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Time-Optimal Path Tracking Problem for Cooperative Manipulators

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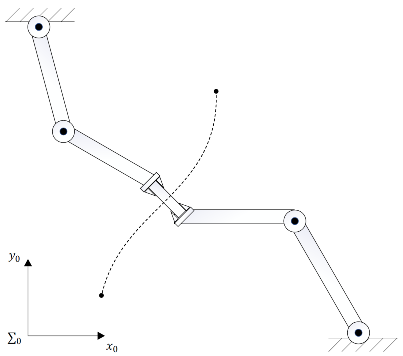


Main Ideas and Contribution

- Time-optimal cooperative transportation of objects by multiple robots
- Cooperative path tracking
- Convex reformulation
- Promising results for two planar manipulators

Manipulator Dynamics

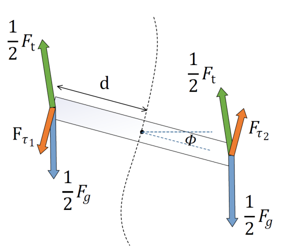
$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \qquad i = 1,2$$

Time-Optimal Path Tracking (Single Manipulator)

$$\begin{aligned}
\underset{T,\, s(\cdot),\, \tau(\cdot)}{\text{minimize}} \quad & T \\
\text{subject to} \quad & \tau(t) = m(s(t))\ddot{s}(t) + c(s(t))\dot{s}(t)^2 + g(s(t)) \\
& s(0) = 0,\ s(T) = 1 \\
& \dot{s}(0) = \dot{s}_0,\ \dot{s}(T) = \dot{s}_T,\ \dot{s}(t) \geq 0 \\
& \underline{\tau}(s(t)) \leq \tau(t) \leq \bar{\tau}(s(t)) \\
& \text{for } t \in [0,T]
\end{aligned}$$

Time-Optimal Cooperative Path Tracking

- Resulting torques form end-effector forces

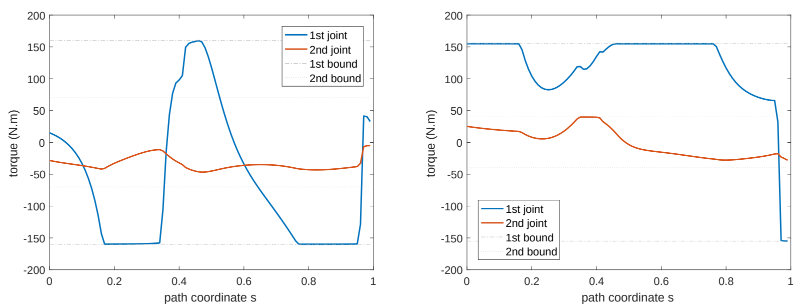


$$\begin{aligned}
\bar{\tau}_i(s) = &\left(\bar{J}_i(q_i(s))^T R_0^i \bar{m}_i(s)\right)\ddot{s} \\
&+ \left(\bar{J}_i(q_i(s))^T R_0^i \bar{c}_i(s)\right)\dot{s}^2 + \left(\bar{J}_i(q_i(s))^T R_0^i \bar{g}\right)
\end{aligned}$$

- Convex formulation

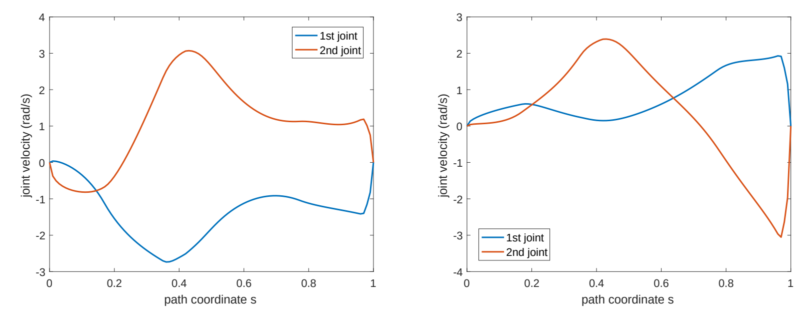
$$\begin{aligned}
\underset{a(\cdot),\, b(\cdot),\, \tau_1(\cdot),\, \tau_2(\cdot)}{\text{minimize}} \quad & \int_0^1 \frac{1}{\sqrt{b(s)}} ds \\
\text{subject to} \quad & \tau_i(s) = \left(m_i(s) - \bar{J}_i(q_i(s))^T R_0^i \bar{m}_i(s)\right) a(s) \\
& \quad + \left(c_i(s) - \bar{J}_i(q_i(s))^T R_0^i \bar{c}_i(s)\right) b(s) \\
& \quad + \left(g_i(s) - \bar{J}_i(q_i(s))^T R_0^i \bar{g}\right), \quad i = 1,2 \\
& b(0) = \dot{s}_0^2,\ b(1) = \dot{s}_T^2 \\
& b'(s) = 2a(s),\ 0 \leq b(s) \leq \bar{b}(s) \\
& \underline{\tau}_i(s) \leq \tau_i(s) \leq \bar{\tau}_i(s), \quad i = 1,2 \\
& \text{for } s \in [0,1]
\end{aligned}$$

Joint Torques



(a) First manipulator (b) Second manipulator

Joint Velocities



(a) First manipulator (b) Second manipulator

Future Work

- Constraints such as rate of torque change (non-convex)
- More manipulators and distributed optimization
- More realistic grasping

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