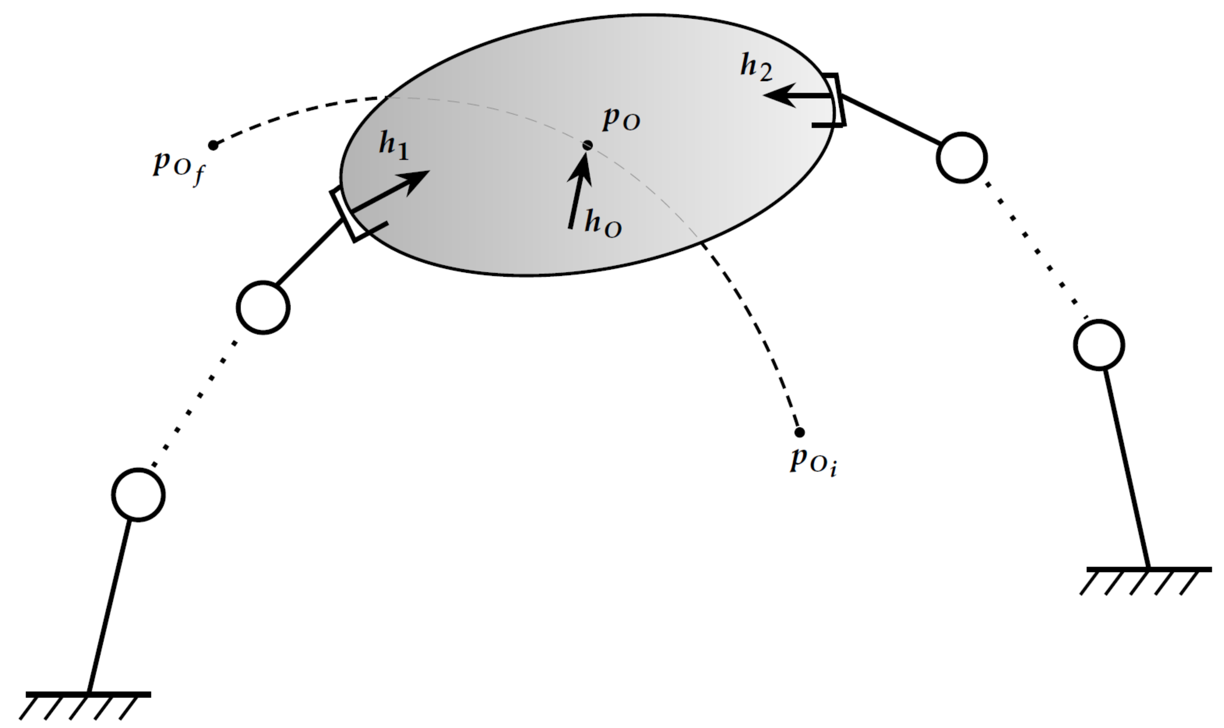
Time-Optimal Cooperative Path Tracking for Multi-Robot Systems

Hamed Haghshenas (hamed.haghshenas@liu.se), Anders Hansson, Mikael Norrlöf



Contributions

- Formulating the time-optimal cooperative path tracking problem as a convex optimization problem and subsequently as an SOCP.
- Proposing a new approach for obtaining internal force-free load distributions.

Time-optimal cooperative path tracking

Problem

- An object is rigidly grasped by multiple manipulators. The objective is to minimize the traversal time required to move the object with a desired orientation along a prescribed geometric path, where the path is given for the centre of mass of the object.
- The prescribed geometric path and the object's orientation are given as functions of the scalar path coordinate s.

Dynamics

Manipulator: $M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i - J_i^T(q_i)h_i$

Object: $M_o(x_o)\dot{v}_o + C_o(x_o,\dot{x}_o)v_o + g_o = h_o$

Relationship between forces h_o and h_i

$$h_o = G(s)h, \qquad h = [h_1^T,\ldots,h_N^T]^T$$

Solution: $h = G^{\dagger}h_o + Vh_I$

Coupled dynamics

$$\tilde{\tau}(s) = \big(\tilde{m}(s) + \tilde{J}^T(s)G^{\dagger}(s)m_o(s)\big)\ddot{s} + \big(\tilde{c}(s) + \tilde{J}^T(s)G^{\dagger}(s)c_o(s)\big)\dot{s}^2 + \tilde{g}(s) + \tilde{J}^T(s)G^{\dagger}(s)g_o + \tilde{J}^T(s)V(s)h_I$$

Convex formulation

$$\begin{aligned}
\underset{a(\cdot),b(\cdot),\tilde{\tau}(\cdot)}{\text{minimize}}\quad & \int_0^1 \frac{1}{\sqrt{b(s)}}ds\\
\text{subject to}\quad & \tilde{\tau}(s) = \big(\tilde{m}(s) + \tilde{J}^T(s)G^{\dagger}(s)m_o(s)\big)a(s)\\
& \quad + \big(\tilde{c}(s) + \tilde{J}^T(s)G^{\dagger}(s)c_o(s)\big)b(s)\\
& \quad + \tilde{g}(s) + \tilde{J}^T(s)G^{\dagger}(s)g_o + \tilde{J}^T(s)V(s)h_I,\\
& b'(s) = 2a(s),\\
& b(0) = \dot{s}_0^2,\quad b(1) = \dot{s}_T^2,\quad b(s) \ge 0,\quad b(s) \le \bar{b}(s),\\
& \underline{\tau}_i(s) \le \tau_i(s) \le \overline{\tau}_i(s),\quad i \in \mathcal{N},\\
& \forall s \in [0,1].
\end{aligned}$$

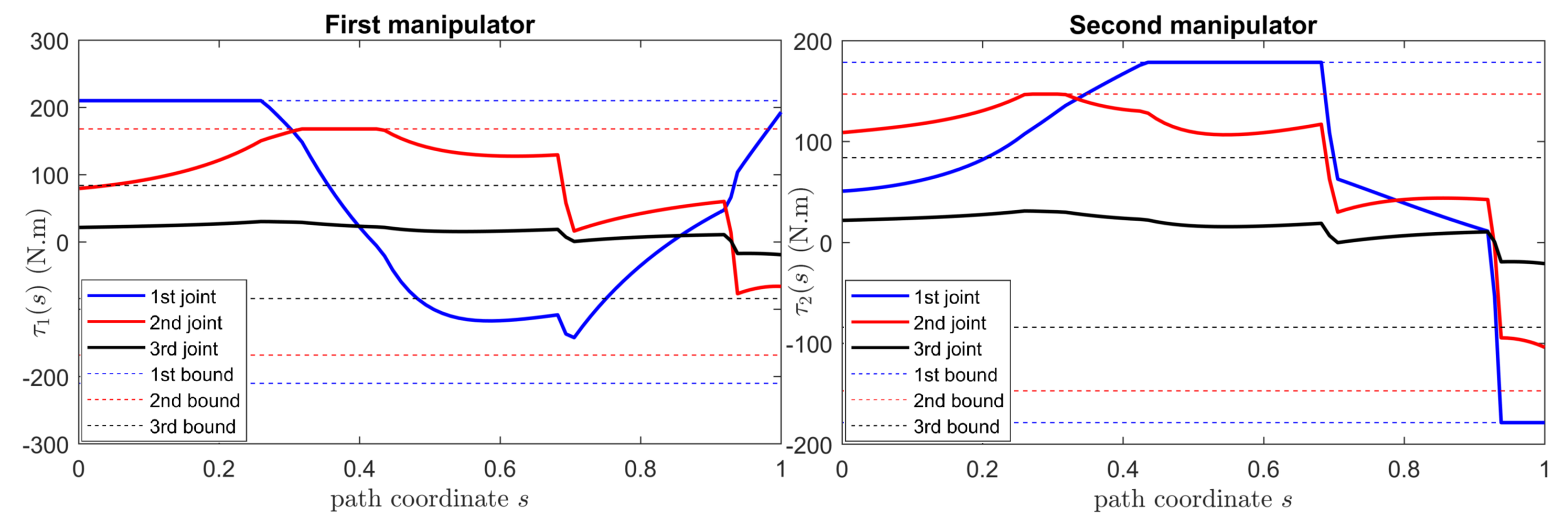


Figure 1: Joint torques $\tau_1(s)$ and $\tau_2(s)$

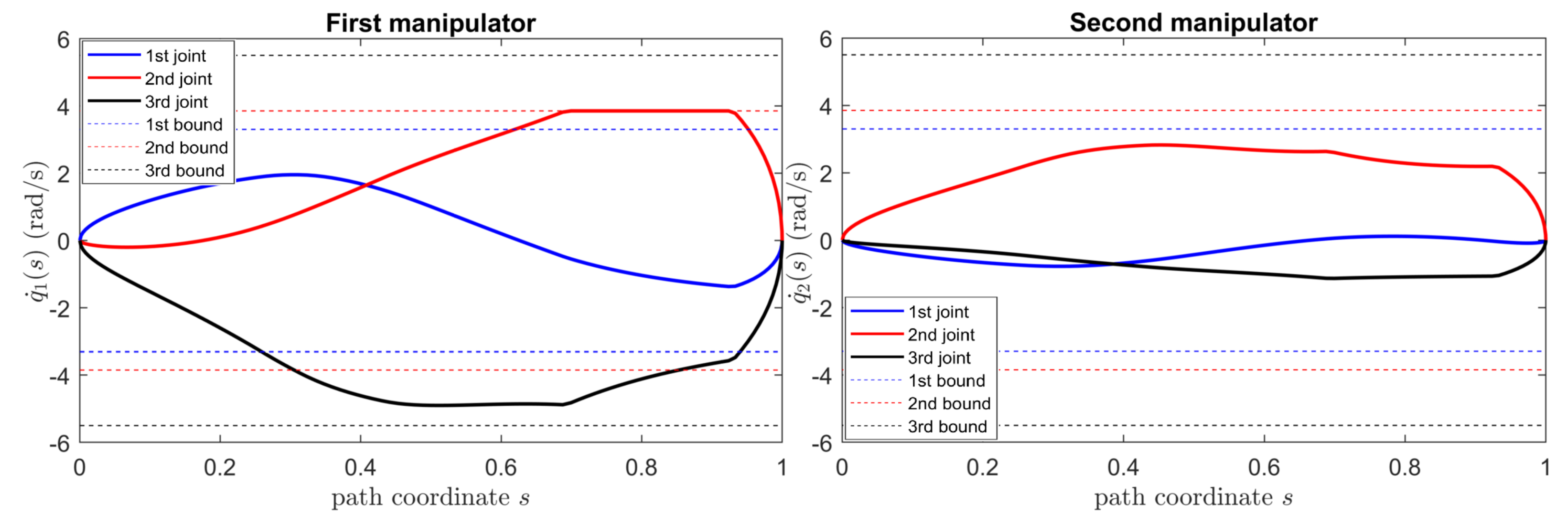
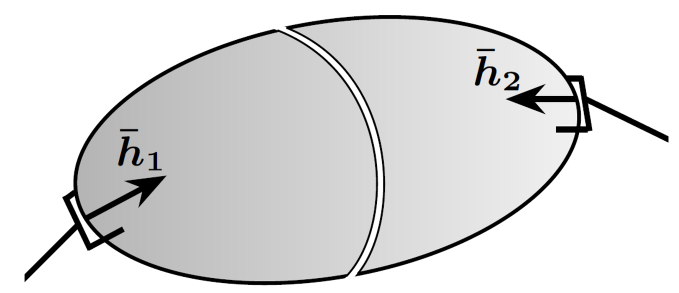


Figure 2: Joint velocities $\dot{q}_1(s)$ and $\dot{q}_2(s)$

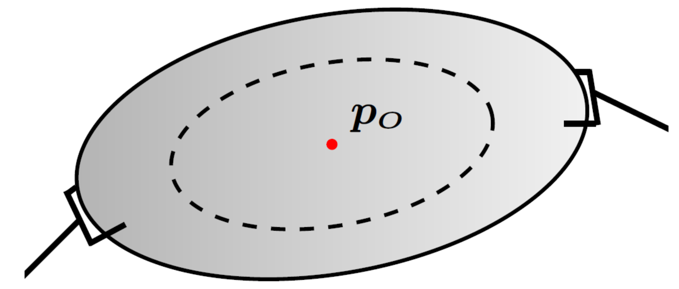
Internal force-free load distributions

Problem: How to choose $G^{\dagger}$ so that resulting force distribution from $h = G^{\dagger}h_o$ becomes free of internal forces.

- In this approach, the object is divided into several parts and the forces required to move the parts according to the desired motion of the object are computed. Then the pseudo-inverse is chosen in such a way that it results in the same distribution of forces.



- Segments are chosen in such a way that their centres of mass become the same as the object's centre of mass.



- The obtained pseudo-inverse of the grasp matrix is parameterized by coefficients that have the meaning of the inertial parameters of the considered parts of the object.

Three-dimensional convex objects with uniform mass density

$$G^{\star} = \begin{bmatrix} \frac{m_{O_1}}{m}I_3 & 0_{3\times3}\\ -\frac{m_{O_1}}{m}S(p_{1O}) & \lambda_1 I_3\\ \frac{m_{O_2}}{m}I_3 & 0_{3\times3}\\ -\frac{m_{O_2}}{m}S(p_{2O}) & \lambda_2 I_3 \end{bmatrix},\quad \lambda_1 = \Big(\frac{m_{O_1}}{m}\Big)^{\frac{5}{3}},\quad \lambda_2 = 1 - \Big(\frac{m_{O_1}}{m}\Big)^{\frac{5}{3}}$$

Table 1: Minimal traversal time for some values of the mass ratios.

m_{O_1}/m	m_{O_2}/m	Minimal traversal time (sec)
0.15	0.85	0.855
0.35	0.65	0.637
0.55	0.45	0.684
0.85	0.15	1.001

Future work

- Incorporate constraints that do not preserve convexity
- Devise a distributed algorithm for solving the optimization problem
- Extend the results to scenarios with nonconvex objects

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LINKÖPING UNIVERSITY
Division of Automatic Control