Decentralized Data Fusion: Information, Consistency and Bandwidth Aspects

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Background

In decentralized data fusion the *data incest* problem arises, which, if not dealt with correctly, implies that *common information* is used multiple times. In general, the result of reusing information is an estimate for which *consistency* cannot be guaranteed. Unfortunately, keeping track of the common information is generally a cumbersome, or even impossible, task. This suggests fusion rules such as *covariance intersection* (CI) to be used.

Covariance Intersection

The fusion of the estimates $(\hat{\mathbf{x}}_1, \mathbf{P}_1)$ and $(\hat{\mathbf{x}}_2, \mathbf{P}_2)$ is given by

$$\mathbf{P}_f^{-1} = \omega \mathbf{P}_1^{-1} + (1-\omega)\mathbf{P}_2^{-1}$$
$$\mathbf{P}_f^{-1}\hat{\mathbf{x}}_f = \omega \mathbf{P}_1^{-1}\hat{\mathbf{x}}_1 + (1-\omega)\mathbf{P}_2^{-1}\hat{\mathbf{x}}_2$$

where $(\hat{\mathbf{x}}_f, \mathbf{P}_f)$ forms the fused estimate and $\omega \in [0, 1]$.

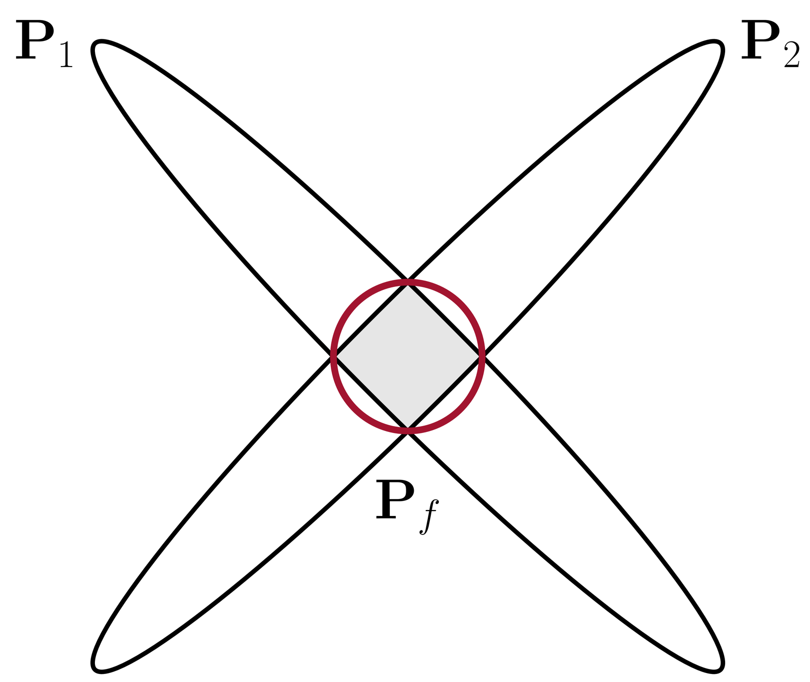

Illustration: CI fusion. Despite what common information is implicitly shared by $\mathbf{P}_1$ and $\mathbf{P}_2$, the optimally and consistently fused covariance matrix will have an ellipse that lie inside the grey shaded area. This area is completely enclosed by the ellipse of the CI fused covariance matrix $\mathbf{P}_f$ (the red ellipse).

Problem Formulation

The problem is to consistently fuse estimates in a decentralized setup, having multiple fusion nodes, with bandwidth limitations and unknown common information. In this context consistency is defined as

$$\mathbf{P} - \mathrm{E}[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^\intercal] \succeq 0$$

where $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ is the true error of the state estimate calculated as the deviation from the true state $\mathbf{x}$ and $\mathbf{P}$ is the corresponding covariance of the estimate.

Diagonal Covariance Approximation

The amount of data exchanged can be reduced by approximating the covariance matrix $\mathbf{P}$ using its diagonal, *i.e.* $\mathbf{D}$. This generally leads to an inconsistent estimate, which can be handled in different ways, as described below. The advantage is that the degrees of freedom of the $n \times n$ covariance matrix is reduced from $n(n+1)/2$ down to n.

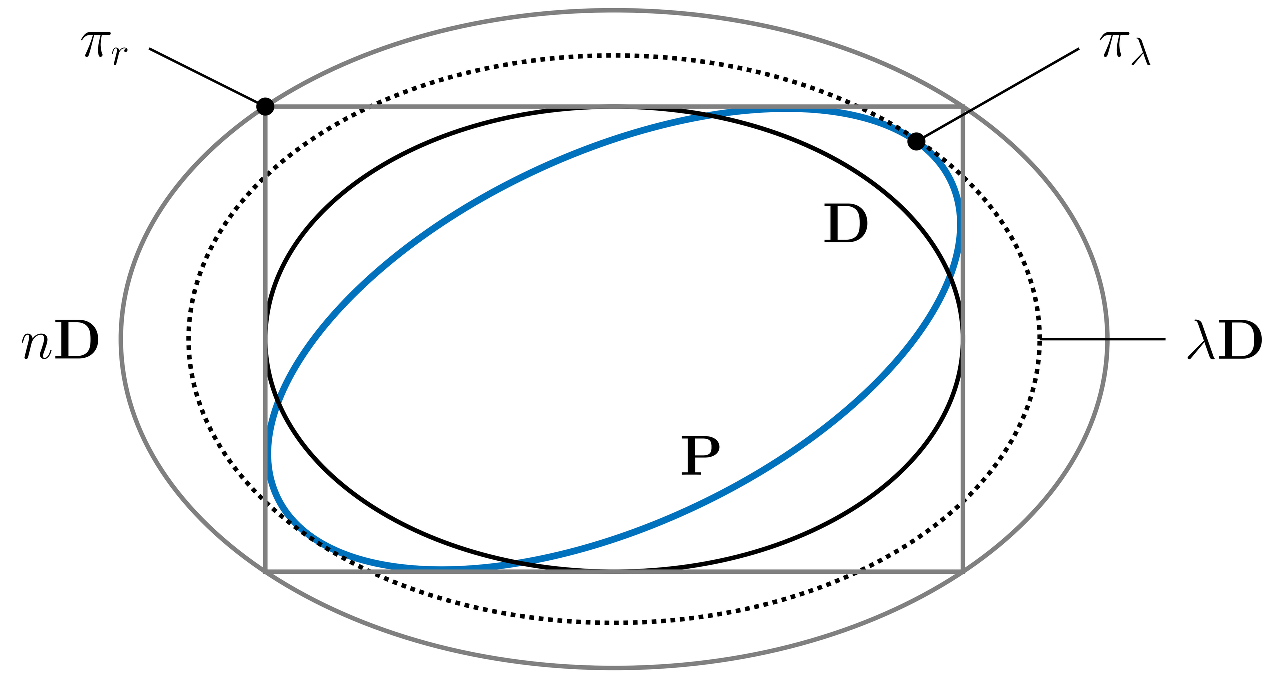

Illustration: Covariance ellipses of $\mathbf{P}$ (the blue ellipse) and $\mathbf{D}$ (the inner black ellipse). The outer ellipses are $\mathbf{D}$ scaled by different factors.

Recovering Consistency

Examples of how consistency can be recovered are given below:

- Scale the matrix $\mathbf{D}$ with the dimensionality n. The geometrical interpretation is that an axis aligned covariance ellipse, which tangents the rectangle that perfectly encloses $\mathbf{D}$, will enclose all potential matrices $\mathbf{P}$ having $\mathbf{D}$ as its diagonal approximation. This is related to finding the point π_r in the figure above.
- Scale the matrix $\mathbf{D}$ with the largest eigenvalue λ of $\mathbf{D}^{-1/2}\mathbf{P}\mathbf{D}^{-1/2}$. This is related to finding the point π_λ in the figure above.
- Let $\mathbf{c} = (c_1, \dots, c_n)$ denote a vector of scaling factors where each element $c_i \geq 1$. Construct the matrices $\mathbf{C} = \mathrm{diag}(\mathbf{c})$ and $\mathbf{D}_c = \mathbf{C}\mathbf{D}$, and solve
$$\underset{\mathbf{c}}{\text{minimize}} \quad f(\mathbf{D}_c(\mathbf{c}))$$
$$\text{subject to } \mathbf{D}_c - \mathbf{P} \succeq 0$$
- If a diagonal matrix $\mathbf{D}_d$ is constructed, where the i:th diagonal entry of $\mathbf{D}_d$ is the absolute sum of row i of $\mathbf{P}$, *i.e.*
$$d_{ii} = \sum_j |p_{ij}|$$
then $\mathbf{D}_d - \mathbf{P}$ will automatically be a symmetric diagonally dominant matrix and as such be positive semi-definite.

Source: Forsling et al. (2019). *Consistent Distributed Track Fusion Under Communication Constraints.* In: Proceedings of the 22nd International Conference on Information Fusion (FUSION).

Information Projections

Questions: How and by which amount can the bandwidth demands be reduced by selection of certain information projections? By how much is the fusion gain then decreased?

Motivating Example

Consider the information matrices $\mathcal{I}_1$ and $\mathcal{I}_2$ defined as

$$\mathcal{I}_1 = \mathbf{P}_1^{-1} = \sum_{i=1}^{2} \alpha_i \mathbf{u}_i \mathbf{u}_i^\intercal \qquad \mathcal{I}_2 = \mathbf{P}_2^{-1} = \sum_{i=1}^{2} \beta_i \mathbf{v}_i \mathbf{v}_i^\intercal$$

where α_i and β_i are eigenvalues, and $\mathbf{u}_i$ and $\mathbf{v}_i$ are the corresponding eigenvectors, for each of the respective information matrices, see figure below. The gain from fusing $\mathcal{I}_1$ and $\mathcal{I}_2$ will not be significantly higher than the gain from fusing the most informative projections $\alpha_1\mathbf{u}_1$ and $\beta_1\mathbf{v}_1$.

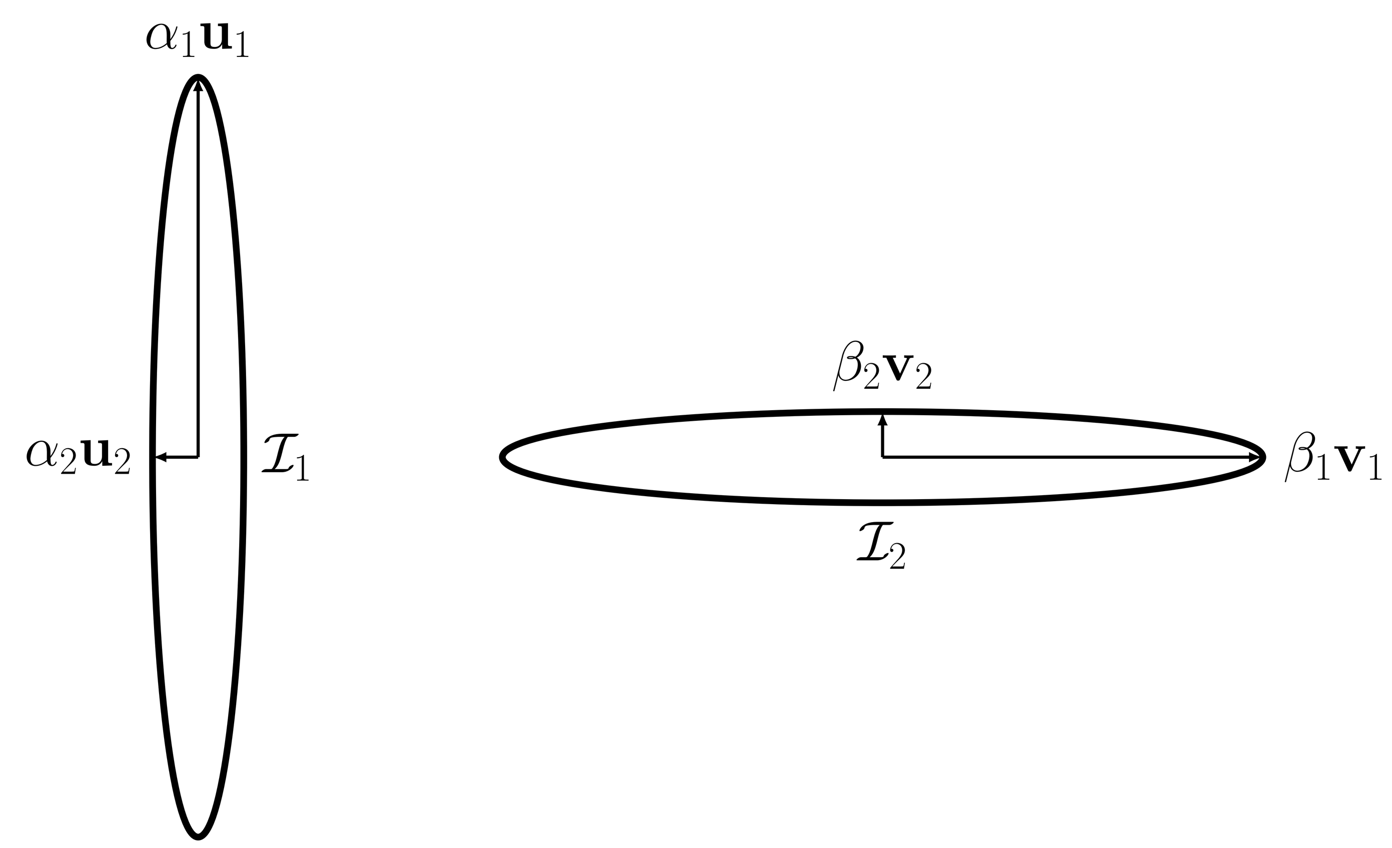

Illustration: The information ellipses of two information matrices decomposed into their eigenvalues and eigenvectors. Each eigenvector can be regarded as a projection of information.

Considered Approach

- Break down information into its **components**.
- **Estimate** which information projections are **most valuable** for the neighbouring nodes.
- Exchange only projections that yield at least a certain amount of **fusion gain** for the neighbouring nodes.

Source: Ongoing work.

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