Feedback Linearization or Jacobian Linearization?

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Summary

Linear Quadratic (LQ) control design is applied to a non-linear system using both Feedback Linearization (FL) and Jacobian Linearization (JL).

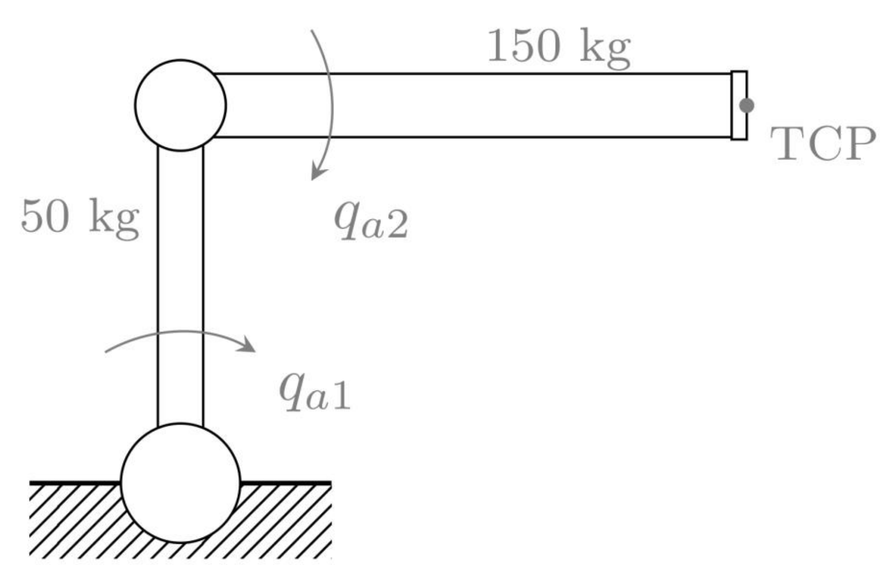
The system in question is an experimentally verified model of an industrial manipulator, and the control task is disturbance rejection.

Taylor expansion of LQ loss functions is used to achieve equivalent controller tunings at selected working points.

Simulation model

Describes two links of an industrial manipulator and features

- non-linear motor friction,
- non-linear gear stiffness,
- time delays,
- quantization, and
- measurement noise.



Control design model

$$\ddot{q}_a = -M_1^{-1}(q_a)(C(q_a, \dot{q}_a) + G(q_a) + \tau(q_a, q_m))$$
$$\ddot{q}_m = M_2^{-1}(N^{-1}\tau(q_a, q_m) + u)$$

$$P : \dot{x} = f(x) + Bu, \quad x = \begin{bmatrix} q_a^T & \dot{q}_a^T & q_m^T & \dot{q}_m^T \end{bmatrix}^T$$

LQ-control of an Industrial Manipulator Benchmark

Disturbances

The system is subjected to two disturbances, one frequency-varying motor disturbance and one external force pulse.

Performance measure

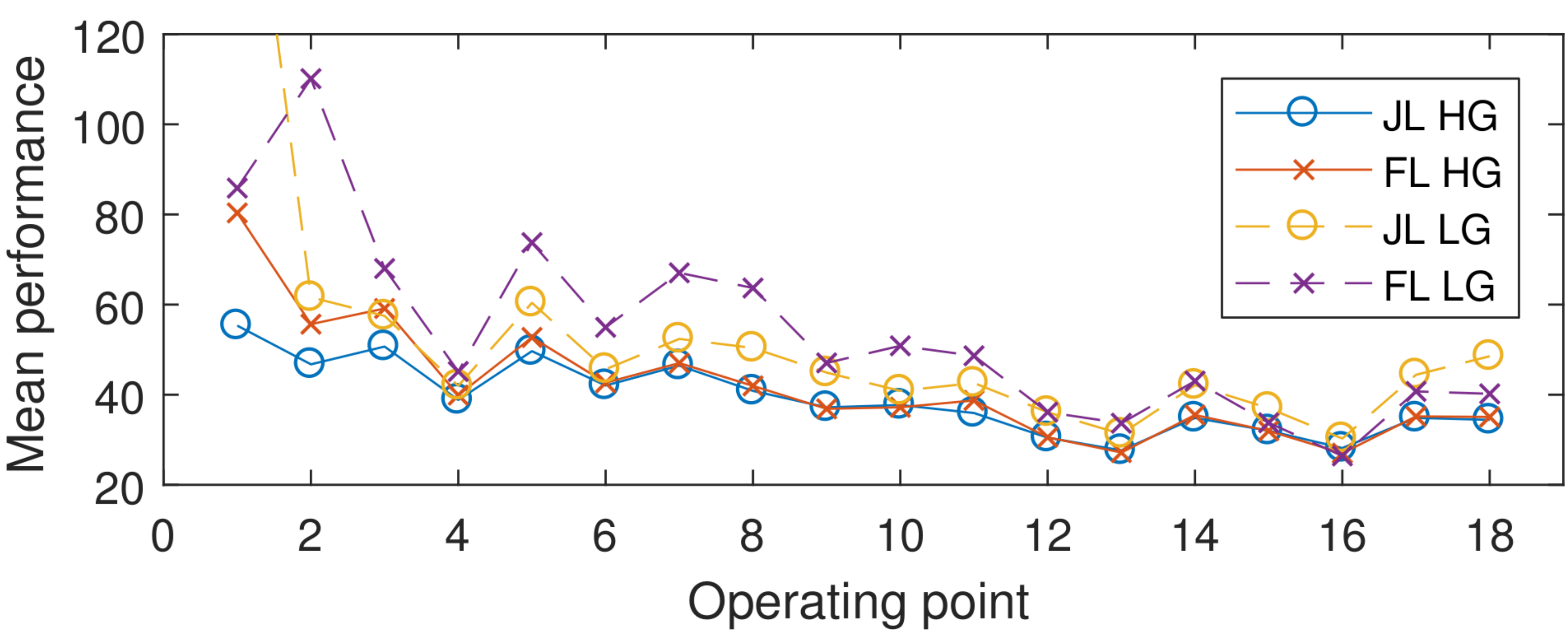
The suitability of the controllers for industrial use is evaluated using a performance measure based mainly on:

- tool position error,
- settling time,
- overall motor torque, and
- motor torque due to measurement noise.

Results

Simulations were run repeatedly with random disturbance parameters at 18 different operating points, with controllers designed for each point.

The controllers were tuned with both a high (HG) and low gain (LG).



Jacobian Linearization

$$P_\delta : \dot{x} = f'_x(x_0)x_\delta + Bu, \quad x_\delta = x - x_0$$

Feedback Linearization

$$P_{_F} : \dot{x} = f(x) + B(\alpha(x) + \beta(x)v)$$

$$y = T(x), \quad y = \begin{bmatrix} q_a^T & \dot{q}_a^T & \ddot{q}_a^T & q_a^{(3)T} \end{bmatrix}^T$$

$$P_{_F} : \dot{y} = A_{_F} y + B_{_F} v$$

Equivalent LQ controller tuning

We have two LQ loss functions: $V_\delta = x_\delta^T Q x_\delta + u^T R u$

$$V_{_F}(y_\delta, v) = y_\delta^T Q_{_F} y_\delta + v^T R_{_F} v + y_\delta^T N_{_F} v$$

Using Taylor expansion we find that the following choice of weight matrices makes the loss functions equal to the second order around a working point:

$$Q_{_F} = Q + H^T \tilde{Q} H, \quad R_{_F} = \tilde{R}, \quad N_{_F} = H^T \tilde{N}$$

$$\begin{aligned} \tilde{Q} &= \alpha'_x(x_0)^T R \alpha'_x(x_0) \\ \tilde{R} &= \beta(x_0)^T R \beta(x_0), \\ \tilde{N} &= 2\alpha'_x(x_0)^T R \beta(x_0) \end{aligned}$$

However, with the much simpler choice

$$Q_{_F} = Q, \quad R_{_F} = \tilde{R}, \quad N_F = 0$$

nearly identical performance is achieved.

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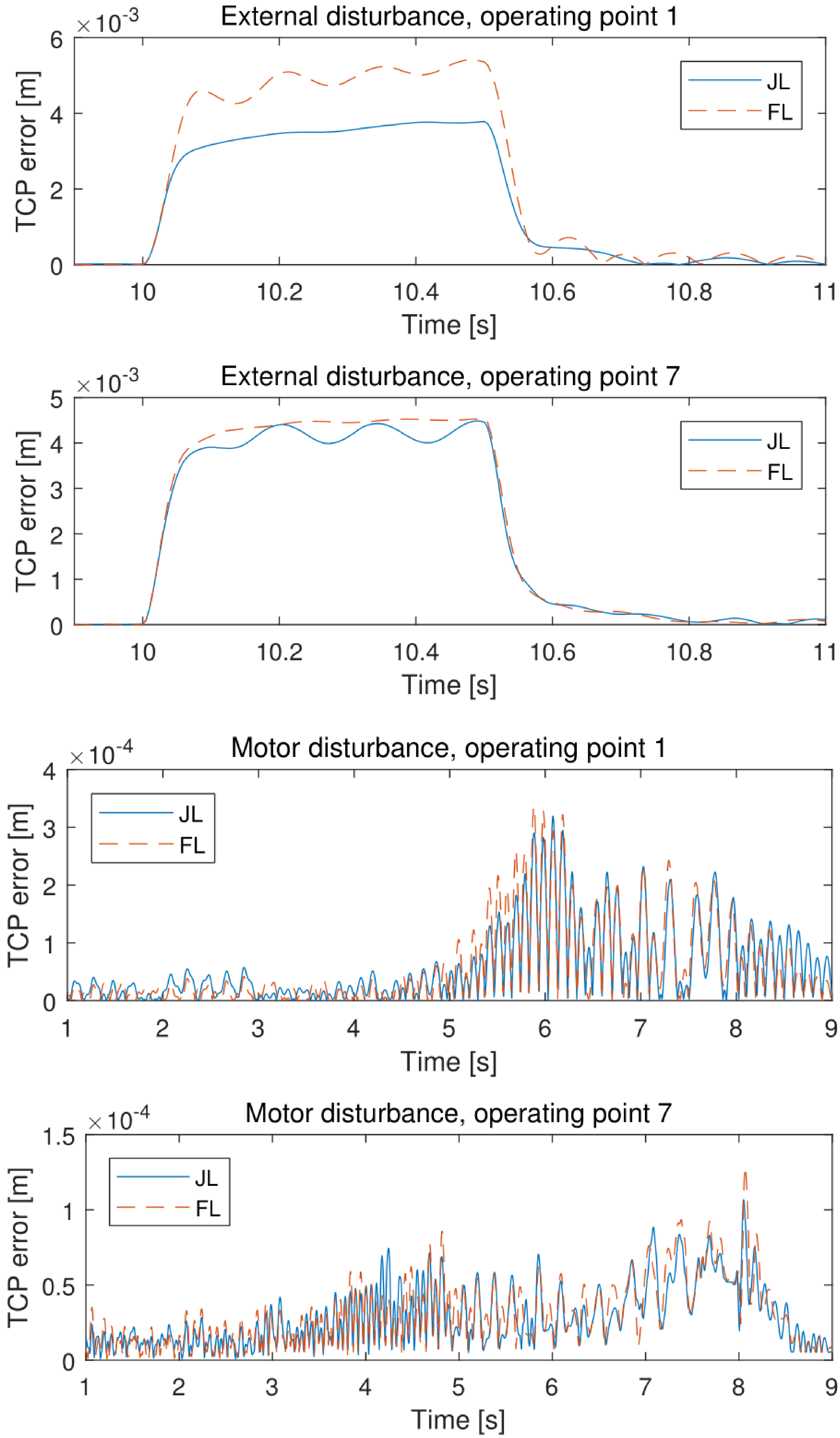
External disturbance, operating point 1
×10^-3
TCP error [m]
Time [s]
JL
FL
External disturbance, operating point 7
×10^-3
TCP error [m]
Time [s]
JL
FL
Motor disturbance, operating point 1
×10^-4
TCP error [m]
Time [s]
JL
FL
Motor disturbance, operating point 7
×10^-4
TCP error [m]
Time [s]
JL
FL