### Abstracts

#### Density-dependent feedback in age-structured populations

Jonathan Andersson, Linköping University, Sweden

In biological populations, density-dependent regulation represents change in individual fitness caused by changes in population size or density. The negative density-dependency, often explained by intra-specific competition and overcrowding effect, is characterised by decline in fitness with increase in populations size or density. In sharp contrast with this is the positive densitydependency, or the Allee effect, characterised by increase in fitness with increase in population size. The Allee effect could result from improved chances to find a mate and increased cooperation. We have modelled this interaction through an improved variant of the McKendrick von Foerster equation with density dependent death and birth rate. The death and birth rate depend on two weighted sizes of the population, one for each rate. The death rate has been skipped the usual assumption of being strictly increasing with respect to the population size, allowing for the Allee effect, but is appropriately required to be infinite for infinite populations. The birth rate is assumed to be bounded. In the case of a non-strictly positive density dependent death rate we have proved boundedness of solution and derived conditions for extinction in terms of the reproductive number and the number of newborns. If the net reproductive rate is less then one then the solution will go to zero if the number of newborns ever get small enough.

This work is a joint collaboration with Vladimir Kozlov (Linköping University), Sonja Radosavljevic (Stockholm University), Vladimir Tkachev and Uno Wennergren (Linköping University)

### The evolution of trait variance creates a tension between species and functional diversity

Gyuri Barabas, Division of Theoretical Biology, IFM, Linköping University

It seems intuitively obvious that species diversity begets functional (trait) diversity: the more plant species there are, the more varied their leaf chemistry will be; more species of crops provide more kinds of food etc. In this talk I argue that the evolution of trait variance challenges this view. In a trait-based eco-evolutionary model, I show that when species richness is low, individual species evolve large trait variation, while in species-rich communities species avoid competition by evolving narrow trait breadths. This effect can be so strong that, even though there are more species in the latter case, they will cover a smaller fraction of the possible trait values than in the species-poor but intraspecifically more variable former one. That is, the expected positive relationship between species and functional diversity is overhauled. This result proved extremely robust to changes in model setup and parameterization. I finish by presenting and discussing empirical data consistent with this claim.

# Consequences of species extinctions on ecosystem service delivery in a large ecological network – a Bayesian network approach

Anna Eklöf, Division of Theoretical Biology, IFM, Linköping University

### Mathematical analysis of SIR models with coinfection and density dependence: degree of complexity

Samia Ghersheen, Vladimir Kozlov, Vladimir Tkachev, Uno Wennergren Linköping University, Linköping, Sweden

Many pathogens that infect humans can coexist within hosts. Therefore coinfection of individual hosts by multiple infectious agents is frequently observed. The study of complete dynamics of this phenomenon involves many complexities. We formulated an SIR model with the coinfection of the two infectious agents in a single host population is considered. The model includes the environmental carry capacity in each class of population. Different special cases of this model are analyzed and several threshold conditions are obtained which describes the establishment of disease in the population. We prove that for small carrying capacity K there exist a globally stable disease free equilibrium point. Furthermore, we establish the continuity of the transition dynamics of the stable equilibrium point, i.e. we prove that (1) for small values of K there exists a unique globally stable equilibrium point, and (b) it moves continuously as K is growing (while its face type may change). This indicate that carrying capacity is the crucial parameter and increase in resources in terms of carrying capacity promotes the risk of infection.

### Hierarchical Bayesian modeling for ecology and epidemiology

Tom Lindström, Division of Theoretical Biology, IFM, Linköping University

Hierarchical Bayesian Models (HBMs) are increasingly used in biological research, facilitated by access to computational power and new algorithms. In my presentation, I will describe what HBMs are on a conceptual level and highlight key concepts and benefits of the statistical framework. Using examples from my own research within the fields of ecology and epidemiology, I will demonstrate how this flexible framework allows us to construct statistical models tailored for specific data and systems and structured in line with our understanding of the underlying processes. Importantly, HBMs aptly handle uncertainty at different levels in a transparent way. For example, the framework can provide parameter estimates at the individual and population level, or separate the underlying process of interest from the observation of the data. Further, uncertainty can be incorporated at the appropriate level when fitted models are used for prediction. I will also show how HBMs can improve parameter estimation through the process of "borrowing strength", and in some instances circumvent prior sensitivity.

# Transport optimization models to help with the Swedish biobased circular economy

Geneviève Metson<sup>1</sup> and Nils-Hassan Quttineh<sup>2</sup> Larger project participants also include: Karin Tonderski<sup>1</sup>, Roozbeh Feiz<sup>3</sup>, Usman Akram<sup>1</sup>, and Uno Wennergren<sup>1</sup> <sup>1</sup>Division of Theoretical Biology, IFM, Linköping University <sup>2</sup>Division of Optimization theory, MAI, Linköping University <sup>3</sup>Division of Environmental Technology and Management, IEI, Linköping University

Nutrient (nitrogen, phosphorus, and potassium) recycling and biogas production can be compatible practices, and both are faced with the challenge of optimally transporting heavy organic waste. In previous work we developed spatiallyexplicit datasets (5km<sup>2</sup>) of crop nutrient demand and recyclable manure for Sweden and used an optimization model to estimate the transport distances and costs associated with redistributing excess nutrients in manure. Currently we are adding biogas plants to scenarios to determine how this changes costs, as well as benefits, associated with organic waste recycling. Through this scenario development however we have had to nimbly adjust datasets and model assumptions to ensure realism making direct comparisons of costs and transport distances difficult across scenarios. Here we present a few examples of how such changes in data and assumptions affect the interpretation and utility of our findings and future research directions.

### Social-ecological traps from dynamical systems perspective

Sonja Radosavljevic, Stockholm University

Social-ecological systems are human-nature coupled systems. Endogenizing human role in ecological modelling (or nature's role in economic modelling) by making it a model variable leads us to social-ecological models. This also allows us to treat social-ecological systems as complex adaptive systems, where every component of the system adapts to changes caused by other components. In the mathematical terms this means that we are studying multidimensional dynamical systems, which may be characterized by positive or negative feedback or combination of both. Multistability which sometimes arise due to the positive feedback in social-ecological literature is represented as 'regime shifts' and 'social-ecological traps'. The former presuppose existence of two stable equilibrium points and forcing the system into one or the other basin of attraction. The latter means that one of the equilibrium points is "undesired" in some sense, mostly because it represents a situation when human well-being is at a low level. To illustrate our findings, I will show a model of poverty trap in agroecological settings and a model of squid fishery with relations between fishers and traders. In the first case, our analysis, based on properties of dynamical systems, shows how interplay of assets, phosphorus, water and soil dynamics give rise to poverty and how agricultural interventions can be used for poverty alleviation. In the second case, I will discuss properties of the extended bioeconomic model.

# Optimal Control of a Threatened Wildebeest-Lion Prey-Predator System in the Serengeti Ecosystem

Thadei Sagamiko<sup>1&4</sup>, Nyimvua Shaban<sup>1</sup>, Curthbet Nahonyo<sup>2</sup> and Oluwole Makinde<sup>3</sup> <sup>1</sup>Department of Mathematics, University of Dar es Salaam <sup>2</sup>Department of Zoology and Wildlife Conservation, University of Dar es Salaam <sup>3</sup>Faculty of Military Science, Stellenbosch University <sup>4</sup>Department of Mathematics, Linköping University

We develop a two-species prey-predator model in which prey is wildebeest and predator is lion. The threats to wildebeest are poaching and drought while to lion are retaliatory killing and drought. The system is found in the Serengeti ecosystem. Optimal control theory is applied to investigate optimal strategies for controlling the threats in the system where anti-poaching patrols are used for poaching, construction of strong bomas for retaliatory killing and construction of dams for drought control. The possible impact of using a combination of the three controls either one at a time or two at a time on the threats facing the system is also examined. We observe that the best result is achieved by using all controls at the same time, where a combined approach in tackling threats to yield optimal results is a good approach in the management of wildlife populations.

References: Chakraborty, K., Chakraborty, M. and Kar, T. 2011 Optimal control of Harvest and Bifurcation of a Prey-Predator Model with stage structure. *Applied Mathematics and Computation*, 217: 8778-8792.; Dubey, B. (2007) A Prey-Predator Model with a Reserved Area. *Nonlinear Analysis: Modeling and Control*, 12: 479-494. ; Peterson MJ 1991. Wildlife parasitism, Science and Management policy. *Wildlife management* **55**:782-789.; Pontryagin L, Boltyanskii V, Gamkrelidze R and Mishchenko E 1962. The mathematical Theory of Optimal process. *Wiley, New York*.

### The Mathematics of Climate Tipping Points

Ivan Sudakov, University of Dayton, USA

A tipping element for the Earth's climate system may be the melting of the summer Arctic sea ice pack, which is occurring at a precipitous rate that has far outpaced the projections of large-scale climate models. As permafrost thaws, some portion of its organic matter will be decomposed by microorganisms, emitting large amounts of greenhouse gases into the atmosphere. While this is another potential tipping point in the climate system and one of the most likely to occur, model estimates do not agree in magnitude or even sign. These types of critical phenomena are of increasing interest as the climate system warms, and are crucial for predicting its stability. In this talk, I will give examples of how bifurcation and stochastic theories are powerful tools that can be used to address such questions. Ultimately this work will advance the representation of tipping elements in climate models, which will provide better predictions of the fate of Earth systems and the response of ecosystems.

### Biodiversity, extinctions and limit evolution structures in food webs

S. Vakulenko, Saint Petersburg, Russia

We investigate the formation of stable ecological networks where many species share the same resource. We show that such a stable ecosystem naturally occurs as a result of extinctions. We obtain an analytical relation for the number of coexisting species, and we find a relation describing how many species that may become extinct as a result of a sharp environmental change. We introduce a special parameter that is a combination of species traits and resource characteristics used in the model formulation. This parameter describes the pressure on the system to converge, by extinctions. When that stress parameter is large, we obtain that the species traits are concentrated at certain values. This stress parameter is thereby a parameter that determines the level of final biodiversity of the system. Moreover, we show that the dynamics of this limit system can be described by simple differential equations. This is a joint work with V. Kozlov and U. Wennergren, published in Phys. Review E, 2017.

### **Periodic Boundaries in Ecological Settings**

Uno Wennergren, Division of Theoretical Biology, IFM, Linköping University

In ecology the dynamical systems may become unpleasantly large, from a mathematical perspective. There are also no sharp system boundaries. Typically, ecosystems exist on our planet and hence the spatial domain is a sphere. Species interactions exist in a spatial context and hence the interactions themselves are expressed on a sphere. Thereby both interactions and the spatial heterogeneity are large systems with no strict boundaries. I present how periodic boundaries have been applied to solve the spatial problem and also how periodic boundaries can be applied to the infinity that is inherent in networks that appears when formulating species interactions. In both these the PBC's are solutions to reduce the complexity but still describe the infinity that is inherent in both space and species interactions. I will also try to sketch how one may include both the infinity of space and species interactions by PBC's.