Obtaining Consistent Parameter Estimators for Second-Order Modulus Models

Fredrik Ljungberg and Martin Enqvist

Introduction

This work deals with the issue of obtaining consistent parameter estimators in nonlinear regression models where the regressors are second-order modulus functions, which is a structure that is often used in models of marine vessels. It is shown that the accuracy of an instrumental variable estimator can be improved by conducting experiments where the **input signal has a static offset** of sufficient amplitude and the **instruments are forced to have zero mean**.

Taylor alternative

Abkowitz (1964):
Taylor-series expansion

$$\tau_j(\xi) = \tau_j(\xi_0) + \sum_{i=1}^{2n_\nu+n_{act}} \Big(\frac{\partial \tau_j(\xi)}{\partial \xi_i}\Big|_{\xi_0} \Delta_\xi + \frac{1}{2}\frac{\partial^2 \tau_j(\xi)}{(\partial \xi_i)^2}\Big|_{\xi_0} \Delta_\xi^2 + \frac{1}{6}\frac{\partial^3 \tau_j(\xi)}{(\partial \xi_i)^3}\Big|_{\xi_0} \Delta_\xi^3 \Big)$$

$$j = 1, \cdots, n_\nu$$

Port/starboard symmetry

$\Longrightarrow$ no second-order terms.

Fedyaevsky and Sobolev (1963):
Second-order modulus functions

$$(m - X_{\dot u})\dot u = X_{|u|u}|u|\,u + (1-t)T + (m + X_{vr})vr + X_{\delta\delta}\delta^2 + X_{ext}$$

$$(m - Y_{\dot v})\dot v + (mx_G - Y_{\dot r})\dot r = -(m - Y_{ur})ur + Y_{uv}uv + Y_{|v|v}|v|\,v + Y_{|v|r}|v|\,r + Y_\delta\delta + Y_{ext}$$

$$(mx_G - N_{\dot v})\dot v + (I_z - N_{\dot r})\dot r = -(mx_G - N_{ur})ur + N_{uv}uv + N_{|v|v}|v|\,v + N_{|v|r}|v|\,r + N_\delta\delta + N_{ext}$$

Port/starboard symmetry thanks to modulus operator.

Motivating example

System: $\begin{cases} x(k+1) = n_0 x(k)\big|x(k)\big| + f_0 u(k) + w(k) \\ y(k) = x(k) + e(k) \end{cases}$

Model: $\hat y(k|\theta) = ny(k-1)\big|y(k-1)\big| + fu(k-1)$

Zero-centered input $(u(k) = \tilde u(k),\ \bar E\{\tilde u(k)\} = 0)$

Asymptotic LS estimate: $\lim_{N\to\infty} \hat\theta_N^{LS} = \cdots = \begin{bmatrix} \frac{n_0 \bar E\{x|x|(x+e)|x+e|\}}{\bar E\{x^4+6x^2e^2+e^4\}} \\ f_0 \end{bmatrix} \neq \begin{bmatrix} n_0 \\ f_0 \end{bmatrix}$

Pick noise-free regressors as instruments

$$\zeta(k) = \Big[x(k-1)\big|x(k-1)\big|\ u(k-1)\Big]^T$$

Asymptotic IV estimate: $\lim_{N\to\infty} \hat\theta_N^{IV} = \cdots = \begin{bmatrix} \frac{n_0 \bar E\{x^4\}}{\bar E\{x|x|(x+e)|x+e|\}} \\ f_0 \end{bmatrix} \neq \begin{bmatrix} n_0 \\ f_0 \end{bmatrix}$

Excitation offset $(u(k) = \bar u + \tilde u(k),\ \bar E\{\tilde u(k)\} = 0)$

Subract mean from instruments

$$\zeta(k) = \Big[x(k-1)^2 - \bar E\{x(k-1)^2\}\ u(k-1) - \bar E\{u(k-1)\}\Big]^T$$

Asymptotic IV estimate: $\lim_{N\to\infty} \hat\theta_N^{IV} = \cdots = \begin{bmatrix} n_0 \\ f_0 \end{bmatrix}$

Lemma: Consistency

System: $\begin{cases} x(k+1) = \Phi^T\Big(\big[x^T(k)\ u^T(k)\big]^T\Big)\theta_0 + w(k) \\ y(k) = x(k) + e(k) \end{cases}$

Model: $\hat y(k) = \Phi^T\Big(\big[y^T(k-1)\ u^T(k)\big]^T\Big)\theta$

Common assumptions:

- $f = \Phi^T(.)\theta$ sec.-ord. mod.
- w, e zero mean and *i.i.d.*
- Global identifiability
- Open-loop experiments
- ζ, w, e mutually indep.
- $E\{\zeta(k)\Phi^T(k)\}$ full rank

Key assumptions:

- The input excites the system so that

  $$\big|x_i(k)\big| > \eta_i > \big|e_i(k)\big|,\ i = 1, \cdots n$$

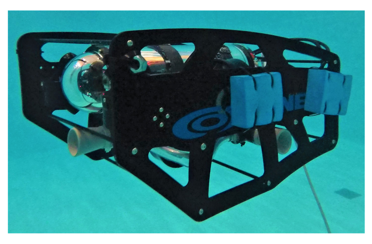
- $\bar E\{\zeta(k)\} = 0$

$\Longrightarrow$ **IV consistent estimator of** θ.

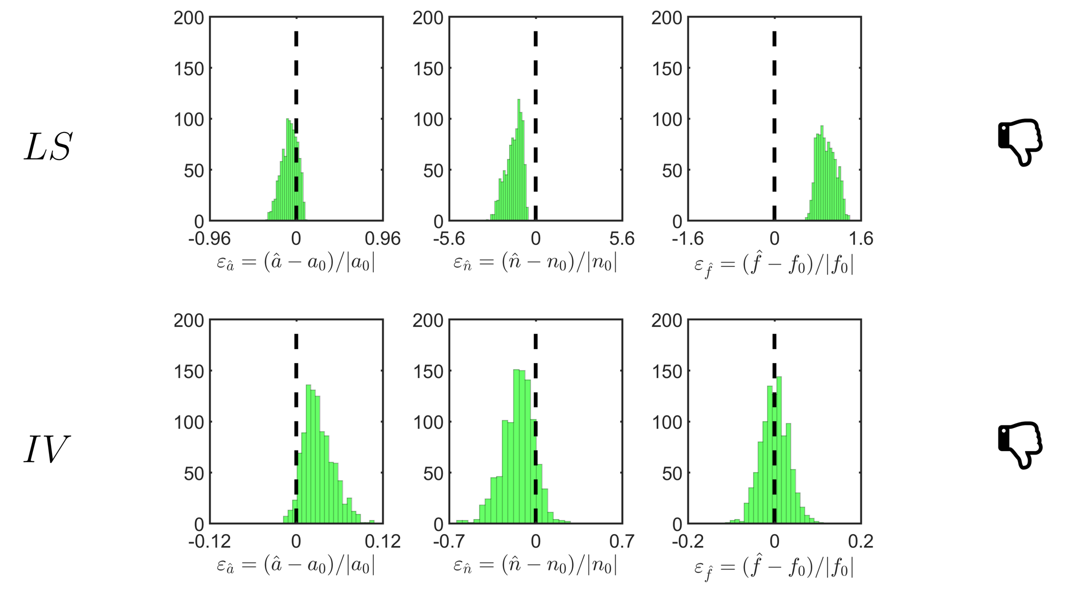
Yaw-rate simulation

$$r(k+1) = a_0 r(k) + n_0 r(k)\big|r(k)\big| + f_0\tau(k) + w(k),$$
$$y(k) = r(k) + e(k),$$

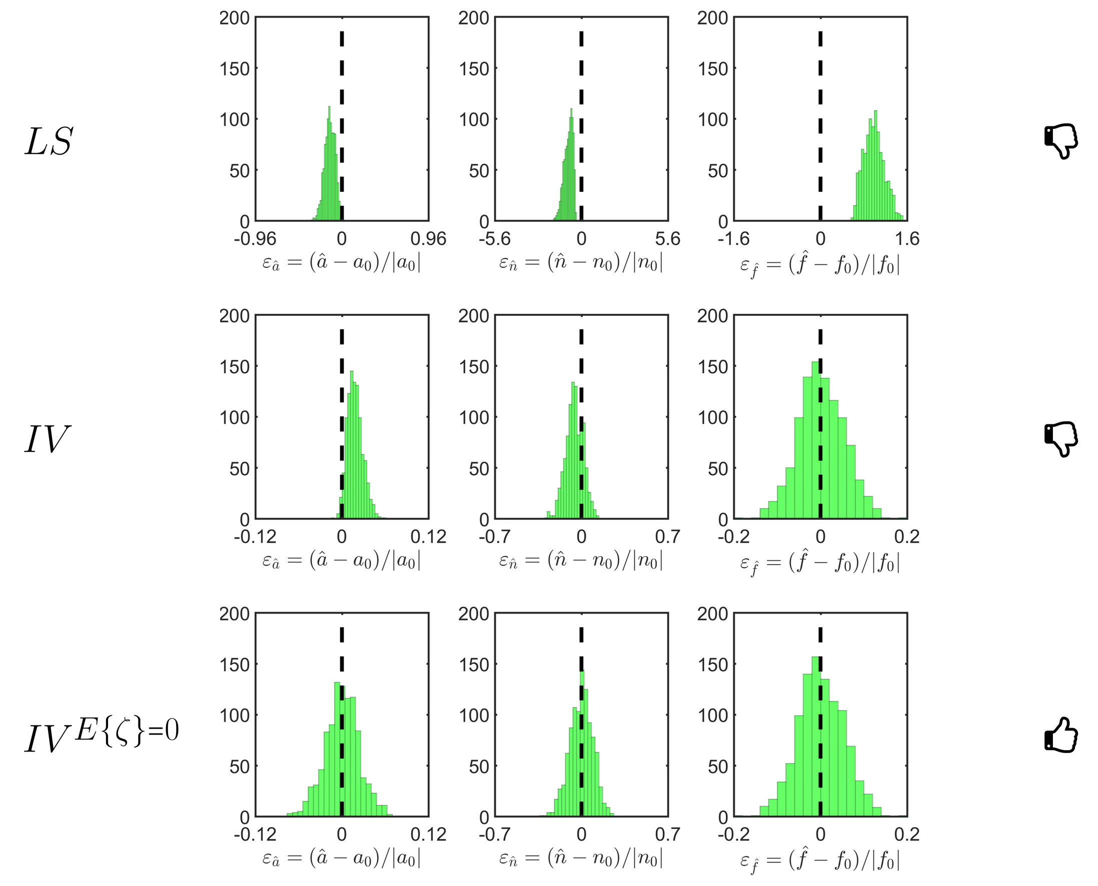
The model is describing the yaw motion of an underwater vehicle (Fossen 2011).



$\hat\theta - \theta_0$ **histograms**
(zero-centered input)



$\hat\theta - \theta_0$ **histograms**
(excitation offset)



Surface-vessel simulation

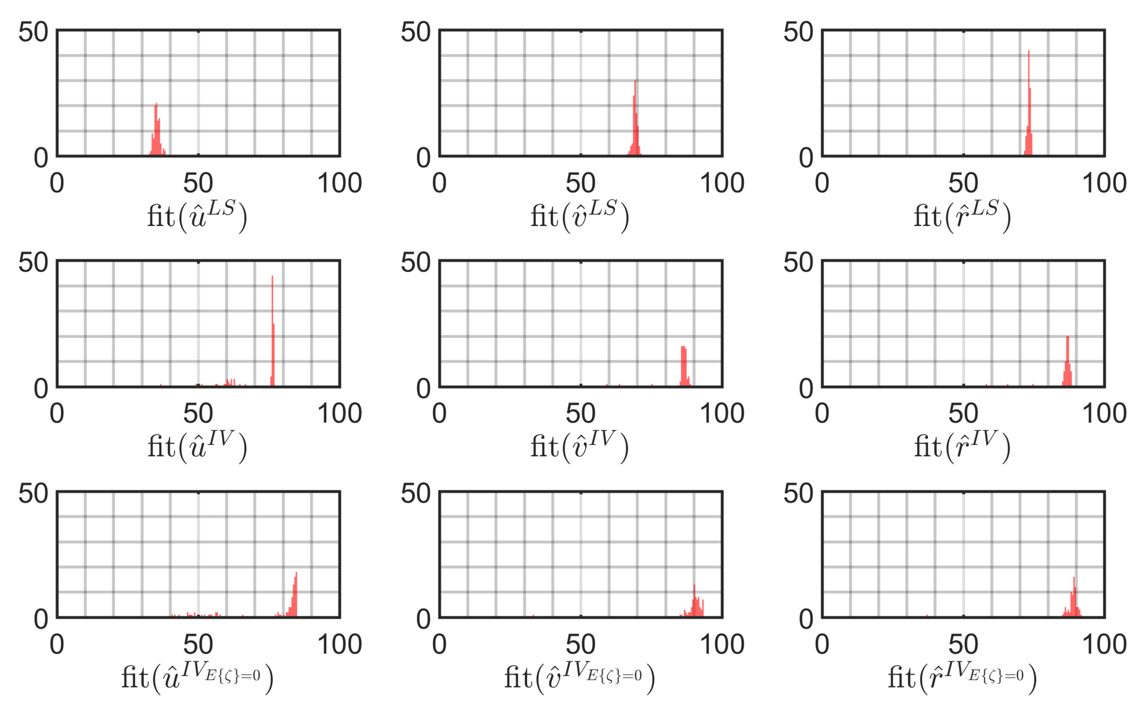
6-DOF system (surge, sway, heave, roll, pitch, yaw):

$$M\dot\nu + C_{RB}\nu + N(\nu)\nu = \tau_{\text{act}} + \tau_{\text{wind}} + \tau_{\text{wave}}$$
$$y = H\nu + e,$$

3-DOF second-order modulus model (surge, sway, yaw):

$$y(k+1) = \Phi^T\Big(\big[y^T(k)\ \tau_{act}^T(k)\big]^T\Big)\theta,\quad \theta \in \mathbb{R}^{17}$$

Fit histograms
(excitation offset)



Possible future work and ideas

- A more thorough investigation of usefulness of the proposed method for limited data sets.
- Consider the case where only parts of the state vector can be measured directly.
- Understand how to deal with cubic terms (Abkowitz' model structure).

LINKÖPING UNIVERSITY

LINKÖPING UNIVERSITY
Division of Automatic Control