Optimal control of cranes with geometric path constraints

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Crane operations in marine ports

In this work we deal with the problem of loading and unloading a ship as fast as possible, see Fig.1. The very nature of this task is to solve a time-optimal problem subject to geometric constraints.

This is problematic if solved by time discretization as the length over which the discretization is performed is not known. Moreover, since the stack height is a function of the space, these geometric constraints are non-linear and non-convex. Thus, we attempt to alleviate these problems by reformulating the optimal control problem and parameterizing it in spatial coordinates.

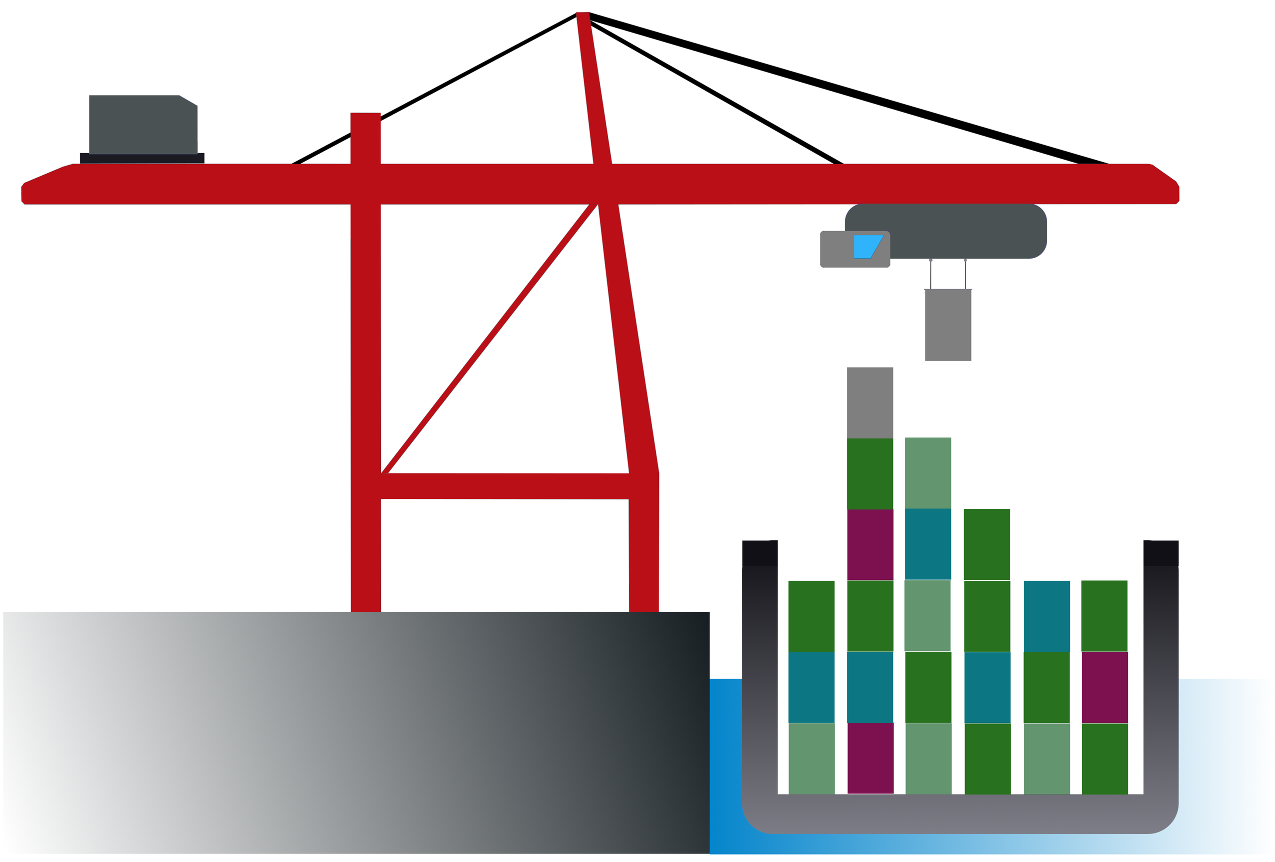

Figure 1: On and off-loading process.

Optimal time with geometric path

Consider the schematic shown in Fig. 2 and the following equations of motion:

$$(m_1+m_2)\ddot{y}(t)+m_2 l(t)(\ddot{\theta}(t)\cos(\theta(t))-\dot{\theta}^2(t)\sin(\theta(t)))+ m_2\ddot{l}(t)\sin(\theta(t))+2m_2\dot{l}(t)\dot{\theta}(t)\cos(\theta(t))=F_t(t)$$

$$m_2\ddot{l}(t)-m_2 l(t)\dot{\theta}^2(t)-m_2 g\cos(\theta(t))+m_2\ddot{y}(t)\sin(\theta(t))=-F_h(t)$$

$$l(t)\ddot{\theta}(t)+2\dot{l}(t)\dot{\theta}(t)+g\sin(\theta(t))+\ddot{y}(t)\cos(\theta(t))=0,$$

We then define the following state variables $x_1 = y$, $x_2=\dot{y}$, $x_3=\theta$, $x_4=\dot{\theta}$, $x_5=l$ and $x_6=\dot{l}$.

The time T it takes to go from initial to final position can be written as

$$T=\int_0^{t_f} dt = \int_0^{x_{2_f}} \frac{dx_1}{x_2},$$

and the state equations become

$$\frac{dt}{dx_1}=\frac{1}{x_2}$$
$$\frac{dx_2}{dx_1}=\frac{f_2}{x_2}$$
$$\frac{dx_3}{dx_1}=\frac{x_4}{x_2}$$
$$\frac{dx_4}{dx_1}=\frac{-2x_6x_4-g\sin(x_3)}{x_2x_5}-\frac{\cos(x_3)}{x_2x_5}f_2$$
$$\frac{dx_5}{dx_1}=\frac{x_6}{x_2}$$
$$\frac{dx_6}{dx_1}=\frac{m_2x_5x_4^2+m_2g\cos(x_3)-F_h}{x_2m_2}-\frac{\sin(x_3)}{x_2}f_2$$

where

$$f_2=\frac{2m_2\cos(x_3)x_6x_4+m_2g\cos(x_3)\sin(x_3)-m_2\sin(x_3)x_5x_4^2}{m_1} - \frac{m_2g\sin(x_3)\cos(x_3)}{m_1} + \frac{\sin(x_3)F_h+m_2x_5x_4^2\sin(x_3)-2m_2x_6x_4\cos(x_3)+F_t}{m_1}.$$

Now, the state vector is $x(x_1)=[t,x_2,x_3,x_4,x_5,x_6]^T$ and the time-optimal problem becomes

$$\begin{aligned}
\text{minimize } & T=\int_0^{x_{1_f}}\frac{1}{x_2(x_1)}dx_1\\
\text{subject to } & \dot{x}(x_1)=f(x_1,x(x_1),u(x_1))\\
& t(0)=0\\
& x_i(0)=0,\ x_i(x_{i_f})=x_{i_f},\ i=2,\dots,n\\
& t(x_1)\geq 0\\
& x_2(x_1)\geq 0\\
& \theta_{min}(x_1)\leq x_3(x_1)\leq\theta_{max}(x_1)\\
& l_{min}(x_1)\leq x_5(x_1)\leq l_{max}(x_1)\\
& u_{min}(x_1)\leq u(x_1)\leq u_{max}(x_1),
\end{aligned}$$

where $u(x_1)=[F_t,F_h]^T$ and n is the system dimension.

The most immediate outcome of this reformulation is that the positions, velocities and accelerations are now related to one another through the parametrization of the path. Additionally, the objective function is now convex. On the hand, $x_2(x_1)=0$ at $x_1(0)$ and $x_1(x_{1_f})$ leads to a convergence issue. To circumvent this, we obtain t analytically after performing linearizations in the system equations and use this solution to compute the time it takes to move along the first and last intervals.

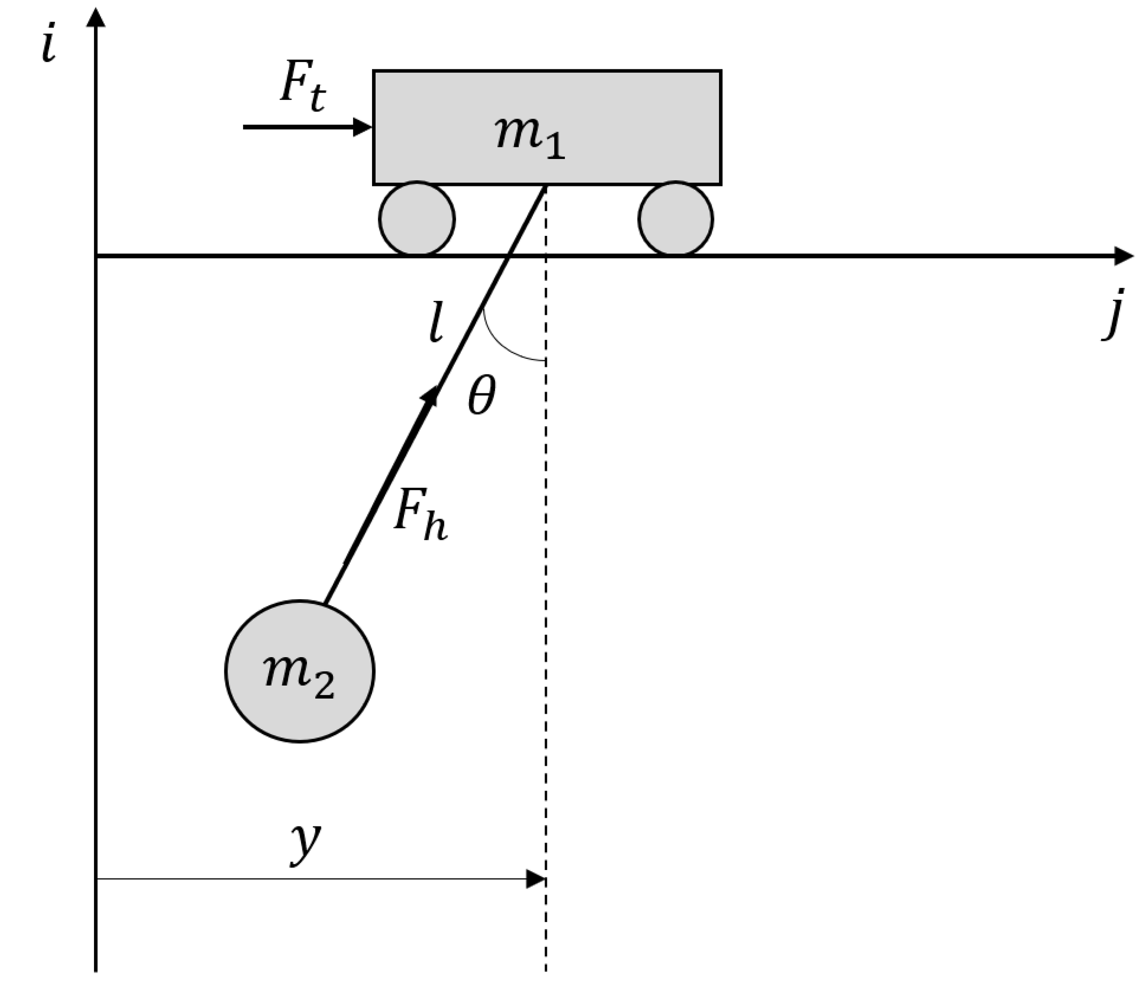

Figure 2: Two-dimensional schematic representing the trolley and payload motions.

Future work

The next steps in this research are:

- Evaluate the model linearizations.
- Implement the controller and perform experiments in a crane simulator.
- Implement this strategy in an actual crane.

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