2001.

**Professor Maciej Błaszczak, A. Mickiewicz University, Poznań,**

*From bi-Hamiltonian geometry to separation of variables*

#### Abstract:

The separation of variables belongs to the most important methods of solving nonlinear ordinary differential equations of Hamiltonian type. It is known from the 19th century, when Hamilton and Jacobi proved that given a set of appropriate coordinates, the so called separated coordinates, it is possible to solve a related dynamical system by quadratures. Unfortunately in the 19th century and most of the 20th century, for a number of models of classical mechanics the separated variables were either guessed or found by some ad hoc methods. A fundamental progress in this field was made in 1985, when Sklyanin adopted the method of soliton systems, i.e. the Lax representation, to systematic derivation of separated variables. In his approach, the appropriate Hamiltonians appear as coefficients of the spectral curve, i.e. the characteristic equation of the Lax matrix. Recently, a new constructive separability theory was constructed, based on a bi-Hamiltonian property of integrable systems. In this seminar we briefly summarise the results of the theory in the simplest case of one-Casimir Poisson pencils and illustrate it on some examples. We also make a few comments on the relation between a separation curve of bi-Hamiltonian approach and a spectral curve of Sklyanin approach.

#### Friday 4 May 2001.

**Pontus Andersson,**

*Random permutations and random circuits*

#### Abstract:

Let  $L_k$  be the length of the  $k$ -th longest cycle in a random (uniformly distributed) permutation of  $n$  elements. It is well known that  $(L_1/n, L_2/n, \dots)$  converges in distribution to a Poisson--Dirichlet distribution as  $n$  tends to infinity. I will give a survey of generalizations and variations of this result, including results on excursion lengths in Brownian motion, factorization of random integers and polynomials, and component sizes of certain random combinatorial structures. The focus will be on combinatorial structures; in particular, I will present a result on the lengths of the circuits obtained by randomly decomposing the set of edges of the complete directed or undirected graph into circuits.

#### Friday 11 May 2001.

**Hans Lundmark,**

*Driven Newton equations and time-dependent separable potentials*

#### Abstract:

I will show how to solve the system of ordinary differential equations  $\ddot{x} = -\nabla V(x, t)$  for certain time-dependent potentials  $V(x, t)$ , where  $x \in \mathbb{R}^n$ . This involves a somewhat nonstandard application of the Hamilton--Jacobi method, using new types of separation coordinates. The result comes from a study of driven Newton systems  $\dot{q} = M(q)$ ,  $q \in \mathbb{R}^{m+n}$ , with integrals of motion of cofactor type. These concepts, as well as a few basic facts needed from analytical mechanics, will all be explained in an elementary way during the talk.

#### Friday 18 May 2001.

**Dr. Mikhail Shapiro, KTH,**

*Hurwitz numbers and Hodge integrals*

#### Abstract:

We shall discuss a relation between the classical problem of counting ramified coverings of the sphere with fixed branch points and the theory of moduli spaces of complex genus curves with marked points. One of the applications is a new proof by A. Okounkov and R. Pandharipande of the famous Witten's conjecture.

#### Monday 28 May 2001.

**Andreas Axelsson, Australian National University, Canberra,**

*Boundary value problems for Dirac operators on Lipschitz domains with applications to Maxwell's equation*

*Abstract:*

This talk will be about boundary value problems like

$$\begin{aligned} \mathbf{D}F(x) + \epsilon_0 F(x) &= 0 \text{ in } \Omega \\ f_{\text{normal}}(y) &= g(y) \text{ on } \partial\Omega \end{aligned}$$

Here  $\Omega \subset \mathbf{R}^n$  is a bounded Lipschitz domain,  $F: \Omega \rightarrow \wedge \mathbf{R}^{n+1}$  is a function with values in the full exterior algebra, with boundary trace  $f \in L^2(\partial\Omega; \wedge \mathbf{R}^{n+1})$ , and  $D = \sum_{k=1}^n e_k \partial/\partial x_k$  is the Dirac operator on  $\mathbf{R}^n$ .

Conditions on the normal part  $g$  ensuring that the solution  $F$  maps  $\Omega \rightarrow \wedge^k \mathbf{R}^{n+1}$  for given  $k=0, \dots, n+1$ , and how this applies to Maxwell's equation, will be discussed.

**2 Friday 1 June 2001.**

**Professor Grigori Mints, Stanford University,**

*Epsilon substitution: past and future*

*Abstract:*

The epsilon substitution method introduced by Hilbert provides numerical realizations of existential sentences. It attracted interest of J. von Neumann, H. Weyl, W. Ackermann and other logicians. After original setbacks and successes in number-theoretic setting before 1941 further progress was made in the 1990s for mathematical analysis (second order arithmetic). We describe original formulation in the framework of Hilbert's program, subsequent change of emphasis to verifiable computer programs, results obtained for predicative subsystems of analysis and the most recent progress for impredicative part.

**2 Wednesday 5 September 2001.**

**Professor Daniel Lesnic, University of Leeds,**

*Inverse Problems with Applications*

*Abstract:*

The field of inverse problems is a relatively new area of mathematical research, having its origins in the fundamental papers of Tikhonov in the mid-1960's. As with any new area of mathematics, one can ask the question why did it start, when it did and why not sooner? In the case of inverse problems, the answer is one of historical prejudice meeting scientific pressure. The historical prejudice dates back to Hadamard (1923) who claimed that the only problems of physical interest were those that had a unique solution depending continuously on the given data. Such problems were called *well-posed* and problems that were not well-posed were labeled *ill-posed*. In particular, ill-posed problems connected with partial differential equations of mathematical physics were considered to be of purely academic interest. The success of aerospace industry caused scientists to ask the question how can one determine the temperature and the heat flux at the outer nozzle of a re-entry vehicle in the atmosphere from measurements taken inside the rocket.

Other applications have since then been found in studying the surface conditions at the exhaust of a rocket or jet engine, the motion of a projectile over a gun barrel surface, the sliding of a piston in the engine of a highly volatile combustion chamber, melting and ablation, freezing, quenching or casting of a material process, wave scattering from obstacles, the backward continuation in time, galvanic corrosion in engineering structures, such as in an underground pipeline, of f-shore structures and chemical plants, etc.

In this talk, I shall formulate and present methods of solution of several types of inverse problems for partial differential equations which will include inverse boundary value problems, inverse initial value problems and inverse coefficient identification problems. In particular, the determination of the temperature and heat flux at a known or unknown inaccessible boundary of a metal in heat conduction, the determination of underspecified boundary conditions in Stokes flows and plate bending, the determination of the initial temperature of boiled water, the determination of the permeability of rocks in porous media, thermal properties of heat conductor, properties of beams and plates in elasticity, coefficients of wave equation, etc. will be discussed.

**2 Wednesday 12 September 2001.**

**Anders Björn,**

*Removable singularities for Hardy spaces of analytic functions*

*Abstract:*

In this talk we will study removable singularities for the *Hardy spaces*  $H^p(G) = \{f \text{ analytic in } G : |f|^p < u \text{ for some } u \text{ harmonic in } G, 0 < p < \infty, \text{ and } H^0(G) = \{f \text{ analytic in } G : f \text{ is bounded}\}$ . A compact set  $K$  is a *removable singularity* for  $H^p(G-K)$  if all functions in  $H^p(G-K)$  have analytic continuations to all of  $G$ .

The talk will be elementary and will contain both the history of the problem and recent results. Recently (August 2001) Xavier Tolsa announced the complete solution of the long standing problem (it goes back at least to 1888) of characterizing the removable singularities for bounded analytic functions, i.e. for the functions in  $H^0$ . This characterization will also be mentioned.

**2 Wednesday 19 September 2001.**

**Bjarne Bøe, Universitetet i Bergen and Institut Mittag-Leffler,**

*Interpolation in the Bloch Space*

*Abstract:*

The Bloch space consists of the holomorphic functions in the unit disk  $D$  satisfying  $|f(z) - f(w)| \leq C b(z, w)$ , where  $b$  is the hyperbolic metric. We say that a sequence  $\{z_n\}$  in  $D$  is interpolating if whenever  $\{w_n\}$  satisfies  $|w_n - w_m| \leq C b(z_n, z_m)$ , then we can find  $f$  in the Bloch space having  $f(z_n) = w_n$ . We give a characterisation of such sequences. This is joint work with Artur Nicolau.

**2 Wednesday 26 September 2001.**

**Dr. Jana Björn, Lunds tekniska högskola,**

*Poincaré inequalities, admissible weights and PDEs on metric spaces*

*Abstract:*

In 1982, Kenig--Fabes--Serapioni singled out a few conditions sufficient for extending Moser's iteration technique to weighted degenerate equations. In the 1990s, it was shown that two conditions are enough -- the doubling condition  $\mu(2B) < C \mu(B)$  and the  $(1, p)$ -Poincaré inequality

$$\mu(B)^{-1} \int_B |u - u_B| d\mu \leq C \text{diam}(B) (\mu(B)^{-1} \int_B |\text{grad } u|^p d\mu)^{1/p}.$$

Here,  $\mu$  is a measure (on  $\mathbf{R}^n$  or a metric space),  $u_B = \mu(B)^{-1} \int_B u d\mu$  and  $\int$  stands for integration. Measures satisfying the above two conditions are called  $p$ -admissible and allow a reasonable definition of weighted Sobolev spaces. One can then study weighted elliptic equations and prove many classical results about weak solutions in this setting. It is even possible to consider PDEs on metric measure spaces satisfying the doubling condition and the Poincaré inequality. Even though there are many examples of admissible weights, it is in general difficult to verify whether a given measure is  $p$ -admissible or not. I shall present some results (old and new) which make it possible to generate new admissible measures.

**2 Wednesday 3 October 2001.**

**Professor Grigori Rozenbloum, Chalmers,**

*Pseudodifferential operators with operator valued symbols and unusual index formulas*

*Abstract:*

The index formula for pseudodifferential operators involves integration of certain differential forms, Chern-Simons characters, containing the trace of some combination of the symbol and its derivatives. When one tries to carry over the index formula to the case of operators acting on functions with values in an infinite-dimensional Hilbert space, one cannot use this expression for the integrand since it contains the trace of non-trace-class operators. The methods of non-commutative geometry enable us to find a regularisation for the characteristic classes thus constructing new, unusual index formulas for a new, rather wide class of pseudodifferential operators with operator valued symbols. Such operators arise in the analysis on manifolds with singularities.

**2 Wednesday 10 October 2001.**

**Dr. Frank Kutzschebauch, Uppsala,**

$\mathbf{C}^n$  - a manifold with huge symmetry group

*Abstract:*

We start with some general remarks about the holomorphic symmetries (=automorphisms=bijjective holomorphic selfmaps) of complex manifolds which of course form a group under composition.

The affine space  $\mathbf{C}^n$  for  $n \geq 2$  is special in the sense that its group of holomorphic automorphisms is infinitely dimensional. This group is not well understood yet and there are many (often completely open) questions about it. We will talk about two of them, their history (partial results) and even remark on their analogs in the polynomial case.

**1. Linearization problem:** Can any automorphism  $f$  of  $\mathbf{C}^n$  which is of finite order  $k$  (applying  $f$  a finite number  $k$  of times gives the identity map) be made linear after a suitable change of coordinates? More generally is the same true for any compact group of automorphisms?

**2. Embedding problem:** Given an embedding  $f: \mathbb{C} \rightarrow \mathbb{C}^n$ . Can one change coordinates so that the embedding becomes the standard one, i.e. does there exist a holomorphic automorphism  $a$  of  $\mathbb{C}^n$  such that

$a \circ f(t) = (t, 0, 0, \dots, 0)$  for all  $t$  in  $\mathbb{C}^n$ ?

Finally we come to discuss the negative solution to Problem 1 found by H. Derksen and the speaker using the negative solution to Problem 2 found by Forstneric, Globevnik, Rosay and Rudin. This method gives a connection to the (holomorphic version of the) famous Zariski cancellation problem and one of the most challenging problems in this area arises: How to detect  $\mathbb{C}^n$  among complex manifolds?

**2 Wednesday 17 October 2001.**

**Dr. Krzysztof Marciniak, ITN, Norrköping,**

*From the Darboux theorem to separation of variables in bi-Hamiltonian systems*

*Abstract:*

The well-known theorem of Darboux about local structure of Poisson manifolds states that it is (locally) always possible to choose coordinates on a Poisson manifold in such a way that the corresponding Poisson operator becomes canonical. It means that every Hamiltonian system of differential equations can be cast into a canonical form. In case of bi-Poisson manifolds (that is manifolds equipped with a pair of Poisson operators that are compatible) one can often introduce so called Darboux-Nijenhuis coordinates that cast the corresponding bi-Hamiltonian systems into a form that allows us to solve them by the method of separation of variables. I will illustrate these concepts on the example of recently investigated bi-cofactor systems of Newton equations. No prerequisites will be necessary.

**2 Wednesday 24 October 2001.**

**Johan Wästlund,**

*Trick taking games*

*Abstract:*

I will begin with a brief outline of the classical theory of combinatorial games. This includes the group structure of the set of all games with the normal playing convention, that is, last move wins. Then I will talk about trick taking games, which is a family of card games. These games do not in general satisfy the normal playing convention. In fact, the move order is not even alternating, since the player who wins a trick plays first in the next one. Each of these games gives rise to an abelian semigroup. I will show how some games can be analyzed and solved by investigating the algebraic structure of this semigroup. In particular, it turns out that the game of "femkort" (five-card), which is well known and popular at least in Sweden, is isomorphic to the set of integers, while two-person whist gives rise to a more complicated semigroup that among other things encodes properties of the technique known to bridge players as elimination and throw-in."

**2 Wednesday 31 October 2001.**

**Gunnar Aronsson,**

*On two moving boundary problems, arising in polymer processing*

*Abstract:*

Injection moulding and compression moulding are two production processes in polymer engineering, which give rise to moving boundary problems with highly nonlinear effects. One wants to understand the flow problem, when a polymer melt expands in a more or less complicated geometry. Two asymptotic solution approaches will be briefly presented, one for each problem. Then the solution for the injection moulding case (formsprutning) will be discussed in some detail. It leads to some interesting geometric considerations. Some nice computer graphics will be shown.

**2 Wednesday 7 November 2001.**

**Professor Ernie Kalnins, University of Waikato, New Zealand,**

*Superintegrability in classical and quantum mechanics*

*Abstract:*

The concept of classical and quantum quadratic superintegrability is introduced. The particular properties that characterise this concept are then discussed. The problem of the classification of such systems in two dimensions is then analysed in detail giving a complete result for spaces of constant curvature. The corresponding problem on a two dimensional space of non-constant curvature follows from this result. Finally we show how to generate extra constants for a classical Hamiltonian system which can be solved by separation of variables. In so doing new families of superintegrable systems are generated.

**2 Wednesday 14 November there were two colloquia. They were part of the GRSweden-conference.**

**2 Wednesday 14 November 2001.**

**Dr Håkan Andreasson, Chalmers,**

*Global existence of solutions to the Einstein equations with symmetry*

*Abstract:*

The cosmic censorship conjecture was proposed by Roger Penrose in the sixties and is considered to be one of the most important open questions in general relativity. I will try to explain what this conjecture is all about (it has to do with the nature of spacetime singularities) and we will see that an important step in proving cosmic censorship is to prove global (geometric) existence of solutions to the Einstein equations. This is a very hard problem in general and a way to simplify it is to make symmetry assumptions. I will present a global existence result for matter spacetimes with a certain kind of symmetry (Gowdy symmetry). As opposed to spherical symmetry this symmetry class is sufficiently non-restrictive to admit for gravitational waves.

**2 Wednesday 14 November, 14.20-15.20.**

**Professor Claes Uggla, Karlstads Universitet,**

*Dynamical systems in Cosmology*

*Abstract:*

The evolution of the Universe is ruled by gravity. Our best theory of gravity is general relativity (GR), and hence one uses Einstein's field equations to produce cosmological models. What scenarios are possible for the early Universe and what is the eventual fate of the Universe according to GR? To address issues like these, it has turned out to be fruitful to use a dynamical systems approach to Einstein's field equations. To illustrate how dynamical systems ideas are applied in GR, I will start by discussing spatially homogeneous and isotropic Friedmann-Lemaître models, which have been remarkably successful in explaining many cosmological observations. First I will give a qualitative picture of the possible features such models exhibit by using simple potential diagrams. Subsequently I will use a dynamical systems approach and give a comparison between the two pictures. Thereafter I will outline how GR exhibits a hierarchical structure that allows one to build increasingly complex models using dynamical systems methods. I will finish by discussing some recent results and speculate about the generic features of cosmological models in GR at very early and late times.

**2 Wednesday 21 November 2001.**

**Timo Koski,**

*A model for predictive mixtures and for classification of sequences*

*Abstract:*

Detection of protein sequence homologies can be done by using mixtures of Dirichlet distributions. These are statistical models for motifs in multiple alignments of protein sequences. We derive this mixture using an assumption of infinite exchangeability and predictive sufficiency. By this argument it is immediate that we are dealing with predictive classification of protein sequences in the sense of predicting a portion of a sequence based on a motif. Finally a result about the distribution of the score based on an exchangeable representation is outlined.

**2 Wednesday 28 November 2001.**

**Professor Kimmo Eriksson, Mälardalens Högskola,**

*Lecture hall partitions*

*Abstract:*

The theory of integer partitions is both accessible, surprising and entertaining. I will give an overview of famous partition identities, from classical results of Euler, via Rogers and Ramanujan, to modern achievements of Andrews, Bressoud and others, and ending with the so called Lecture Hall partition identities of Eriksson and Bousquet-Mélou.

**2 Friday 30 November 2001.**

**Professor Mikhail Shubin, Northeastern University, Boston,**

*Magnetic Schrödinger operators: discreteness of spectrum and strict positivity criteria*

*Abstract:*

Discreteness of spectrum of a self-adjoint operator means that its spectrum consists of isolated eigenvalues with finite multiplicity. Sufficient conditions of discreteness of spectrum for Schrödinger operators in  $\mathbb{R}^n$  were given by H. Weyl (1910) for  $n=1$  and K. Friedrichs (1934) for arbitrary  $n$ . A necessary and sufficient condition for Schrödinger operators with positive

potentials in terms of the Wiener capacity was given by A. Molchanov (1953).

We give a necessary and sufficient condition for the discreteness of spectrum for the magnetic Schrödinger operators with positive scalar potential in  $\mathbf{R}^n$ . It is given in terms which include energy of the magnetic field in balls of fixed radius and a capacity term characterizing the scalar potential in these balls.

We also give a necessary and sufficient condition for such operators to be strictly positive, i.e. condition that 0 is not in the spectrum of such an operator.

(This is joint work with V. Kondratiev and V. Maz'ya.)

📅 **Wednesday 5 December 2001.**

**Professor Mikael Passare, Stockholms universitet,**

*Amoebas, polytopes and integrals of rational functions*

*Abstract:*

To any given complex polynomial one has recently associated a geometric-combinatorial concept, its so called *amoeba*. We shall give a verbal and visual account of (mainly two-dimensional) amoebas and their relation to Newton polytopes and moment mappings.

📅 **Wednesday 12 December 2001.**

**Professor Nail Ibragimov, Blekinge tekniska högskola,**

*Lie group analysis of differential equations*

*Abstract:*

The talk is aimed at discussing basic methods from Lie group theory of integration, linearization and nonlinear superposition for nonlinear differential equations. Methods of modern group analysis of partial differential equations such as symmetry of fundamental solutions, group analysis in financial mathematics, approximate symmetries of equations with a small parameter, and Lie-Bäcklund transformation groups with application in celestial mechanics will be also considered.