Time-Optimal Cooperative Path Tracking for Multi-Robot Systems

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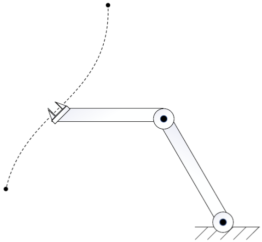
Motion planning

Decoupled approach

- Path planning
  - Task specifications
  - Obstacle avoidance
  - Other robots in a shared workspace
- Path tracking
  - Dynamics
  - Actuator constraints



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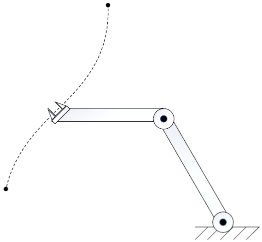
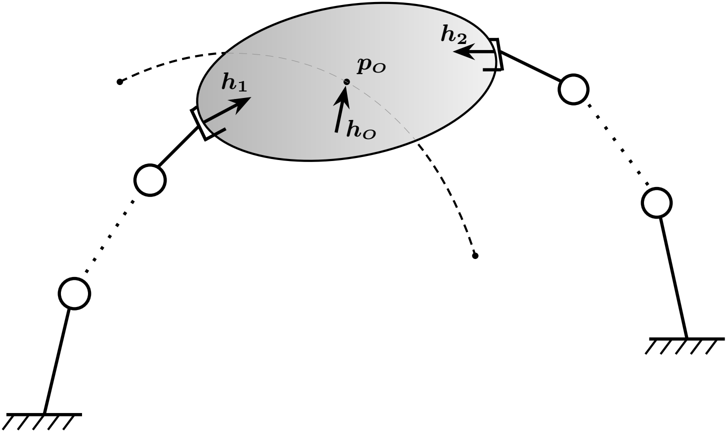


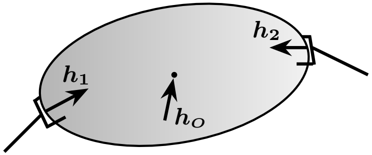
- Assuming that a desired path is given, path tracking can be studied independently.

Time-optimal cooperative path tracking



Internal forces

- Infinite number of forces at grasping points
- A load distribution strategy
- Resulting interaction forces:
    - Motion-inducing
    - Internal forces
        - Squeezing or pulling forces
        - Do not contribute to motion of object
        - Must be avoided
- Open problem: finding all internal force-free distributions
- Can appear in cooperative path tracking problem



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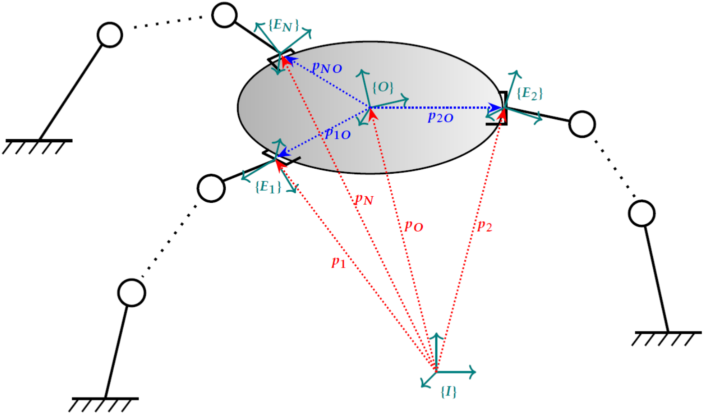
Setup

- The prescribed geometric path and the object's orientation are given as functions of the scalar path coordinate s.



Pose of the ith end-effector:

$$p_i(s) = p_O(s) + p_{iO}(s) = p_O(s) + R_{E_i}(s)p_{iO}^{E_i},$$
$$\phi_i(s) = \phi_O(s) + \phi_{iO}.$$

$p_{iO}^{E_i}$ and ϕ_{iO}: Constant distance and orientation offsets between the reference frames $\{O\}$ and $\{E_i\}$, respectively.

- $q_i(s)$: Using inverse kinematics

Joint velocities and accelerations:

$$\dot{q}_i(s) = q_i'(s)\dot{s},$$
$$\ddot{q}_i(s) = q_i'(s)\ddot{s} + q_i''(s)\dot{s}^2,$$

where $q_i'(s) = \partial q_i(s)/\partial s$, $q_i''(s) = \partial^2 q_i(s)/\partial s^2$, $\dot{s} = ds/dt$ and $\ddot{s} = d^2s/dt^2$.

Manipulator dynamics (ith manipulator)

Joint space model:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i - J_i^T(q_i)h_i,$$

$J_i \in \mathbb{R}^{6\times n_i}$: Geometric Jacobian
$h_i \in \mathbb{R}^6$: Vector of generalized forces exerted by the ith end-effector on the object

$$\tau_i(s) = m_i(s)\ddot{s} + c_i(s)\dot{s}^2 + g_i(s) + J_i^T(s)h_i,$$

where

$$\begin{aligned} m_i(s) &= M_i(q_i(s))q_i'(s), \\ c_i(s) &= M_i(q_i(s))q_i''(s) + C_i(q_i(s), q_i'(s))q_i'(s). \end{aligned}$$

Object dynamics

Newton-Euler formulation:

$$M_O(x_O)\dot{v}_O + C_O(x_O, \dot{x}_O)v_O + g_O = h_O,$$

$h_O \in \mathbb{R}^6$: Generalized forces acting on the object's centre of mass

$$m_O(s)\ddot{s} + c_O(s)\dot{s}^2 + g_O = h_O,$$

where

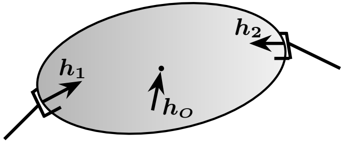
$$m_O(s) = M_O(x_O(s))T_O(s)x'_O(s),$$

$$\begin{aligned} c_O(s) &= M_O(x_O(s))\frac{\partial T_O(s)}{\partial s}x'_O(s) + M_O(x_O(s))T_O(s)x''_O(s) \\ &\quad + C_O(x_O(s), x'_O(s))T_O(s)x'_O(s), \end{aligned}$$

$$T_O(s) = \begin{bmatrix} I_3 & 0_{3\times 3} \\ 0_{3\times 3} & T(\phi_O(s)) \end{bmatrix}.$$

Relationship between h_O and h_i

$$h_O = G(s)h, \qquad h = [h_1^T, \ldots, h_N^T]^T$$



Solution:

$$h = G^\dagger h_O + V h_I,$$

$G^\dagger \in \mathbb{R}^{6N\times 6}$: A right inverse of G (pseudo-inverse)
$V \in \mathbb{R}^{6N\times 6}$: A matrix whose columns span the null space of G
$h_I \in \mathbb{R}^6$: Generally represents the vector of internal forces

- Use of a generic pseudo-inverse of G may lead to internal forces even if $h_I = 0$.

Coupled dynamics

Dynamics of manipulators:

$$\tilde{\tau}(s) = \tilde{m}(s)\ddot{s} + \tilde{c}(s)\dot{s}^2 + \tilde{g}(s) + \tilde{J}^T(s)h$$

Object dynamics:

$$m_O(s)\ddot{s} + c_O(s)\dot{s}^2 + g_O = h_O$$

Relationship between h_O and h:

$$h = G^{\dagger}h_O + Vh_I$$

Coupled dynamics:

$$\begin{aligned}\tilde{\tau}(s) = &\left(\tilde{m}(s) + \tilde{J}^T(s)G^{\dagger}(s)m_O(s)\right)\ddot{s} \\ &+ \left(\tilde{c}(s) + \tilde{J}^T(s)G^{\dagger}(s)c_O(s)\right)\dot{s}^2 \\ &+ \tilde{g}(s) + \tilde{J}^T(s)G^{\dagger}(s)g_O + \tilde{J}^T(s)V(s)h_I\end{aligned}$$

Convex formulation

Optimization variables:

$$a(s) = \ddot{s}(t), \quad b(s) = \dot{s}(t)^2, \quad b'(s) = 2a(s).$$

The additional constraint follows from:

$$\begin{aligned}
\dot{b}(s(t)) &= b'(s)\dot{s}(t), \\
\dot{b}(s(t)) &= \frac{d(\dot{s}(t)^2)}{dt} = 2\ddot{s}(t)\dot{s}(t) = 2a(s)\dot{s}(t).
\end{aligned}$$

Convex formulation of the problem of interest

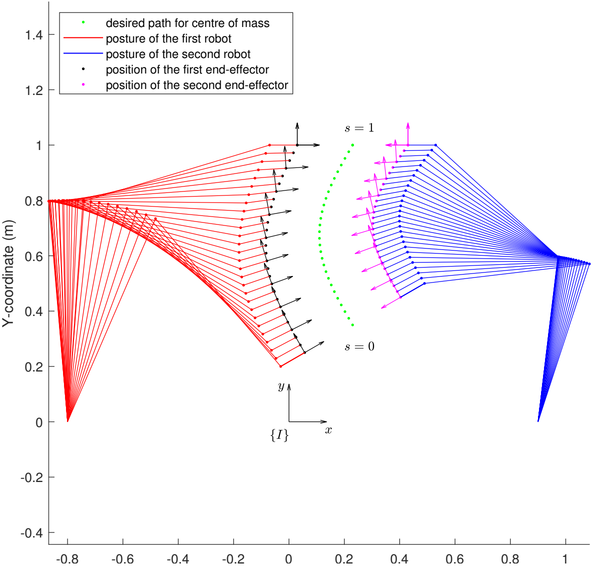
$$
\begin{aligned}
\underset{a(\cdot), b(\cdot), \tilde{\tau}(\cdot)}{\text{minimize}} \quad & \int_0^1 \frac{1}{\sqrt{b(s)}} ds \\
\text{subject to} \quad & \tilde{\tau}(s) = \big(\tilde{m}(s) + \tilde{J}^T(s) G^\dagger(s) m_O(s)\big) a(s) \\
& \qquad + \big(\tilde{c}(s) + \tilde{J}^T(s) G^\dagger(s) c_O(s)\big) b(s) \\
& \qquad + \tilde{g}(s) + \tilde{J}^T(s) G^\dagger(s) g_O + \tilde{J}^T(s) V(s) h_I, \\
& b(0) = \dot{s}_0^2, \\
& b(1) = \dot{s}_T^2, \\
& b(s) \geq 0, \\
& b(s) \leq \overline{b}(s), \\
& b'(s) = 2a(s), \\
& \underline{\tau}_i(s) \leq \tau_i(s) \leq \overline{\tau}_i(s), \quad i \in \mathcal{N}, \\
& \forall s \in [0, 1].
\end{aligned}
$$

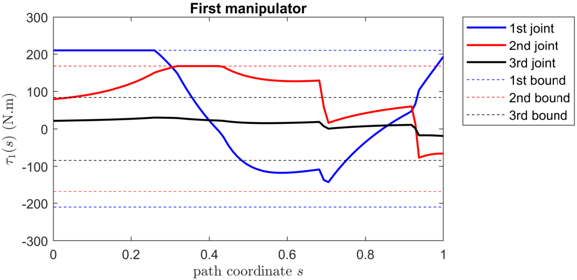
Numerical approach

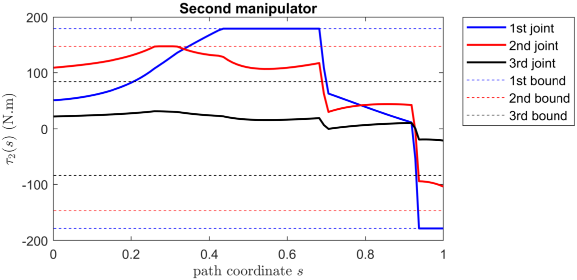
- Direct transcription method
- A large sparse optimization problem

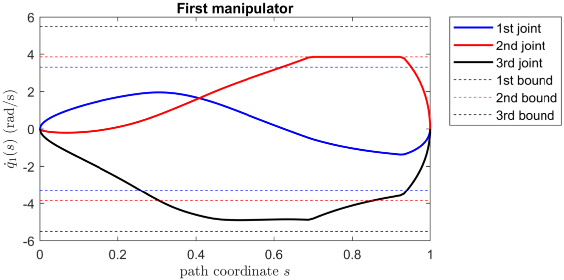
- Can be transformed into an SOCP
- MOSEK, SeDuMi

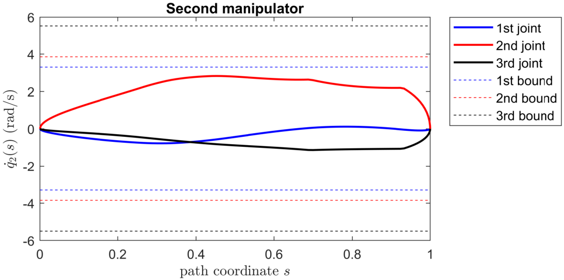
Numerical simulation




First manipulator
300
200
100
0
-100
-200
-300
0
0.2
0.4
0.6
0.8
1
$\tau_1(s)$ (N.m)
path coordinate s
1st joint
2nd joint
3rd joint
1st bound
2nd bound
3rd bound


Second manipulator
200
100
0
-100
-200
0
0.2
0.4
0.6
0.8
1
$\tau_2(s)$ (N.m)
path coordinate s
1st joint
2nd joint
3rd joint
1st bound
2nd bound
3rd bound


First manipulator
6
4
2
0
-2
-4
-6
0
0.2
0.4
0.6
0.8
1
$\dot{q}_1(s)$ (rad/s)
path coordinate s
1st joint
2nd joint
3rd joint
1st bound
2nd bound
3rd bound


Second manipulator
6
4
2
0
-2
-4
-6
0
0.2
0.4
0.6
0.8
1
$\dot{q}_2(s)$ (rad/s)
path coordinate s
1st joint
2nd joint
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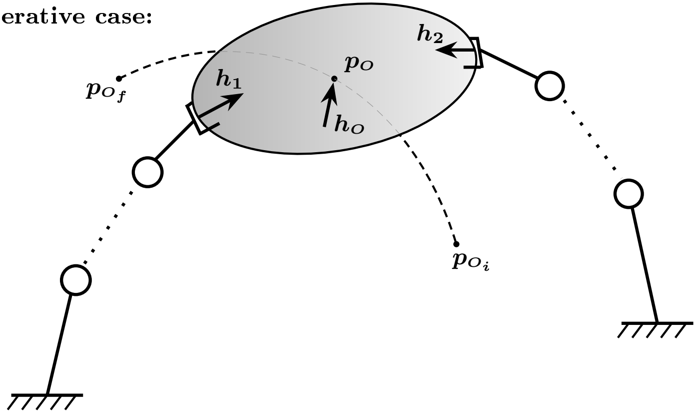
Problem

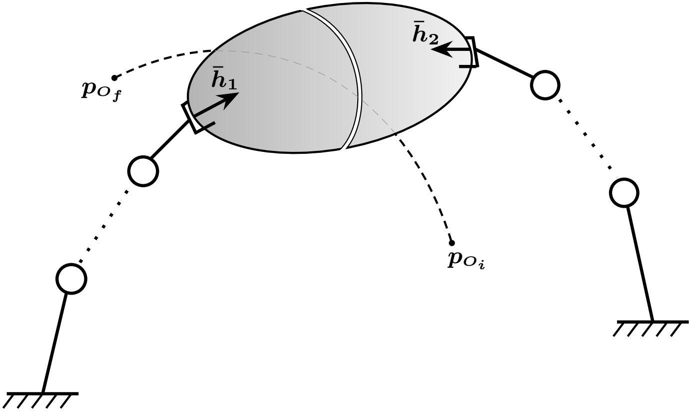
- How to choose $G^\dagger$ so that resulting force distribution from $h = G^\dagger h_O$ becomes free of internal forces.

Cooperative case:



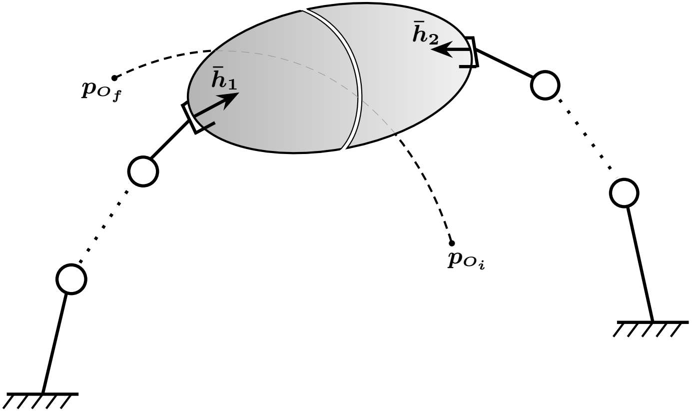
Idea behind the approach

- Object is divided into two parts, each part grasped by one of manipulators
- Infinitesimal distance between pieces, no exchange of forces



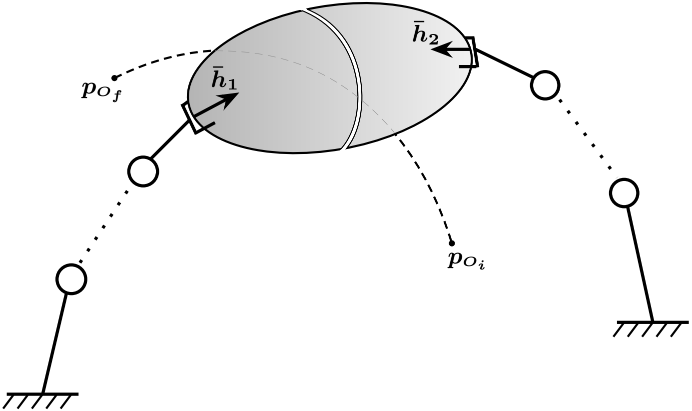
Idea behind the approach

- Each segment must to be moved in the same way that the corresponding part of the object would move in the cooperative case.



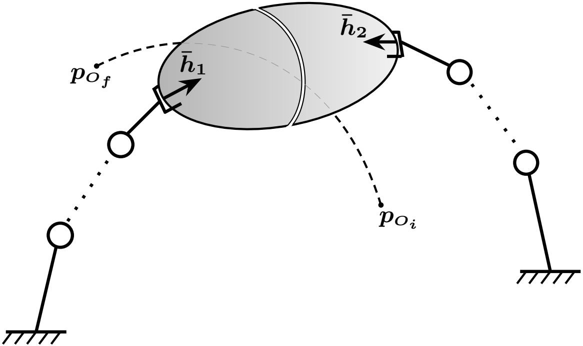
Idea behind the approach

- Once the desired motion of each segment is specified, the forces $\bar{h}_i$ required to fulfill the motion can be obtained.
- No internal forces by design



Idea behind the approach

- Employing the forces $\bar{h}_i$ in the cooperative case will not result in internal forces.
  - Imposed kinematic constraints in both cases are the same.
  - Inertial parameters of the segments together are the same as the inertial parameters of the whole object.



The pseudo-inverse $G^\star$

Solution:

$$G_i^\star = \begin{bmatrix} \frac{m_{O_i}}{m} I_3 & 0_{3\times 3} \\ -\frac{m_{O_i}}{m} S(p_{iO}) & I_{O_i} I_O^{-1} \end{bmatrix},$$

if

$$I_{O_i} = \lambda_i I_O, \quad i \in \{1, 2\},$$

where $\lambda_i, i \in \{1, 2\}$ are some positive coefficients with $\lambda_1 + \lambda_2 = 1$.

Three-dimensional convex objects with uniform mass density:

$$\lambda_1 = \left(\frac{m_{O_1}}{m}\right)^{\frac{5}{3}},$$

$$\lambda_2 = 1 - \left(\frac{m_{O_1}}{m}\right)^{\frac{5}{3}}.$$

Numerical simulation

m_{O_1}/m	m_{O_2}/m	Minimal traversal time (sec)
0	1	1.408
0.15	0.85	0.855
0.25	0.75	0.714
0.35	0.65	0.637
0.45	0.55	0.646
0.5	0.5	0.661
0.55	0.45	0.684
0.65	0.35	0.751
0.75	0.25	0.847
0.85	0.15	1.001
1	0	1.611

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Conclusions and future work

Conclusions:

- Formulating the time-optimal cooperative path tracking problem as a convex optimization problem and subsequently as an SOCP
- Proposing a new approach for obtaining internal force-free load distributions

Future work:

- Incorporate constraints that do not preserve convexity
- Incorporation of the freedom in the choice of the parameters of the pseudo-inverse as optimization variables
- Extend the results to scenarios with nonconvex objects

Thank you for listening!

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