# Estimation of Nonlinear Greybox Models for Marine Applications Presentation of licentiate thesis at LINK-SIC workshop

Fredrik Ljungberg

Division of Automatic Control Department of Electrical Engineering Linköping University



# Research motivation

# Why do we need marine models?

- Facilitation of development.
- Achieving satisfactory model-based control.

#### Main modelling challenges:

- Nonlinear dynamic forces and moments.
- Environmental disturbances, like wind, waves and ocean currents (often non-additive).





### Problem description

**Definition.** A second-order modulus function is a function,  $f : \mathbb{R}^{n+p} \to \mathbb{R}^m$  that can be written as

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \Phi^T(\boldsymbol{x})\boldsymbol{\theta},$$

where each element of the  $p \times m$  matrix  $\Phi(\mathbf{x})$  is on one of the forms  $x_i, |x_i|, x_i x_j, x_i |x_j|$  for  $i, j \leq n$  or zero and  $\theta \in \mathbb{R}^p$  is a vector of coefficients.

#### Main objective

Obtaining <u>consistent estimators</u> of  $\theta$  for the class of models that can be expressed as <u>second-order modulus functions</u>, which are robust to:

- Measurement uncertainty.
- Non-additive environmental disturbances.



# Contributions

- 1. The suggestion of an experiment design where the input signal has a static offset of sufficient amplitude and the instruments in an IV method are forced to have zero mean.
- 2. A method to estimate the first-order moments of system disturbances alongside the model parameters.
- 3. Experimental work.



#### F. Ljungberg, M. Enqvist.

Obtaining Consistent Parameter Estimators for Second-Order Modulus Models. IEEE Control Systems Letters, 3(4):781-786, 10 2019.

#### F. Ljungberg, M. Enqvist.

Consistent Parameter Estimators for Second-order Modulus Systems with Non-additive Disturbances.

In Proceedings of the 21st IFAC World Congress, Berlin, Germany, 2020 (to appear).



# Marine modelling: Undisturbed equations of motion

$$\dot{\eta} = J(\eta)\nu,$$
  
$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau_{\rm act}.$$

#### 6-DOF models:

(surge, sway, heave, roll, pitch, yaw)

$$\eta = \begin{bmatrix} x_n & y_n & z_n & \phi & \theta & \psi \end{bmatrix}^T$$
$$\nu = \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T$$
$$\tau_{\text{act}} = \begin{bmatrix} F_x & F_y & F_z & M_x & M_y & M_z \end{bmatrix}^T$$



Maneuvering models: (surge, sway, yaw)

$$\eta = \begin{bmatrix} x_n & y_n & \psi \end{bmatrix}^T$$
$$\nu = \begin{bmatrix} u & v & r \end{bmatrix}^T$$
$$\tau_{\text{act}} = \begin{bmatrix} F_x & F_y & M_z \end{bmatrix}^T$$



# Marine modelling: Environmental disturbances Disturbed equations of motion:

 $\dot{\eta} = J(\eta)\nu,$  $M_{RB}\dot{\nu} + M_A\dot{\nu}_r + C_{RB}(\nu)\nu + C_A(\nu_r)\nu_r + D(\nu_r)\nu_r + F(\nu_q)\nu_q = \tau_{\text{act}}.$ 

•  $\nu_r = \nu - \nu_c$ , where  $\nu_c$  is the velocity of an ocean current.

• 
$$\nu_q = \nu - \nu_w$$
, where  $\nu_w$  is the wind velocity.  
Disturbances:

$$\nu_c = J^{-1}(\eta)\nu_{c,n},$$
 $\nu_w = J^{-1}(\eta)\nu_{w,n}.$ 

Note: Disturbance effects depend on the ship's attitude.

More about this on my poster!

# Eliminating disturbances: Problem description Challenge: Estimate $\theta$ when

System:  $\begin{cases} \mathbf{x}(k+1) = \Phi^T \left( \begin{bmatrix} \mathbf{x}^T(k) + \mathbf{v}^T(k) & \mathbf{u}^T(k) \end{bmatrix}^T \right) \theta_0 + \mathbf{w}(k), \\ \mathbf{y}(k) = \mathbf{x}(k) + \mathbf{e}(k), \end{cases}$ Model:  $\hat{\mathbf{y}}(k) = \Phi^T \left( \begin{bmatrix} \mathbf{y}^T(k-1) & \mathbf{u}^T(k-1) \end{bmatrix}^T \right) \theta.$ 

#### Main assumptions:

- $f = \Phi^T(.)\theta$  is a second-order modulus function.
- $\mathbf{v}(k)$ ,  $\mathbf{w}(k)$  and  $\mathbf{e}(k)$  are disturbance signals.
- $\mathbf{v}(k)$  is independent of  $\mathbf{x}(l)$  for  $k \ge l$  (non-physical for ships).



# Eliminating disturbances: Solutions

#### Main result

**Challenge:** Developing estimators of  $\theta$  which are consistent despite measurement uncertainty and process disturbances.

**Solution 1.** Experiment design with excitation offset and zero-mean instruments.

Solution 2. Utilizing disturbance measurements  $(\mathbf{y}_2(k) = \mathbf{v}(k))$ .



Requires: Solution 1 (by Lemma 4.1).

Requires: Solution 1 and 2 (by Lemma 4.2).



# Estimating disturbances: Problem description

#### Second-order modulus system in surge:

$$\begin{split} u(k+1) &= u(k) + \mathcal{X}_{u}u_{r}(k) + \mathcal{X}_{|u|u} |u_{r}(k)| u_{r}(k) + \mathcal{W}_{|u|u} |u_{q}(k)| u_{q}(k) + \mathcal{X}_{\mu}\tilde{\tau}_{x}(k), \\ y_{u}(k) &= u(k) + e_{u}(k), \\ y_{\psi}(k) &= \psi(k) + e_{\psi}(k). \end{split}$$
  
Here  $u_{r}(k) &= u(k) - u_{c}(k)$  and  $u_{q}(k) = u(k) - u_{w}(k).$ 

Sought parameters:

$$\theta = \begin{bmatrix} 1 + \mathcal{X}_u & \mathcal{X}_{|u|u} + \mathcal{W}_{|u|u} & \mathcal{X}_{\mu} \end{bmatrix}^T.$$

**Disturbances:** 

$$u_c(k) = \cos(\psi(k)) \nu_{c,NS}(k) + \sin(\psi(k)) \nu_{c,EW}(k),$$
  
$$u_w(k) = \cos(\psi(k)) \nu_{w,NS}(k) + \sin(\psi(k)) \nu_{w,EW}(k).$$



# Estimating Disturbances: Solutions

#### Main result

**Challenge:** Developing estimators of  $\theta$  which are consistent despite non-additive environmental disturbances in the forms of ocean currents and wind. **Solution 1.** Augment the predictor with heading-angle dependent regressors. **Solution 2.** Utilize disturbance (wind) measurements.



Vessel/wind speed

Requires: Solution 1.

Requires: Solution 1 and 2.



# Experimental study: Experiment description

### The studied ship

- Size: Roughly 30 meters long
- Actuation: 2 azimuth thrusters (along centerline)
  - Sensing: GNSS receiver (with two antennas)
    - Propeller-based anemometer on weather vane

### The collected data

- Day 1: Light breeze ( $\approx 3 \text{ m/s}$ )
  - Data used for validation
- Day 2: Fresh breeze ( $\approx 10 \text{ m/s}$ )
  - Data used for estimation
- Additional: 6 shorter experiments per day (6 10 min. each)
  - No ocean currents
  - No excitation offset







# Experimental study: Results using sway data

Estimators:

- Regular LS  $(\hat{\theta}_N^{LS_1})$
- Regular IV  $(\hat{\theta}_N^{IV_1})$
- IV with heading-angle dependent regressors  $(\hat{\theta}_N^{IV_2})$
- IV utilizing wind measurements  $(\hat{\theta}_N^{IV_3})$

Average fit for each estimator:

Estimator	Fit - Sway
$\hat{\theta}_N^{LS_1}$	$50.3056 \pm 4.4771$
$\hat{\theta}_N^{IV_1}$	$63.3711 \pm 8.3876$
$\hat{\theta}_N^{IV_2}$	$71.9800 \pm 2.8652$
$\hat{\theta}_N^{IV_3}$	$70.3522 \pm 3.4760$



# Conclusion and future work

#### Main result

- A framework for estimation of second-order modulus models has been suggested.
- The methods have been analyzed and show promising results in simulations.
- Some ideas have also been tested on real data.

#### Possible future work and ideas

- More focused study on experiment design.
- Connection to disturbance observers.
- Compare with blackbox approach.



# Conclusion and future work

#### Main result

- A framework for estimation of second-order modulus models has been suggested.
- The methods have been analyzed and show promising results in simulations.
- Some ideas have also been tested on real data.

#### Possible future work and ideas

- More focused study on experiment design.
- Connection to disturbance observers.
- Compare with blackbox approach.

#### Acknowledgments

- ABB for collaboration.
- LINK-SIC for funding.





www.liu.se

