Policy Gradient

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What is Policy Gradient?

The most ambitious way of solving an RL problem

- Policy: The agent's decision
- ~~Value function: how good the agent does in a state~~
- ~~Model: The agent's interpretation of the environment~~

Agent's goal: To learn the policy by directly optimizing the total reward.

$$J = \mathbf{E}_{\tau \sim \pi_\theta}[R(T)]$$

How?

Optimization by perturbation:

- Consider a stochastic parametric policy with parameter θ
- Observe the total reward as a result of perturbation
- Optimize the parameters of the policy by finding $\nabla_\theta J$

A simple math rule based on $\nabla_p \log p = \frac{1}{p}$

$$\nabla_\theta \log p = \nabla_p \log p \nabla_\theta p = \frac{1}{p} \nabla_\theta p.$$

$$\boxed{\nabla_\theta p = p \nabla_\theta \log p} \tag{1}$$

We will have a closer look at the following components in J

$$J = \mathbf{E}_{\tau \sim \pi_\theta}[R(T)]$$

- π_θ: The parametric pdf of the policy
- τ: A sampled trajectory and the expectation is defined over the probability of the trajectory
- $R(T)$: The total reward

Why to consider a stochastic policy instead of a deterministic one?

- To enable learning by deviating from the deterministic policy
- If a deterministic policy is considered, the agents remains in a local optimum forever.

How to define the pdf?

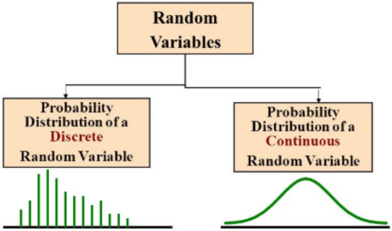
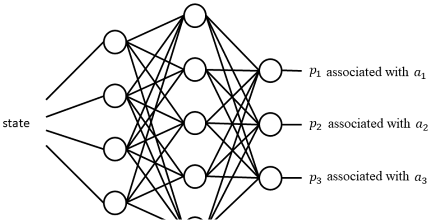


Photo Credit: @ https://towardsdatascience.com/probability-distributions-discrete-and-continuous-7a94ede66dc0

Generates the pdf $\pi_\theta = network(s)$




```
network = keras.Sequential([
        keras.layers.Dense(30, input_dim=n_s, activation='relu'),
        keras.layers.Dense(30, activation='relu'),
        keras.layers.Dense(n_a, activation='softmax')])
```

Continuous action space

Select a Gaussian distribution as the pdf π_θ and generate the mean

$$\pi_\theta = \frac{1}{\sqrt{(2\pi\sigma^2)^{n_a}}} \exp[-\frac{1}{2\sigma^2}(a - \mu_\theta(s))^T(a - \mu_\theta(s))]$$

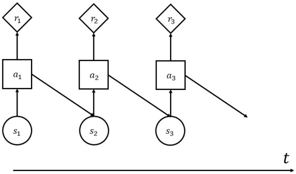
For example, for a linear policy

$$\mu_\theta(s) = \theta s$$

Discrete:
Parameterize the pdf

Continuous:
Parameterize the mean of a Gaussian pdf

Sampling a trajectory



Select $a_t \sim \pi_\theta,\ t = 1, \ldots, T$ and step the environment

$$\tau = (s_1,\ a_1,\ ,r_1,\ s_2,\ a_2,\ r_2 \ldots, s_{T+1})$$

The probability of the trajectory

$$P(\tau|\theta) = \prod_{t=1}^{T} p(s_{t+1}|s_t, a_t)p(a_t|\theta).$$

- $p(s_{t+1}|s_t, a_t)$: the model of the environment
- $p(a_t|\theta)$: The pdf π_θ evaluated at a_t.
  - Discrete action space: $\pi_\theta = network(s)$. So, $p(a_t|\theta)$ is obtained by indexing into the output vector network(state).
  - Continuous action space:

$$p(a_t|\theta) = \frac{1}{\sqrt{(2\pi\sigma^2)^{n_a}}} \exp[-\frac{1}{2\sigma^2}(a_t - \mu_\theta(s_t))^T(a_t - \mu_\theta(s_t))]$$

Love math? Dive in!

$$
\begin{aligned}
\nabla_\theta J &= \nabla_\theta \mathbf{E}\,[R(T)] \\
&= \int_\tau \nabla_\theta P(\tau|\theta) R(T) \\
&= \int_\tau P(\tau|\theta) \nabla_\theta \log P(\tau|\theta) R(T), \quad \text{using log-derivative trick} \\
&= \mathbf{E}_{\tau\sim\pi_\theta}[\nabla_\theta \log P(\tau|\theta) R(T)] \\
&= \mathbf{E}_{\tau\sim\pi_\theta}[(\nabla_\theta \textstyle\sum_{t=1}^{T} \log \underbrace{p(s_{t+1}|s_t,a_t)}_{Dynamics} + \nabla_\theta \textstyle\sum_{t=1}^{T} \log p(a_t|\theta)) R(T)] \\
&= \mathbf{E}_{\tau\sim\pi_\theta}[\nabla_\theta \textstyle\sum_{t=1}^{T} \log p(a_t|\theta) R(T)]
\end{aligned} \tag{2}
$$

But how to compute?

Use pre-built cost functions for the classification task in ML.

Policy Gradient:

$$J = \sum_{t=1}^{T} R(T) 1 \log p(a_t|\theta)$$

- Number of actions n_a
- Trajectory length T
- Weight $R(T)$
- State s_t
- Target label for state s_t and action a
- $p(a_t|\theta)$: probability of a_t by the network at state s_t

Classification:

$$J_{wcec} = -\frac{1}{M} \sum_{m=1}^{M} \sum_{c=1}^{C} w_c y_m^c \log h_\theta(x_m, c)$$

- Number of classes C
- Data length M
- Weight w_c
- Image x_m
- Target label for x_m for class c; y_m^c
- $h_\theta(x_m, c)$: The probability of x_m belongs to class c by the network

Summary of optimizing the parameters in the discrete action space

PG is similar to the classification task!

- The network should produce probability
- The cost to be optimized is a weighted cross entropy cost
- The weights are $R(T)$

We can compute $\mathbf{E}_{\tau\sim\pi_\theta}[\nabla_\theta \sum_{t=1}^{T} \log p(a_t|\theta) R(T)]$ easily!

$$\nabla_\theta J = \frac{1}{\sigma^2|\mathcal{D}|} \sum_{\tau\in\mathcal{D}} \sum_{t=1}^{T} (a_t - \mu_\theta(s_t)) \frac{d\mu_\theta(s_t)}{d\theta}^{\dagger} R(T).$$

If we consider a linear policy $\mu_\theta(s_t) = \theta\, s_t$

$$\nabla_\theta J = \frac{1}{\sigma^2|\mathcal{D}|} \sum_{\tau\in\mathcal{D}} \sum_{t=1}^{T} (a_t - \theta\, s_t) s_t^{\dagger} R(T).$$

Discrete:
Assign a cross entropy cost function and let the ML library optimize the parameter!

Continuous:
Use

$$\nabla_\theta J = \frac{1}{\sigma^2 |\mathcal{D}|} \sum_{\tau \in \mathcal{D}} \sum_{t=1}^{T} (a_t - \theta\, s_t) s_t^\dagger R(T)$$

and optimize θ with a gradient algorithm, e.g.

$$\theta = \theta + \alpha \nabla_\theta J.$$

Putting all together

We build/consider a parametric pdf $\pi_\theta(s)$. Then, we iterate:

1. Collect data
   - Observe s and sample $a \sim \pi_\theta(s)$.
   - Apply a and observe r.
   - Add s, a, r to the history.
2. Update the parameter θ
   - We calculate the total reward.
   - We optimize the policy by a gradient algorithm.

PG: The most ambitious way of solving an RL problem

- Directly optimizes the reward for MDP
- No model, no bellman equation
- Random search is a special case
- Can be extremely good or bad
- Take a look at implementation on my github

Crash_course_on_RL/pg_notebook.ipynb

Email your questions to

farnaz.adib.yaghmaie@liu.se