Particle Filtering Approach for Data Association

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Problem Formulation

Data Association (DA) in this work is defined as

- Assigning a set of measurements of landmarks at time t, $Y_t = \{y_t^i\}_{i=1}^{m_t}$, $y_t^i \in \mathbb{R}^{d_y}$, to the correct landmarks, $L = \{l^j\}_{j=1}^{N_l}$, $l^j \in \mathbb{R}^{d_l}$
- Finding a correspondence variable at time t, $C_t = \{c_t^j\}_{j=1}^{N_l}$, $c_t^j \in \mathbb{N}$, such that $c_t^j = i$ if y_t^i is originating from l^j

This problem can be solved by defining a fitness function, $f(C_t, Y_t, L, \theta_t)$, and finding the best fit as in

$$\widehat{C}_t = \arg\max_{C_t} f(C_t, Y_t, L, \theta_t)$$
$$\textbf{s.t. } C_t \in \mathsf{C}$$

where θ_t are possible extra parameters and C is the constraint set for the cerrespondence.

Data Association Example

5 landmarks, $L = \{l^j\}_{j=1}^5$, 2 measurements, $Y_t = \{y_t^i\}_{i=1}^2$. Landmarks number 3 and 5 are measured by the measurements number 2 and 1, respectively.
The correct correspondence is then $c_t^1 = 0,\ c_t^2 = 0,\ c_t^3 = 2,\ c_t^4 = 0,\ c_t^5 = 1$ or more compactly $C_t = \{0,\ 0,\ 2,\ 0,\ 1\}$.

Particle filter is chosen as an algorithm to solve the DA problem.

Particle Filter Algorithm

Input: Prior $p(x_0)$, Transition distribution $p(x_t|x_{t-1})$, Likelihood $p(Y_t|x_t)$, Proposal distribution $\pi(x_t|x_{0:t-1}, Y_{1:t})$, $Y_{1:T}$
Output: $\{\hat{p}(x_t|Y_{1:t})\}_{t=0}^T$
Initialize:
$x_0^i \sim p(x_0),\ w_0^i = \frac{1}{N}, i = 1:N$
for $t = 1$ **to** T
1. $x_t^i \sim \pi(x_t|x_{0:t-1}^i, Y_{1:t}), i = 1:N$ (Proposal Sampling)
2. $\tilde{w}_t^i = w_{t-1}^i \frac{p(Y_t|x_t^i)p(x_t^i|x_{t-1}^i)}{\pi(x_t^i|x_{0:t-1}^i, Y_{1:t})}$ (Weights Update)
3. $w_t^i = \frac{\tilde{w}_t^i}{\sum_{j=1}^N \tilde{w}_t^j}$ (Weights Normalization)
4. $\hat{p}(x_t|Y_{1:t}) = \sum_{i=1}^N w_t^i \delta(x_t^i - x_t)$ (Posterior Estimate)
5. Draw N particles from $\{x_t^i\}_{i=1}^N$ with the probability proportional to their respective weight and set $w_t^i = \frac{1}{N}$, $i = 1:N$

endfor

Data Association Particle Filter (DAPF)

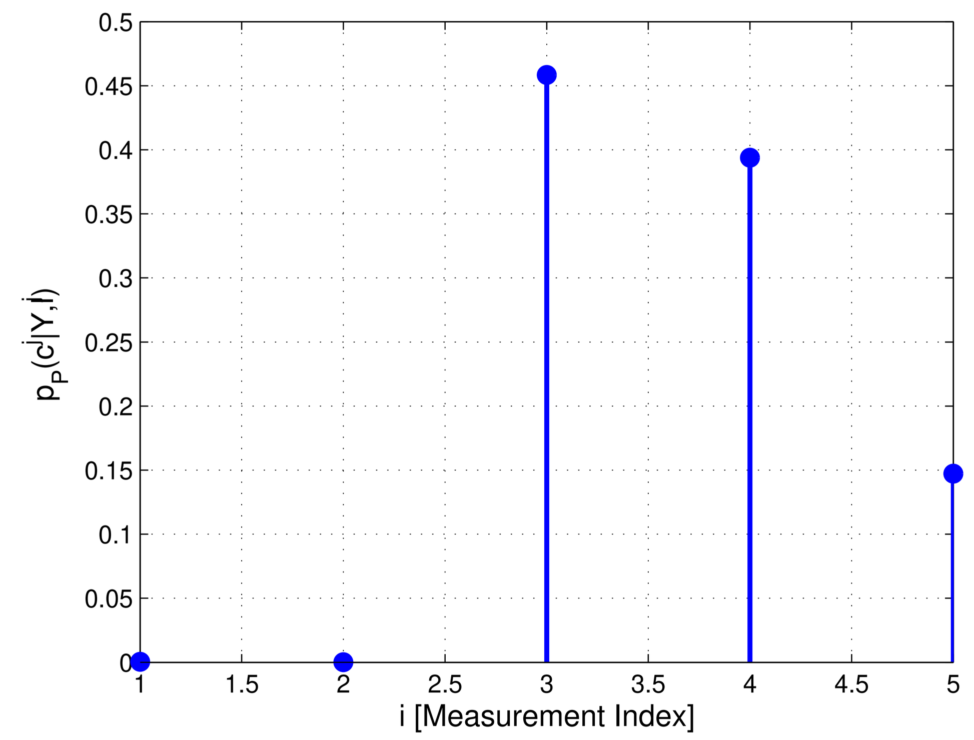
Implemetation boils down to a choice of the three (colored) ditributions.
No exact form exist in the case of data association, but well motivated approximations are used. In this case the transition and proposal densities can be chosen to be the same.

Transition/Proposal Density

Decide which landmarks to use giving the set $\tilde{L}_t \subset \{1, ..., N_l\}$:

$$p_O(\mathbb{I}(c_t^j)|\mathbb{I}(c_{t-1}^j)) = P_O^{\mathbb{I}(c_{t-1}^j)}(1 - P_O)^{1-\mathbb{I}(c_{t-1}^j)}$$
$$p_N(\mathbb{I}(c_t^j)|\mathbb{I}(c_{t-1}^j)) = P_N^{1-\mathbb{I}(c_{t-1}^j)}(1 - P_N)^{\mathbb{I}(c_{t-1}^j)}$$

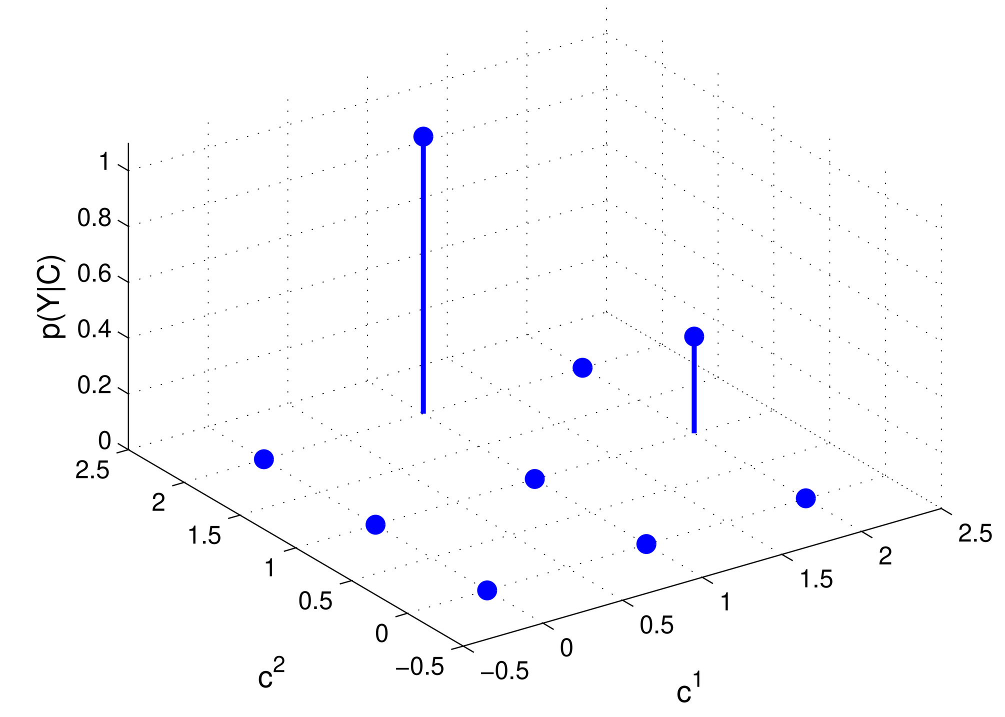
$\mathbb{I}$ is the indicator function
Uniform proposal
$c_t^j \sim \mathsf{U}(1, m_t),\ j \in \tilde{L}_t$
Non-uniform proposal
$c_t^j \sim p_P(c_t^j|Y_t, \tilde{L}_t, \theta_t) = e^{-\mu(y_t^{c_t^j} - h(\theta_t, l^j))^T(y_t^{c_t^j} - h(\theta_t, l^j))}$



Example of non-uniform proposal distribution

Likelihood

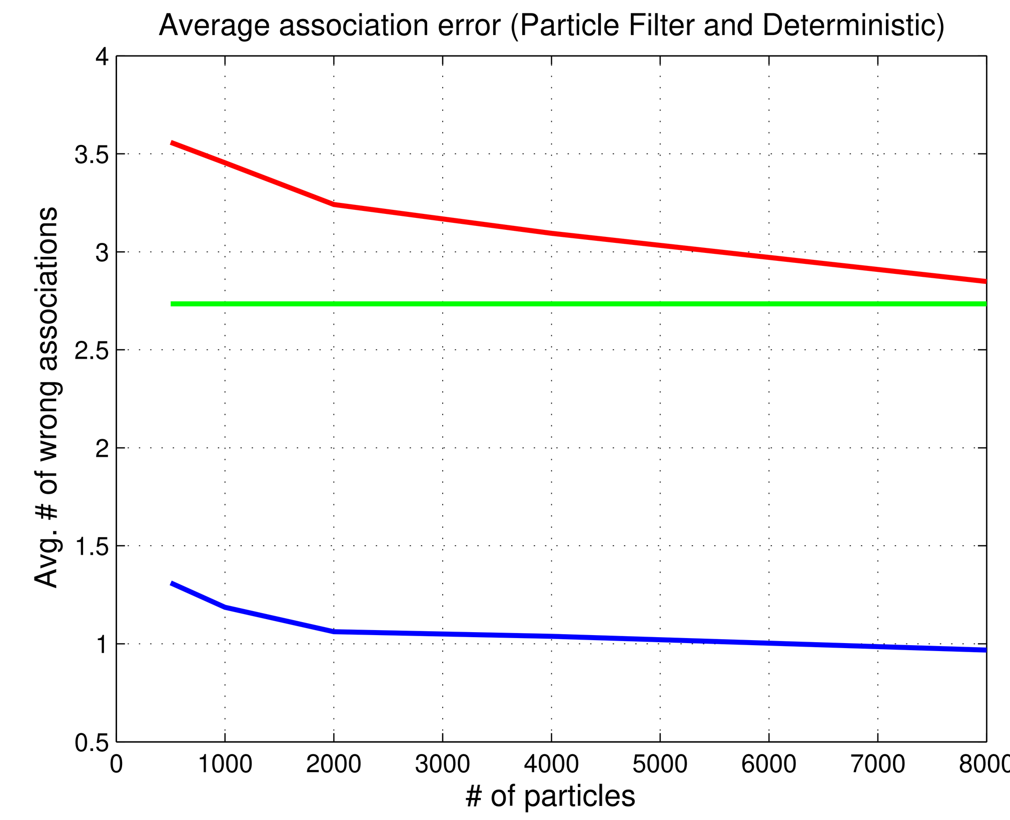
$$p(Y_t|C_t, L, \theta_t) = \prod_{j \in L}\left[e^{-\nu|y_t^{c_t^j} - h(\theta_t, l^j)|\mathbb{I}(c_t^j)} \prod_{k=1}^{m_t}\left(1 - e^{-\nu|y_t^k - h(\theta_t, l^j)|}\right)^{1-\mathbb{I}(c_t^j)}\right]$$



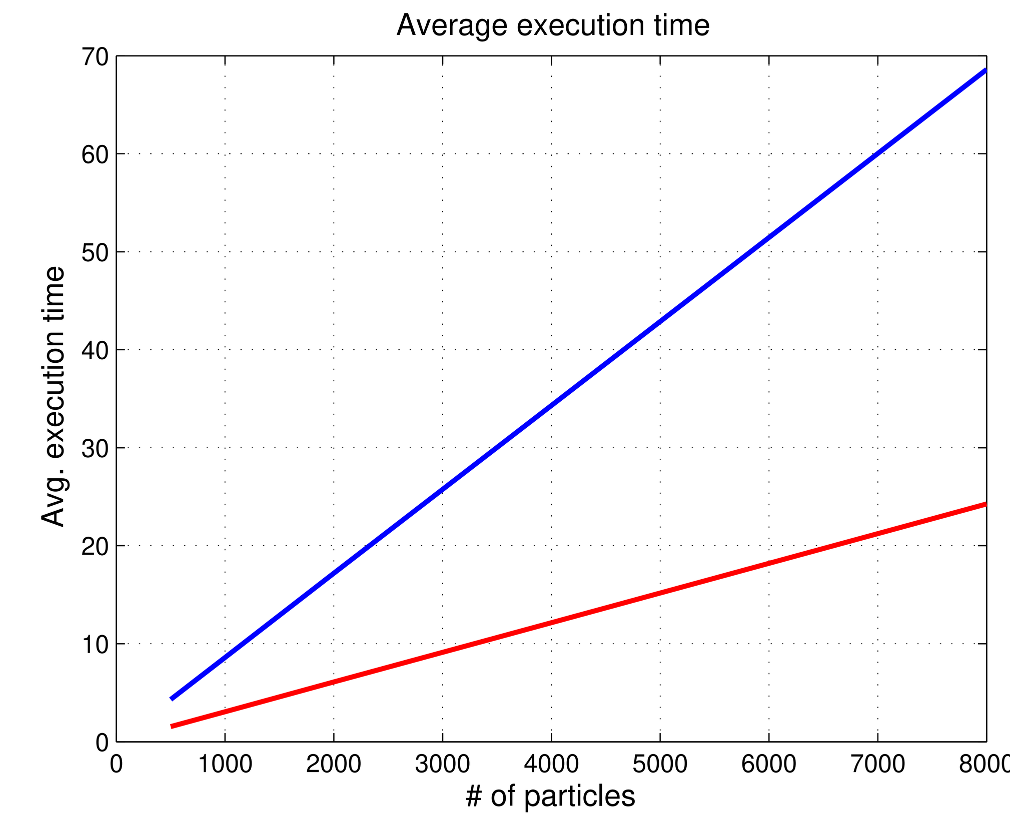
Example of likelihood

Results

The method is evaluated in a 2D simulation environment with 30 landmarks and compared to the Nearest Neighbor. 100 MC simulation are used for each number of particles $N \in \{500,\ 1000,\ 2000,\ 4000,\ 8000\}$ and average association error is evaluated.



Non-uniform proposal **Uniform proposal** **NN**



Conclusions & Future Work

- Performance is evaluated for two different proposal distributions on a small 2D simulation example and compared to NN
- DAPF with non-uniform proposal has the best performance, but with a increased computational cost
- More thorough performance investigation as well as application on real data are the next steps

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