Policy Gradient on Linear Quadratic Problem

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- **Dynamics:**

$$s_{t+1} = As_t + Bu_t + w_t$$

- **State and action:**

$$s_t \in \mathbb{R}^n,$$
$$u_t \in \mathbb{R}^m$$

- **Cost function ($\equiv$ negative of reward):**

$$c_t = s_t^\dagger Q s_t + u_t^\dagger R u_t, \quad Q \geq 0, R > 0$$

Solvability Criterion: Minimize the average cost ($\equiv$ maximize the average reward)

$$\lambda = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} c_t .$$

We consider a linear policy, so the mean of the pdf is selected as

$$\mu_\theta(s) = \theta\, s \tag{1}$$

and the pdf is given by

$$\pi_\theta(s) = \frac{1}{\sqrt{(2\pi\sigma^2)^{n_a}}} \exp[-\frac{1}{2\sigma^2}(a - \theta s)^\dagger (a - \theta s)]$$

1. Collect data
   - Observe s and sample $a \sim \pi_\theta(s)$

```
a = theta s + sigma * np.random.randn(n_a)
```

   - Apply a and observe r.
   - Add s, a, r to the history.
2. Update the parameter θ
   - We calculate the reward and standardize it.
   - We calculate the gradient using

$$\nabla_\theta J = \frac{1}{\sigma^2 |\mathcal{D}|} \sum_{\tau \in \mathcal{D}} \sum_{t=1}^{T} (a_t - \theta\, s_t) s_t^\dagger R(T). \tag{2}$$

   - We optimize the policy by a gradient algorithm (e.g. an ADAM optimizer)

Try the following:

- Run

  Crash_course_on_RL/pg_on_lq_notebook.ipynb

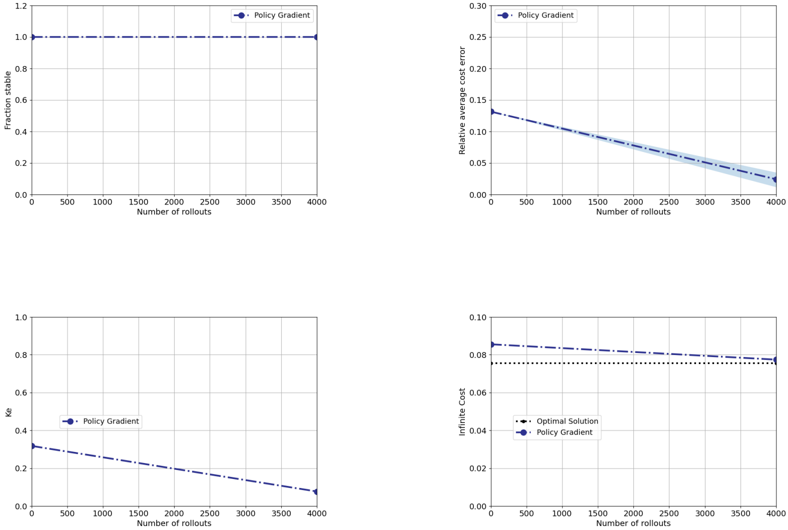
  and verify the median of the error in estimating the optimal gain is $\sim 0.08\%$.

- Set

  'explore_mag=0.000001' in 'Mypgrl.pg_linpolicy'

  and verify that the agent cannot learn the optimal gain by using a deterministic policy in PG.

- Make sure you understand the code!


Policy Gradient
Fraction stable
Number of rollouts
Relative average cost error
Ke
Infinite Cost
Optimal Solution

Email your questions to

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