Ship Modelling for Estimation and Control

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Introduction

As marine vessels are becoming increasingly autonomous, having access to accurate simulation models is turning into an absolute necessity. This holds both for facilitation of development and for achieving satisfactory model-based control. One way to obtain such models is through system identification, i.e., to build models based on measured data. The aim of this poster is to summarize the Ph.D. project *Ship Modelling for Estimation and Control* which deals with this issue.

Ship Modelling

Ship models are typically based on the equations of motion:

$$\dot{\eta} = J(\eta)\nu,$$
$$M_{RB}\dot{\nu} + M_A\dot{\nu}_r + C_{RB}(\nu)\nu + C_A(\nu_r)\nu_r + D(\nu_r)\nu_r + F(\nu_q)\nu_q = \tau_{\text{act}}.$$

- η and ν constitute (generalized) position and velocity.
- ν_r is relative velocity between ship and water.
- ν_q is relative velocity between ship and air.
- The second equation is nonlinear due to C_{RB}, C_A (Coriolis forces) which give quadratic terms as well as D, F (hydrodynamic/aerodynamic damping) which give quadratic terms with absolute values.

Definition. *A second-order modulus function is a function, $f_{som} : \mathbb{R}^{n+p} \to \mathbb{R}^m$ that can be written as*

$$f_{som}(x, \theta) = \Phi^T(x)\theta,$$

where each element of the $p \times m$ matrix $\Phi(x)$ is on one of the forms $x_i, |x_i|, x_i x_j, x_i|x_j|$ for $i, j \leq n$ or zero and $\theta \in \mathbb{R}^p$ is a vector of coefficients.

Parameter Estimation

System:
$$\begin{cases} \mathbf{x}(k+1) &= f_{\text{som}}\left(\begin{bmatrix}\mathbf{x}(k) + \mathbf{R}(k)\mathbf{v}(k) \\ \mathbf{u}(k)\end{bmatrix}, \theta_0\right) + \mathbf{w}(k), \\ \mathbf{y}(k) &= \mathbf{x}(k) + \mathbf{e}(k). \end{cases}$$

Model:
$$\hat{\mathbf{y}}(k,\theta) = \Phi^T\left(\begin{bmatrix}\mathbf{y}(k-1) \\ \mathbf{u}(k-1)\end{bmatrix}\right)\theta.$$

Estimator:
$$\hat{\theta}_N^{IV} = \underset{\theta}{\operatorname{argmin}} \underbrace{\left\| \frac{1}{N}\sum_{k=1}^{N} \mathbf{Z}(k)\left(\mathbf{y}(k) - \hat{\mathbf{y}}(k,\theta)\right)\right\|_2^2}_{=V_N^{IV}(\theta)}.$$

Goal: Find a consistent estimator for θ when...

Scenario 1: $\mathbb{E}\{v(k)\} = 0$.
Interpretation: Bursts of wind gusts/ocean currents.
Solution: Excitation offset and zero-mean instruments.

Scenario 2: $\mathbb{E}\{v(k)\} = \bar{v}$ and $\mathbf{R}(k) = \mathbf{R}_0$.
Interpretation: Bursty wind/current with additional static component and ship motion with fixed attitude.
Solution: Excitation offset, zero-mean instruments and auxiliary disturbance measurement, $y_v(k) = \mathbf{v}(k) + \mathbf{e}_v(k)$.

Scenario 3: $\mathbb{E}\{v(k)\} = \bar{v}$ and $\mathbf{R}(k)$ measured.
Interpretation: Bursty wind/current with additional static component and ship motion with varying attitude.
Solution: Excitation offset, zero-mean instruments and modified predictor

$$\hat{\mathbf{y}}(k,\theta) = \Phi^T\left(\begin{bmatrix}\mathbf{y}(k-1) + \mathbf{R}(k-1)\rho \\ \mathbf{u}(k-1)\end{bmatrix}\right)\theta,$$

where both θ and ρ are estimated.

Remarks

1. The estimate $\hat{\rho}$ contains information about the disturbances.
2. In order to uniquely identify properties regarding *both* wind and currents, a mix of solutions is needed.

Experiment Design


- Most ships are unique (new model needed for each).
- Short commission times (few hours).
- Current solution: standard ship maneuvers.

Goal: Find $u(k)$ which maximizes a scalar criterion of

$$G(N) \triangleq \frac{1}{2}\frac{\partial^2}{\partial\theta^2}V_N^{IV}(\theta) = \left[\frac{1}{N}\sum_{k=1}^{N}\Phi(k)\mathbf{Z}^T(k)\right]\left[\frac{1}{N}\sum_{k=1}^{N}\Phi(k)\mathbf{Z}^T(k)\right]^T.$$

Challenge: Nonconvex problem (requires good initial guess).

Solution part 1:

1. Choose candidate signals $u_1(k), \ldots, u_Q(k)$ (standard maneuvers).
2. Estimate $\bar{G}_1, \ldots, \bar{G}_Q$ (information matrices) based on simulation experiments with a nominal model.
3. Assume that $u_1(k), \ldots, u_Q(k)$ are to be applied in sequence and solve an optimization problem to find the information-optimal mix, *i.e.*, for how long they should be applied w.r.t. each other.

Solution part 2: Solve a lattice-based motion planning problem

$$\begin{aligned} \underset{\{m_k\}_{k=0}^{M-1}, M}{\text{minimize}} \quad & \sum_{k=0}^{M-1} J(m_k) \\ \text{s.t.} \quad & \mathbf{x}_0 = \mathbf{x}_s, \quad \mathbf{x}_M = \mathbf{x}_f, \\ & \mathbf{x}_{k+1} = f(\mathbf{x}_k, m_k), \\ & m_k^p \in \left\{\underbrace{m^1, \cdots, m^Q}_{\text{informative}}, \underbrace{m^{Q+1}, \cdots, m^{Q+B}}_{\text{basic}}\right\}, \\ & c(m_k, \mathbf{x}_k) \in \mathcal{X}_{\text{free}}. \end{aligned}$$

- The signals $u_1(k), \ldots, u_Q(k)$, are used to form motion primitives.
- The ratios found in Solution part 1 are respected by augmenting the state vector with motion primitive counters.

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