Time-optimal control of cranes subject to container height constraints

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Summary

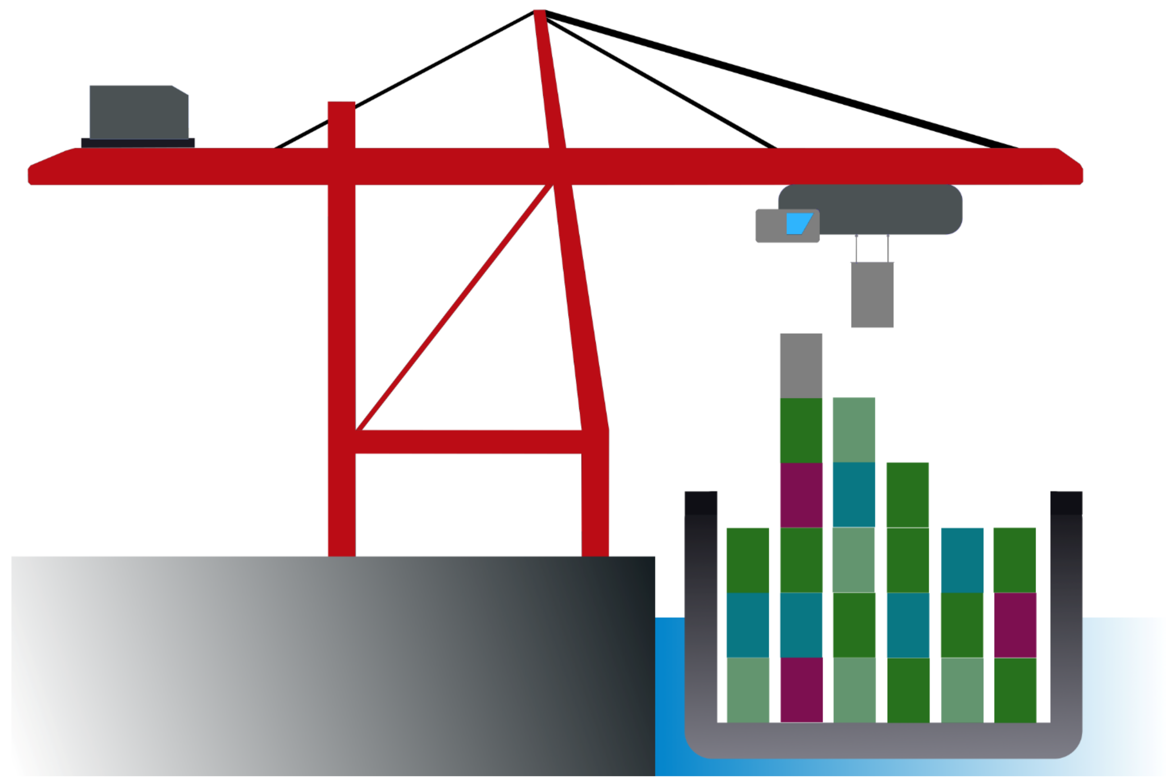
Objective: Move the payload into and onto a container ship in minimum time.

Challenge: Avoid colision with constainer stacks.

Trick: Variable change in an optimal control problem.

Outcome: Non-convex container avoidance constraints become linear bound constraints.

Benefit: No functional representation of the container stack heights is required.



Original problem formulation

The nonlinear state-space representation in the original form is

$$\dot{x}(t) = f(t, x(t), u(t)), \tag{1}$$

where the state variables are

$$\begin{aligned} x_1 &= x_p, \quad x_3 = y_p \quad x_5 = l, \quad x_7 = \theta, \\ x_2 &= \dot{x}_p, \quad x_4 = \dot{y}_p, \quad x_6 = \dot{l}, \quad x_8 = \dot{\theta}, \end{aligned} \tag{2}$$

and the original time-optimal control formulation is written as

$$\begin{aligned} \text{minimize } & T = \int_0^{t_f} 1\, dt \\ \text{subject to } & \dot{x}(t) = f(t, x(t), u(t)) \\ & 0 \le y_p(t) \le h - s(x_p(t)) \leftarrow \text{avoidance constraints} \\ & \vdots \\ & \text{other constraints} \end{aligned} \tag{3}$$

Problem reformulation

1. Use spatial derivatives instead of temporal derivatives

$$\dot{x}_1 = \frac{dx_1}{dt} = x_2 \implies \frac{dt}{dx_1} = \frac{1}{x_2}, \frac{dx_2}{dx_1} = \ldots \tag{4}$$

2. New state vector is $x = [t, \dot{x}_p, y_p, \dot{y}_p, l, \dot{l}, \theta, \dot{\theta}]^T$.
3. With $x_1(x_p) = t(x_p)$ as the cost function

$$\begin{aligned} \text{minimize } & J = t(x_{p_f}) \\ \text{subject to } & x_2 \dot{x}(x_p) = f(x_p, x(x_p), u(x_p)) \\ & 0 \le y_p(x_p) \le h - s(x_p) \leftarrow \text{container constraints} \\ & \vdots \\ & \text{other constraints} \end{aligned} \tag{5}$$

Geometric constraints

1. Time discretization of the container avoidance constraint in (3) leads to

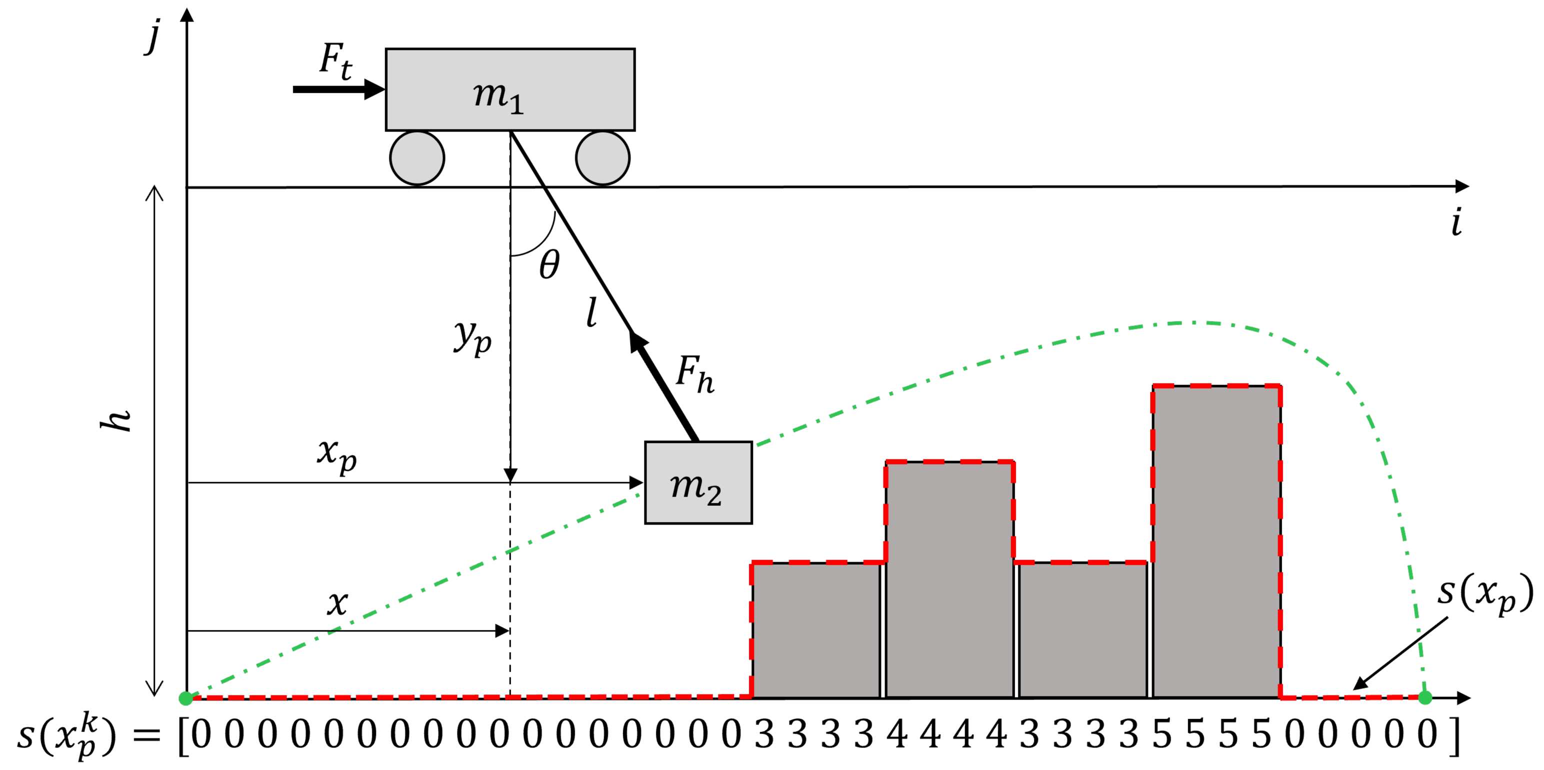
$$0 \le y_p(t^k) \le h - s(x_p(t^k)), \quad ✗$$

where the container profile $s(x_p)$ is generally discontinuous, nonlinear and non-convex.

2. Spatial discretization of the container avoidance constraint in (5) leads to upper bound constraints for $y_p(x_p)$.

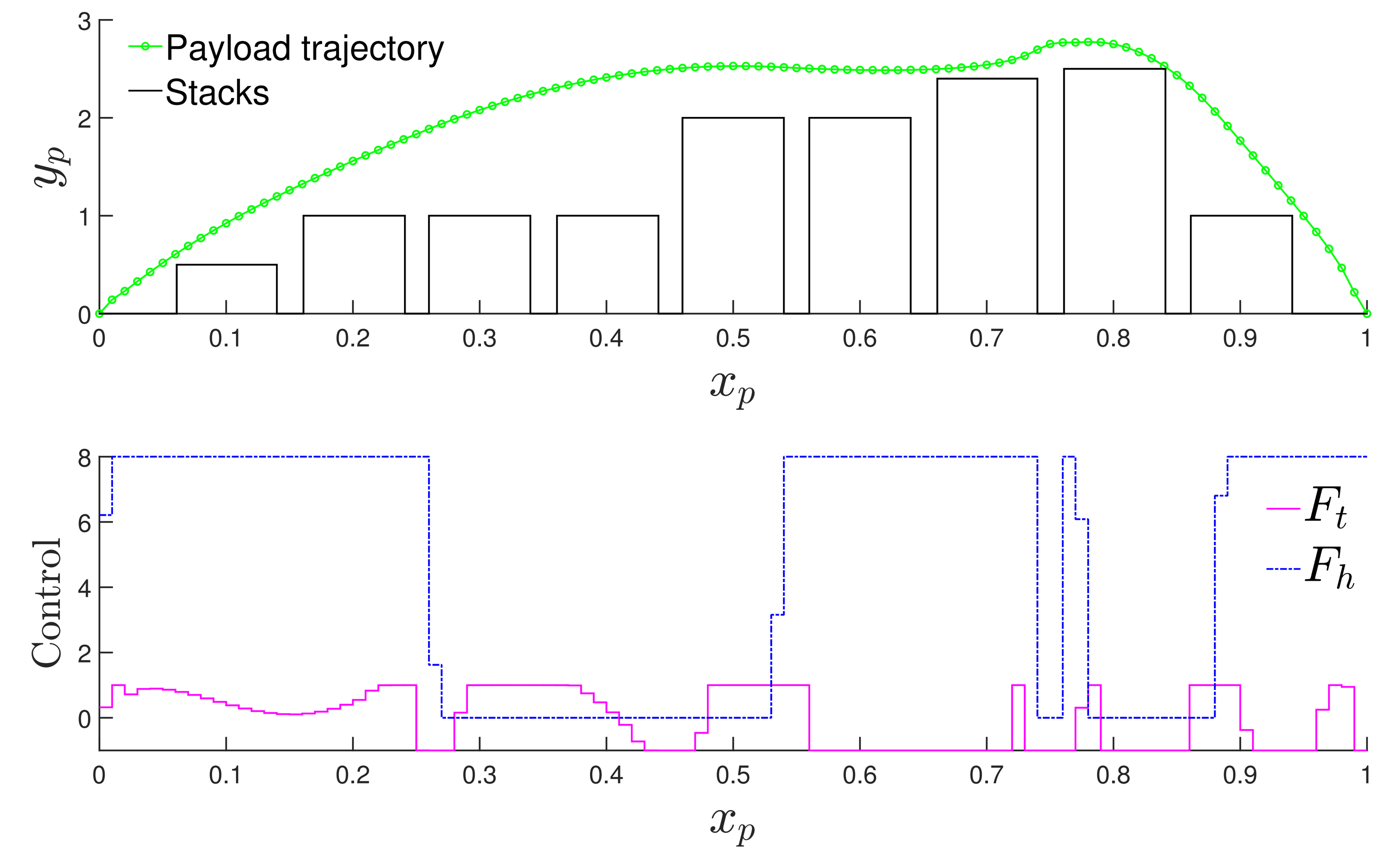
$$0 \le y_p(x_p^k) \le h - s(x_p^k). \quad ✓$$

Note that we no longer need an explicit function $s(x_p)$, but simply function values which can be computed when setting up the numerical model.



Simulation example

To ilustrate and validate the idea, a small scale scenario of stack configuration was simulated.



Future work

- Investigate energy consumption and energy optimal solutions.
- Apply the method in closed loop.
- Go beyond the point-mass assumption.
- More physical and geometric contraints to the setup.

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