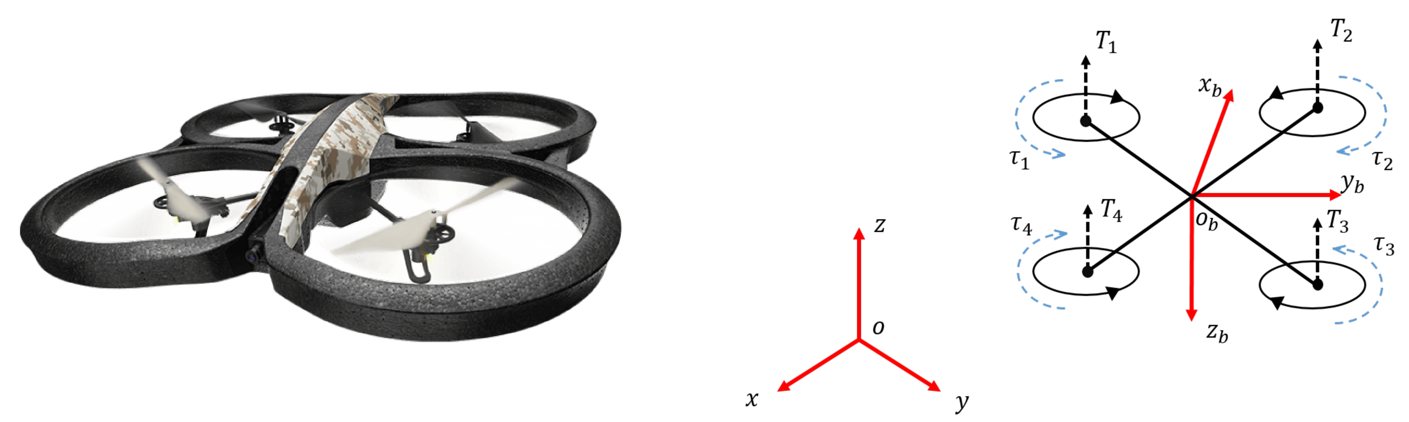
# Modeling and diagnosis of UAVs

Du Ho (du.ho.duc@liu.se)

## Summary

Since the dynamic model of a quadcopter is too complex for estimation purposes, submodels are considered. First, a lateral dynamic model is used to estimate the mass changes of the quadcopter using only measurements from onboard sensors, which leads to a sensor-to-sensor problem. Second, a challenge of a general sensor-to-sensor problem is that it is sometimes not obvious to select which signals to use as input or output of a SISO system. It is shown that an Instrumental Variable (IV) approach gives identical results when estimating the forward and inverse models of a SISO system. Third, the IV method can also provide accurate estimates of the parameters of a Hammerstein model of the vertical dynamics in the closed-loop setting.

## Quadcopter dynamic



In the body-fixed frame, the dynamic equations of a rigid body quadcopter using the Newton-Euler equations are given by

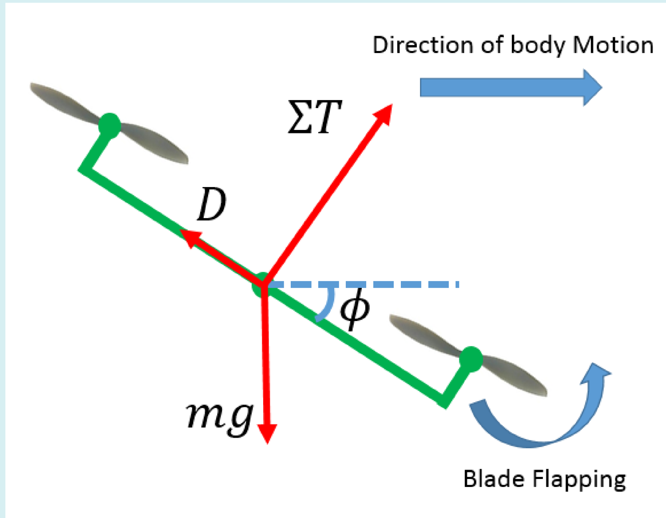
$$\begin{aligned} m\dot{V}_B + m\nu \times V_b &= mR^T g + E_B^F(\Omega) + D_B^F(V_b) \\ I\dot{\nu} + \nu \times (I\nu) &= O_B^\tau(\nu, \Omega) + E_B^\tau(\Omega) \end{aligned} \quad (1)$$

where $m$ is the mass of the quadcopter.

## Submodels of a quadcopter

- The drag effect makes the lateral acceleration linearly dependent on the lateral velocity in the body-fixed frame.
- The vertical thrust equation contains a linear term due to the induced velocity of the air flow interacting with the propellers.

## Sensor-to-sensor problem



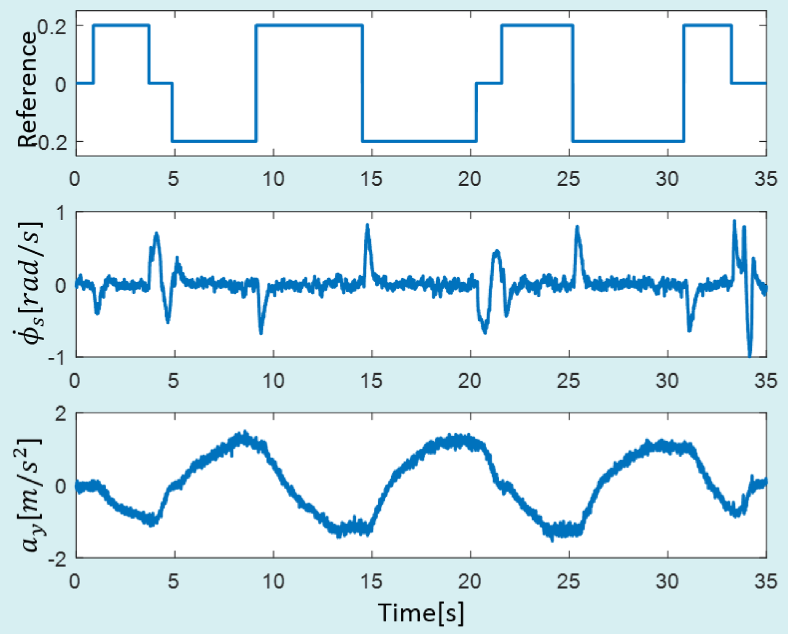

The effect of the relative speed of the blades with respect to free air divides the operating region of a propeller into two areas: a retreating and an advancing blade. The advancing blade has a higher relative velocity than the retreating one, which creates a force imbalance between the two areas.

Projecting (1) onto the lateral plane in the body-fixed frame

$$\dot{v} = g\cos(\theta)\sin(\phi) - \frac{\lambda_1}{m}v \quad (2a)$$

Using measurements from an IMU

$$a_y = \frac{\lambda_1}{m}v + e_{a_y} \quad \dot{\phi}_m = \dot{\phi} + e_{\dot{\phi}} \quad (3)$$

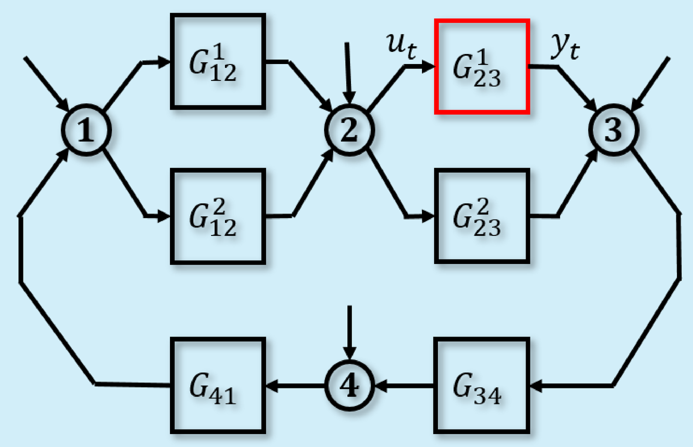



Three different mass datasets are collected.

| $m_{ref}$ | $m_c$ | $\hat{m}_c$ (LS) | $\hat{m}_c$ (EKF) | $\hat{m}_c$ (IV) |
|---|---|---|---|---|
| $455\,g$ | $510\,g$ | $1362.5 \pm 54.9\,g$ | $505.6 \pm 258.8\,g$ | $504.1 \pm 3.9\,g$ |
| | $582\,g$ | $2126.2 \pm 78.9\,g$ | $384.4 \pm 161.2\,g$ | $580.9 \pm 3.8\,g$ |
| $510\,g$ | $455\,g$ | $170.3 \pm 6.9\,g$ | $458.9 \pm 234.8\,g$ | $460.3 \pm 3.4\,g$ |
| | $582\,g$ | $795.8 \pm 25.7\,g$ | $387.3 \pm 187.3\,g$ | $587.5 \pm 3.2\,g$ |
| $582\,g$ | $455\,g$ | $124.5 \pm 4.6\,g$ | $689.7 \pm 289.6\,g$ | $456.1 \pm 3.0\,g$ |
| | $510\,g$ | $373.1 \pm 12.1\,g$ | $766.4 \pm 370.7\,g$ | $505.2 \pm 2.8\,g$ |

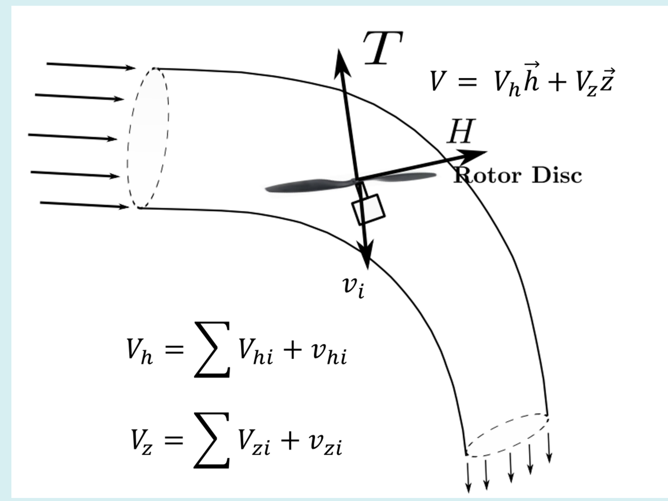
The least-squares and EKF methods give unreliable results while the IV method can detect the changes of mass accurately.

## Errors-in-variables estimation



The IV method gives identical estimates of the forward model $G_{23}^1$ and the inverse model $G_{32}^1$, regardless input-output selection, using finite data.

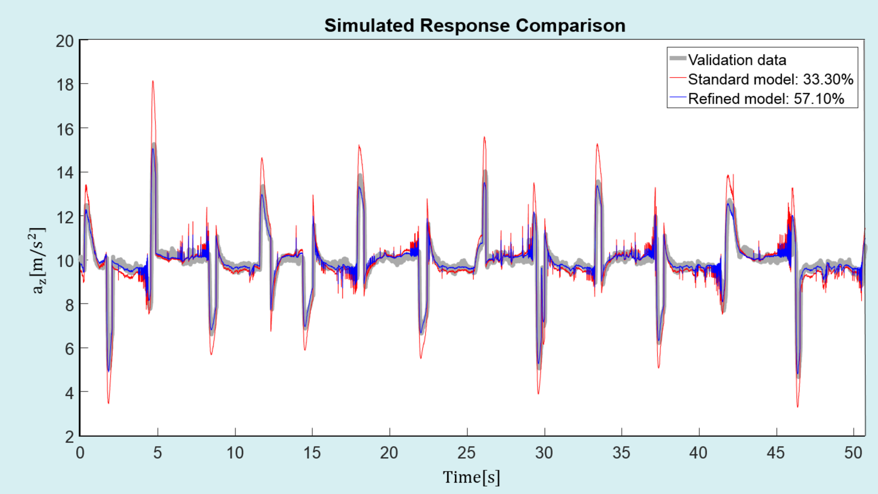
## Nonlinear Hammerstein model




$V_{hi}$ and $V_{zi}$ are the horizontal and vertical velocities of the $i^{\text{th}}$ rotor in the body-fixed frame, and $v_{hi}$ and $v_{zi}$ are the horizontal and vertical induced velocities of the air stream through the $i^{\text{th}}$ rotor.

Projecting (1) onto the $z_b$ axis yields

$$\dot{w} = \underbrace{-\underbrace{\frac{k_1u_t^2}{m}}_{\text{standard thrust}} - \frac{k_2u_t}{m}}_{\text{refined thrust}} - \frac{k_w}{m}w + g\cos\theta\cos\phi \quad (4)$$




The refined model (57.10% model fit) gives a more accurate estimate of the vertical dynamics of the quadcopter than the standard model (33.30% model fit).

## Future work

1. Fault detection and isolation algorithms.
2. Other nonlinear block-oriented models.

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MarineUAS
Innovative Training Network on Autonomous Unmanned Aerial Systems for Marine and Coastal Monitoring

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