Optimal control of cranes subject to container height constraints

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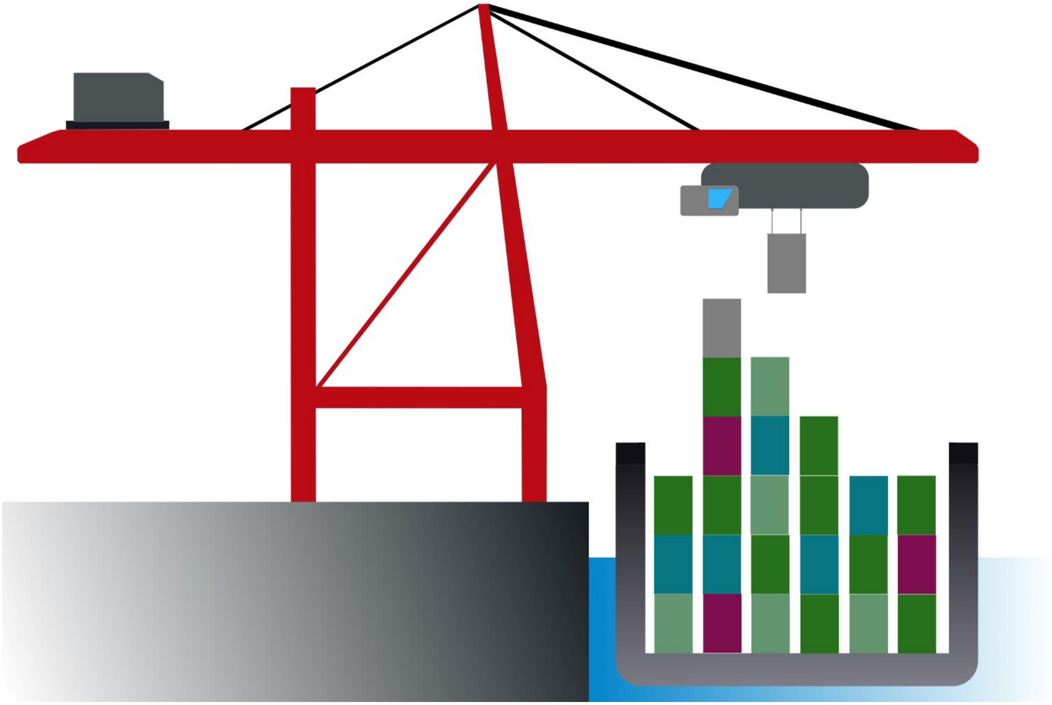
Loading and unloading ships

Objective: Trade-off time and energy when loading a container ship.

Challenge: Avoid collision with container stacks.



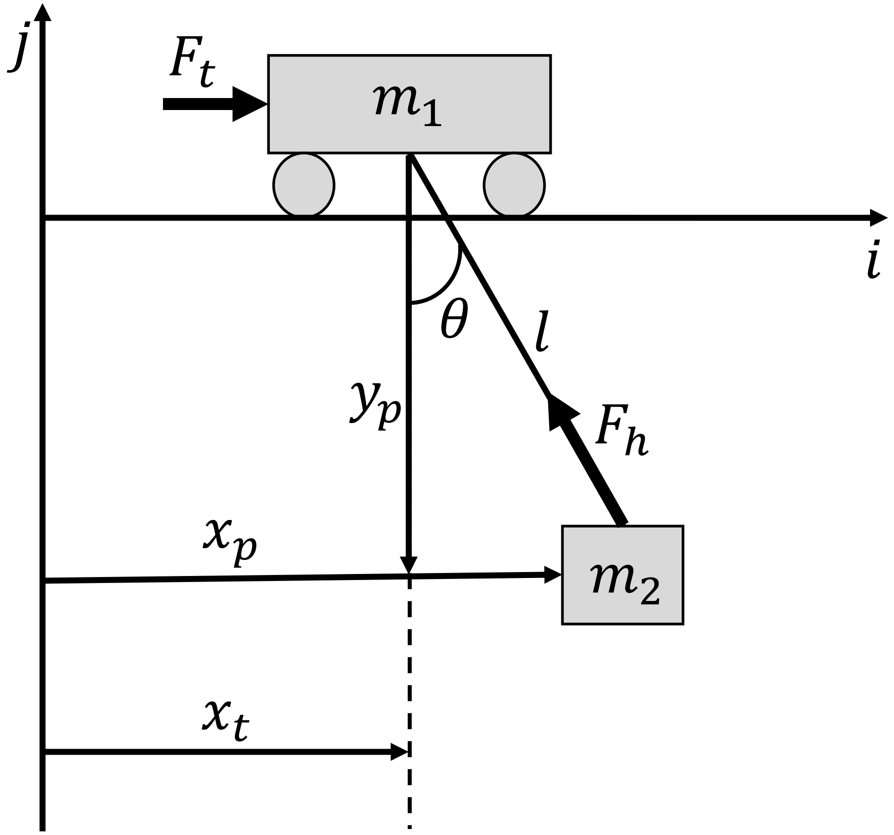
Modelling

First obtain a nonlinear state-space representation of the system

$$\dot{x}(t) = f\big(t, x(t), u(t)\big),$$

where the state variables are

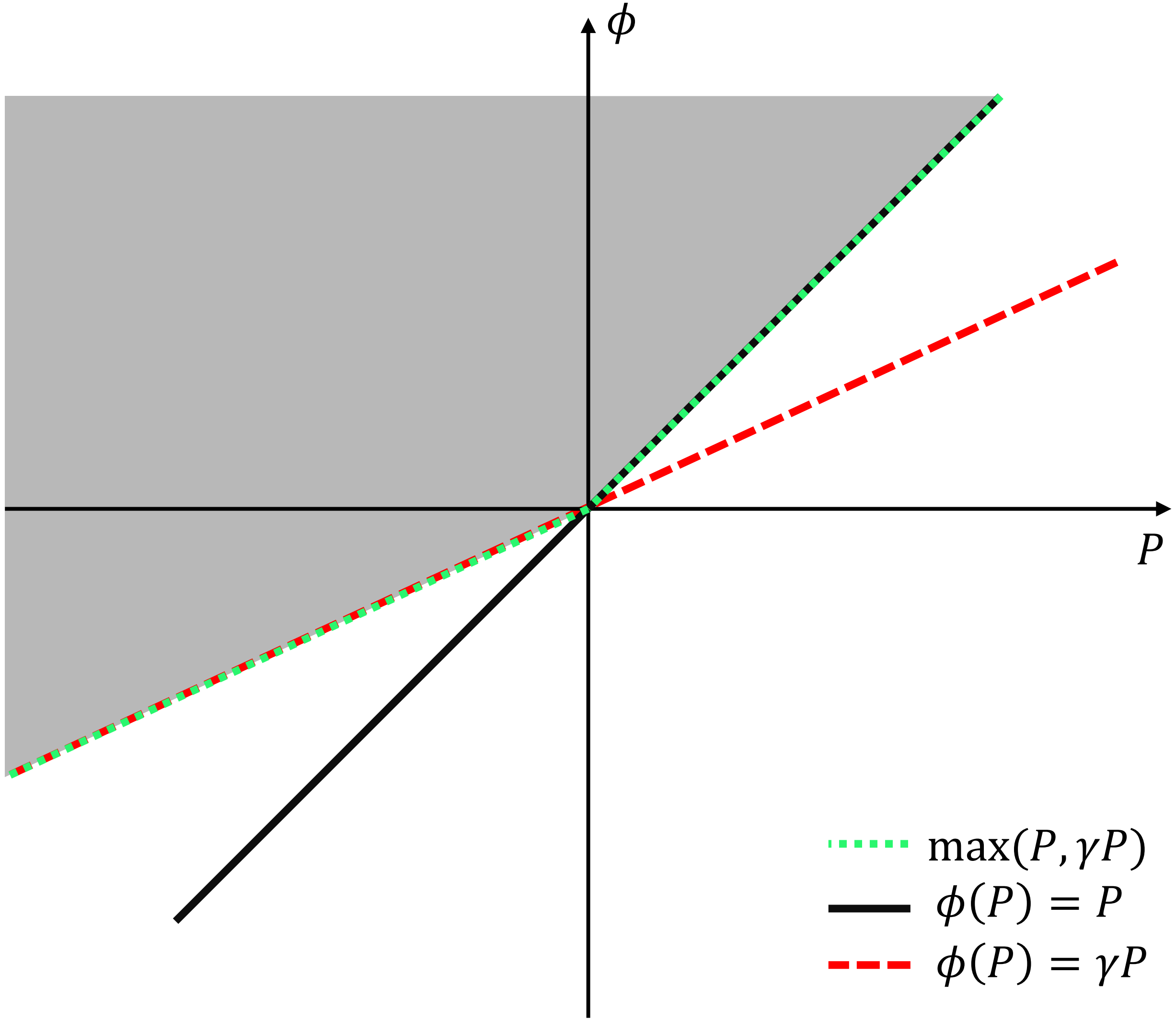
$$x = [x_p, \dot{x}_p, y_p, \dot{y}_p, l, \dot{l}, \theta, \dot{\theta}]^T$$



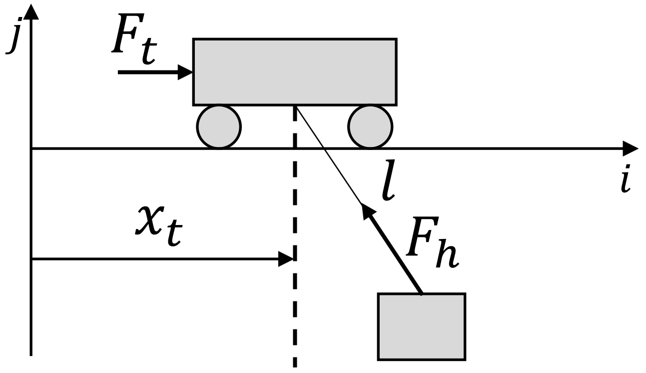
Modelling

If we take energy regeneration into account

$$E(t) = \int_0^t \max\big(P(\tau), \gamma P(\tau)\big)\, d\tau$$



Energy terms



For the STS-crane we consider the energy used to move the trolley and to hoist the payload, so

$$E(t) = \underbrace{\int_0^{t_f} \max\big(P_t(\tau), \gamma_t P_t(\tau)\big)\, d\tau}_{E_t} + \underbrace{\int_0^{t_f} \max\big(P_h(\tau), \gamma_h P_h(\tau)\big)\, d\tau}_{E_h}$$

$$P_t = F_t \dot{x}_t \text{ and } P_h = -F_h \dot{l}$$

Formulation

To avoid a non-smooth integrand, we introduce an auxiliary variable

$$\min_u \int_0^{t_f} \max\big(P(t), \gamma P(t)\big)\, dt$$

$$\Rightarrow$$

$$\begin{aligned} \min_u \quad & \int_0^{t_f} z(t)dt \\ \text{s.t.} \quad & z(t) \geq P(t) \\ & z(t) \geq \gamma P(t) \end{aligned}$$

Time and energy optimal control

The trade-off between time and energy can be done through

$$\min_u \alpha \int_0^{t_f} dt + (1-\alpha) \int_0^{t_f} z_t(t) + z_h(t) dt$$

s.t. $\dot{x}(t) = f\big(t, x(t), u(t)\big)$

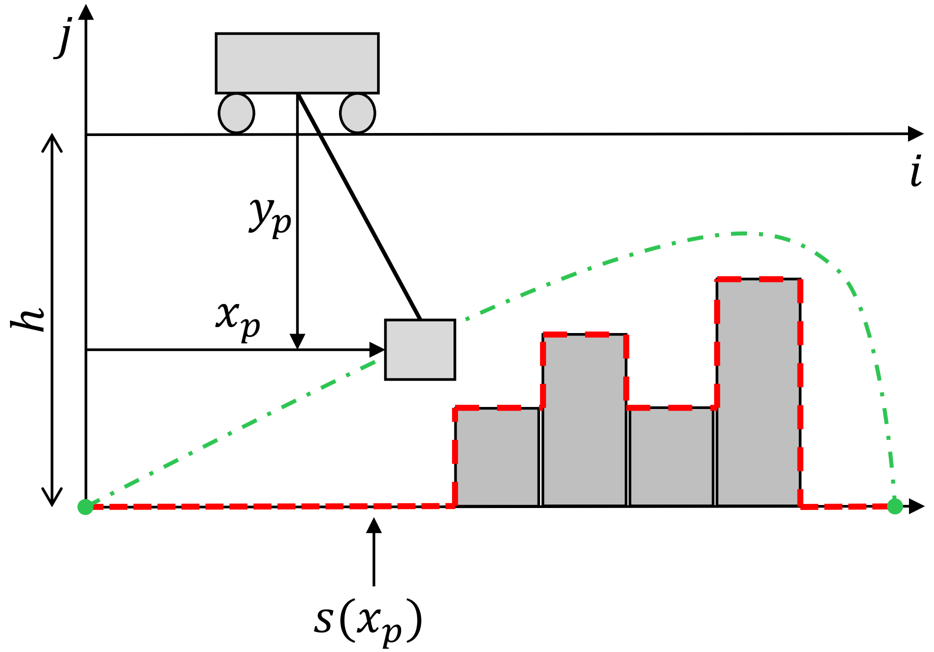
$z_t(t) \geq P_t(t) \qquad z_h(t) \geq P_h(t)$

$z_t(t) \geq \gamma P_t(t) \qquad z_h(t) \geq \gamma P_h(t)$

$0 \leq y_p(t) \leq h - s(x_p(t))$ ← avoidance constraints

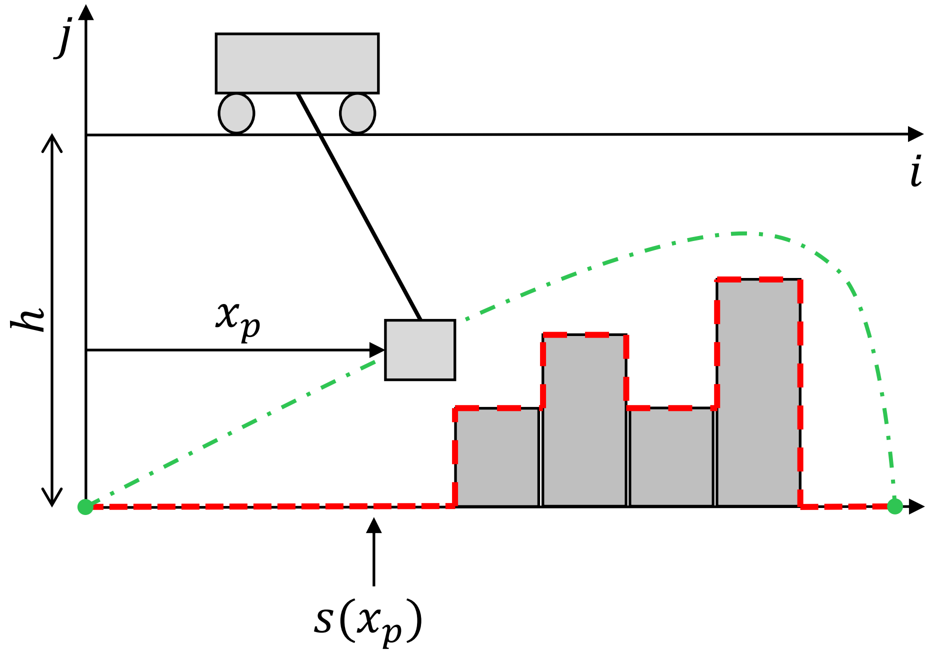
$\vdots$

other constraints



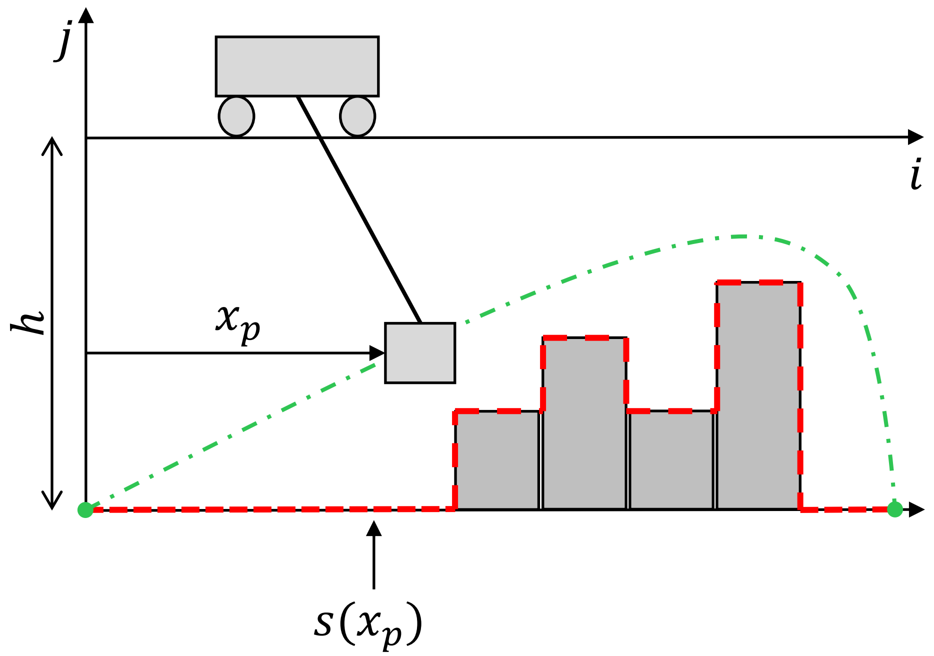
Issues with this formulation

- The constraints need to be constructed from a function $\boldsymbol{s}(\boldsymbol{x_p}(\boldsymbol{t}))$.
- Container height constraints are usually nonlinear and non-smooth function of space.



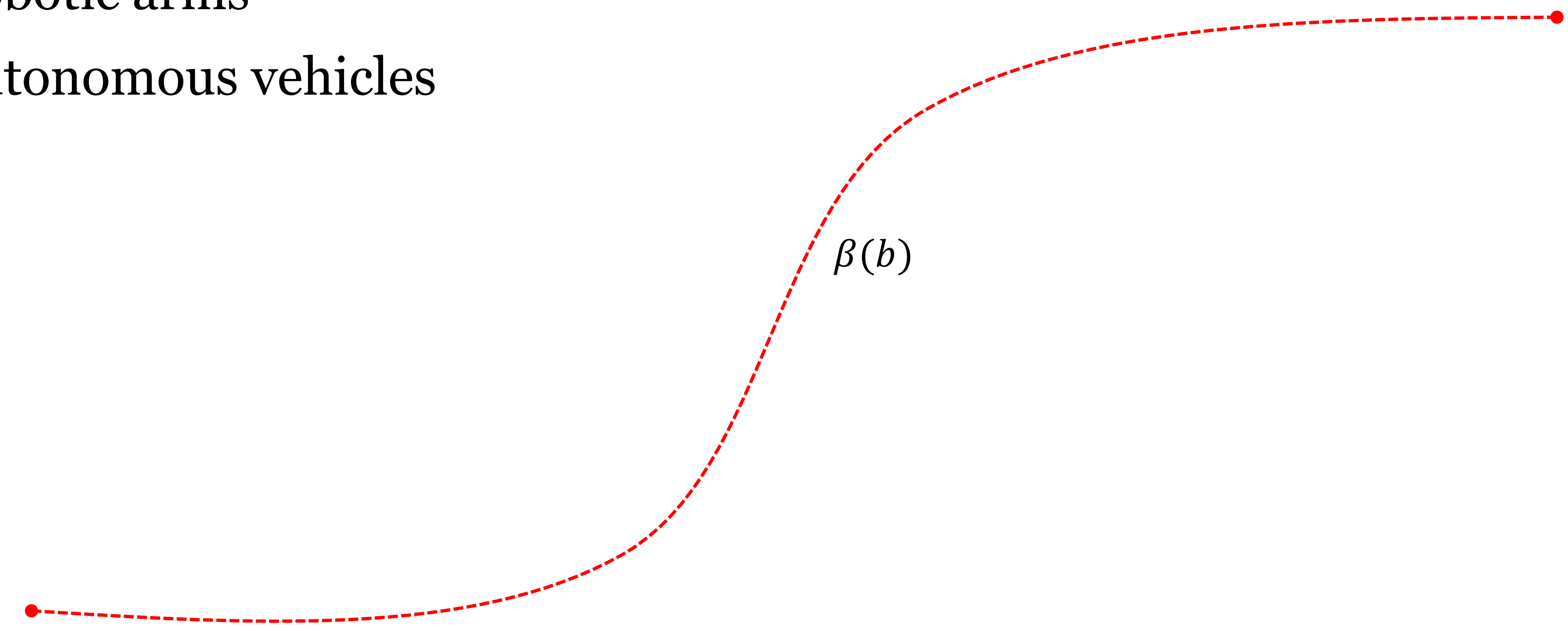
Issues with this formulation

- The stack height at t^k is a function $s(x_p(t^k))$, thus $s(x_p(t))$ must be defined from the current configuration.
- The free variable is also the one being optimized. So, the solution will be influenced by fixed time sampling rate (fixed # of control intervals).



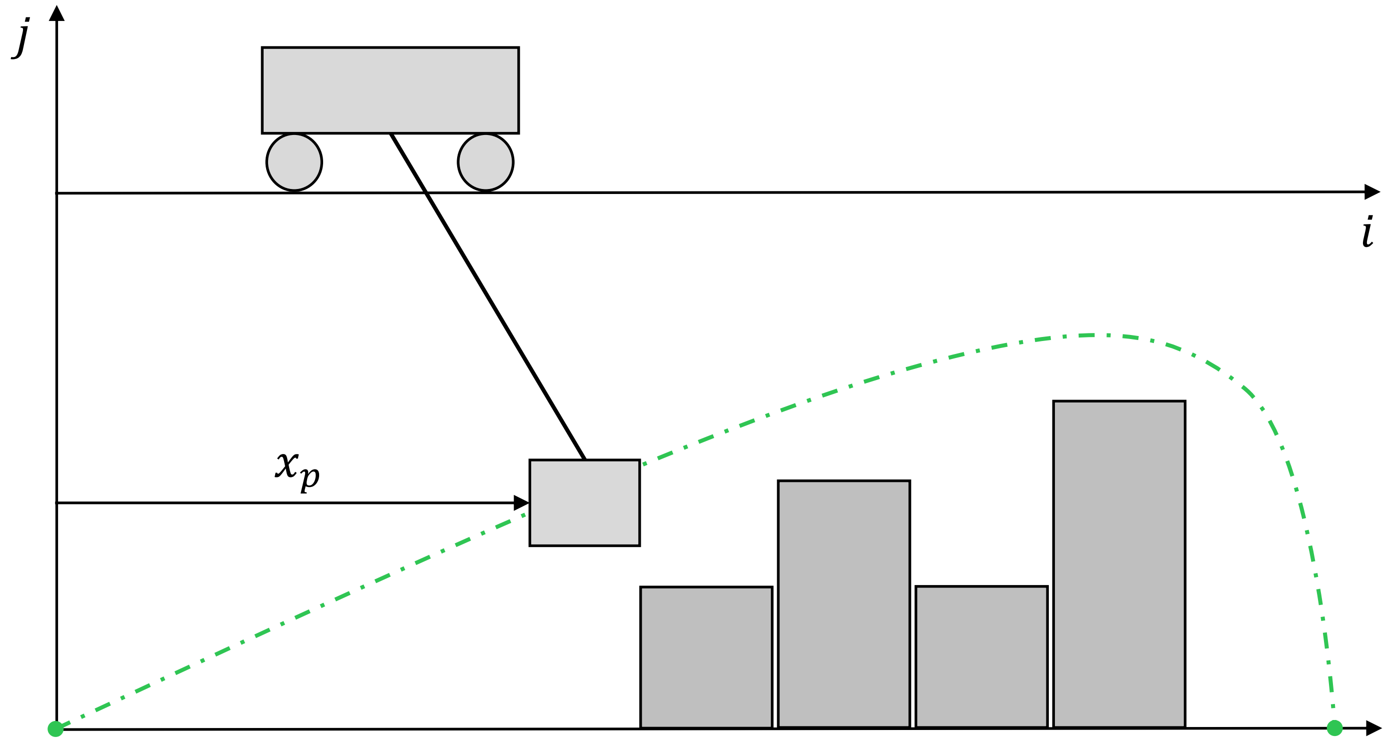
Geometric path

- Robotic arms
- Autonomous vehicles



Geometric path

- Container avoidance



Problem reformulation

Container avoidance constraints are easier described along loading site (x_p).

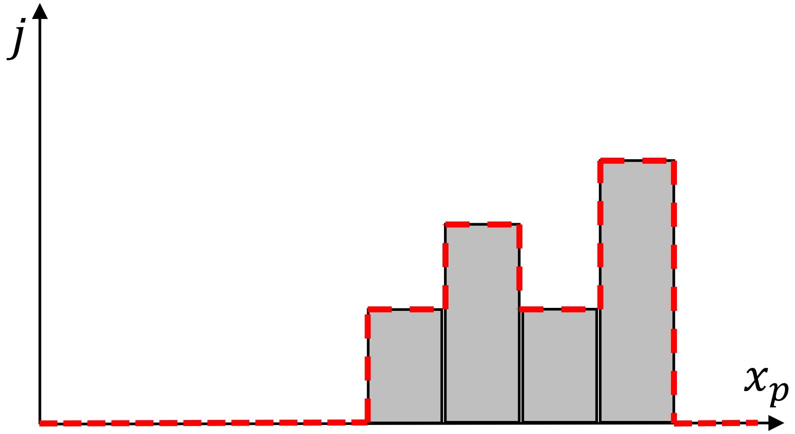
Thus, we change the integration variable

$$\dot{x}_1 = \frac{dx_1}{dt} = x_2 \Rightarrow \frac{dt}{dx_1} = \frac{1}{x_2}, \frac{dx_2}{dx_1} = \cdots$$

and

$$x = \left[t, x_p', y_p, y_p', l, l', \theta, \theta'\right]^T$$

Note: $x' = \frac{dx}{dx_p}$

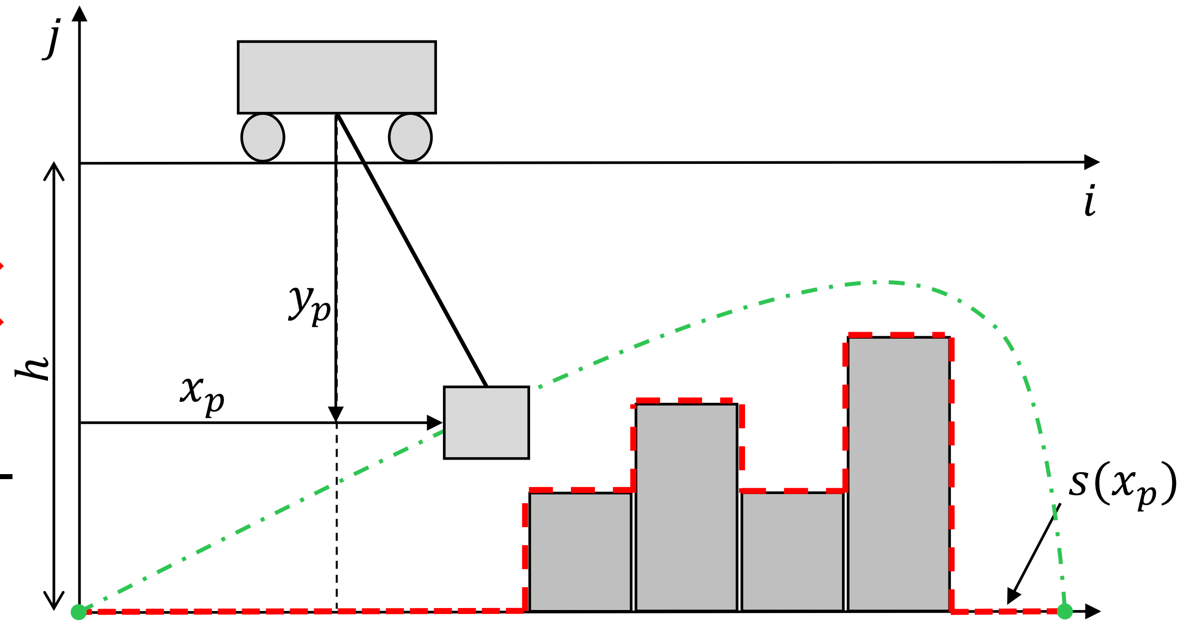


Geometric constraints

- Time discretization before the variable change leads to

$$0 \leq y_p(x_p(t^k)) \leq h - s(x_p(t^k)).$$ ✗

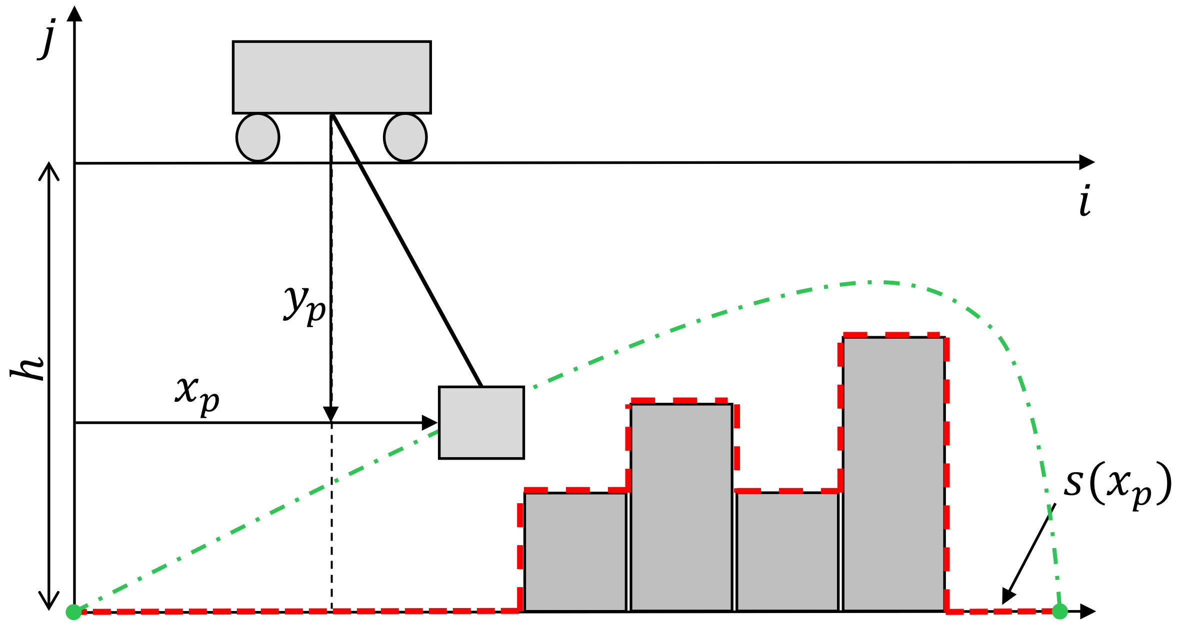
- Where $s(x_p)$ is generally discontinuous, nonlinear and non-convex



Geometric constraints

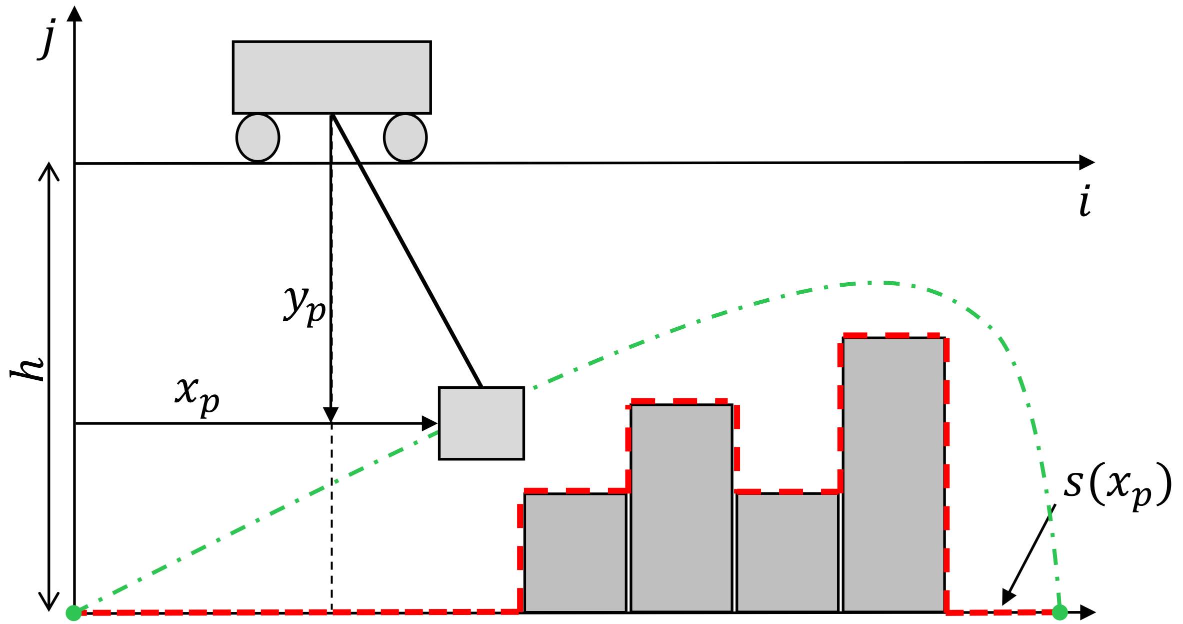
Spatial discretization leads to upper bound constraints in $y_p(x_p)$

$$0 \leq y_p(x_p^k) \leq h - s(x_p^k).$$



Geometric constraints

Note that we no longer need an explicit function $s(x_p)$, but simply function values which can be computed when setting up the numerical model.



$$s(x_p^k) = [00000000000000000333344443333555550000]$$

Additional implications

- A natural choice of the cost function would be

$$J = \alpha \int_{x_{p_0}}^{x_{p_f}} \frac{1}{x_2} dx_p + (1-\alpha) \int_{x_{p_0}}^{x_{p_f}} \frac{z_t(x_p) + z_h(x_p)}{x_2} dx_p \, ,$$

However, $x_1(x_p) = t$ and

$$x_2 x_9' = z_t(x_p) \Rightarrow x_9(x_p) = \int_{x_{p_0}}^{x_{p_f}} \frac{z_t(x_p)}{x_2} dx_p$$

$$x_2 x_{10}' = z_h(x_p) \Rightarrow x_{10}(x_p) = \int_{x_{p_0}}^{x_{p_f}} \frac{z_h(x_p)}{x_2} dx_p$$

Additional implications

- Then…

$$J = \alpha x_1(x_{p_f}) + (1-\alpha)\left(x_9(x_{p_f}) + x_{10}(x_{p_f})\right)$$

New formulation

$$\min_{u} \quad \alpha \int_{0}^{t_f} dt + (1-\alpha) \int_{0}^{t_f} z_t(t) + z_h(t) dt$$

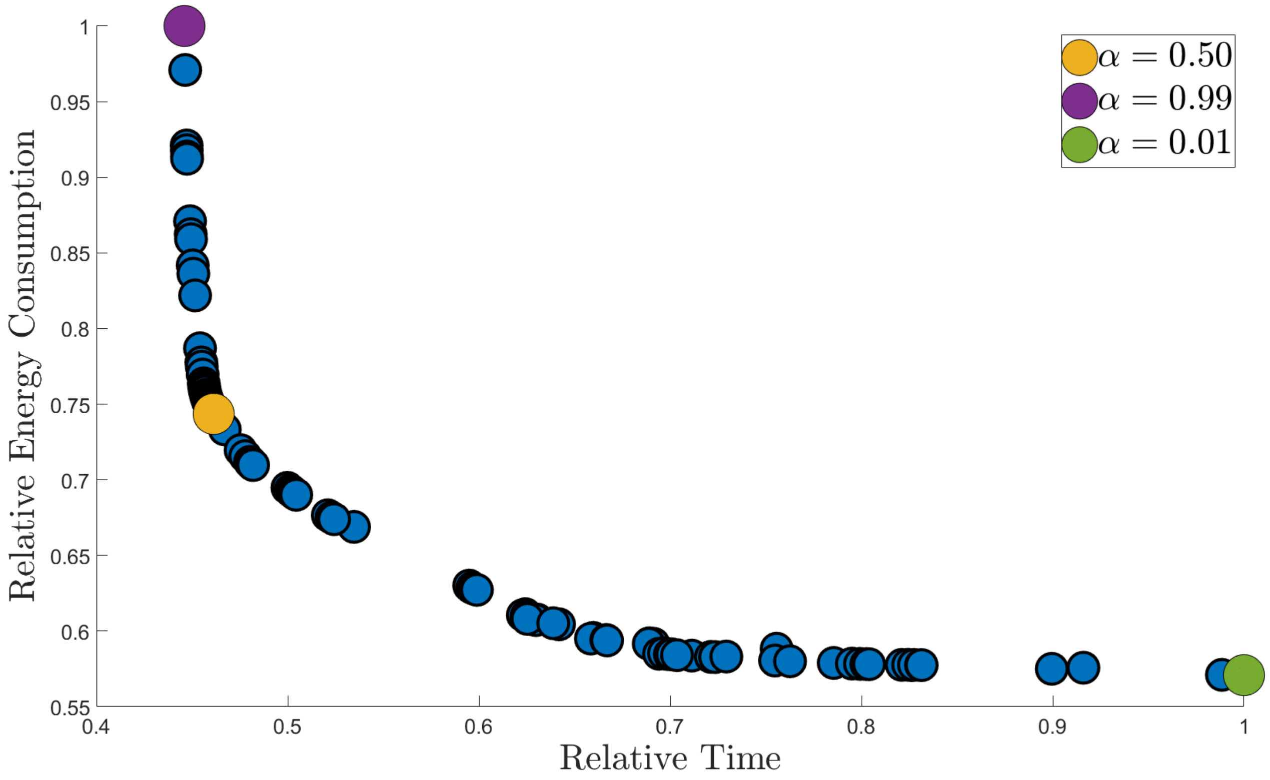
$$\begin{aligned} \text{s.t.} \quad & \dot{x}(t) = f\big(t, x(t), u(t)\big) \\ & z_t(t) \geq P_t(t) \qquad z_h(t) \geq P_h(t) \\ & z_t(t) \geq \gamma_t P_t(t) \qquad z_h(t) \geq \gamma_h P_h(t) \\ & 0 \leq y_p(t) \leq h - s(x_p(t)) \\ & \vdots \\ & \text{other constraints} \end{aligned}$$

avoidance constraints

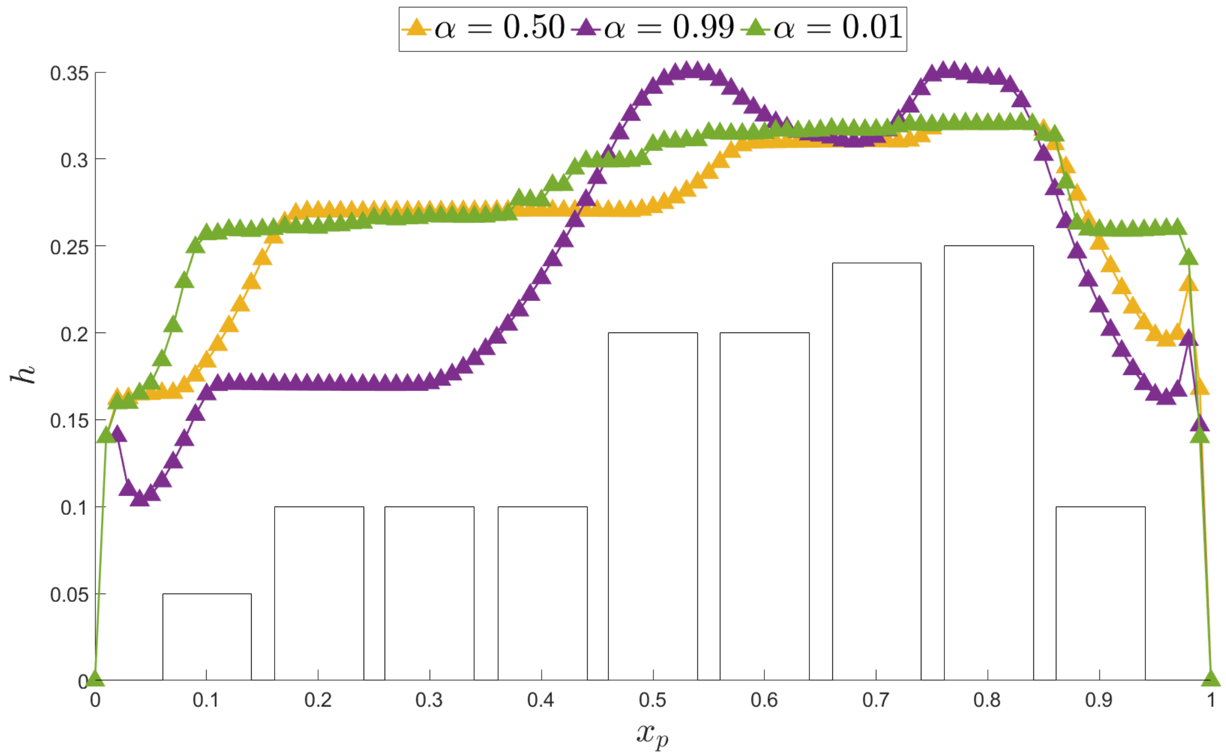
$$\min_{u} \quad \alpha x_1(x_{p_f}) + (1-\alpha)\Big(x_9(x_{p_f}) + x_{10}(x_{p_f})\Big)$$

$$\begin{aligned} \text{s.t.} \quad & x_2 \dot{x}(x_p) = f\Big(x_p, x(x_p), u(x_p)\Big) \\ & z_t(x_p) \geq P_t(x_p) \qquad z_h(x_p) \geq P_h(x_p) \\ & z_t(x_p) \geq \gamma_t P_t(x_p) \qquad z_h(x_p) \geq \gamma_h P_h(x_p) \\ & 0 \leq y_p(x_p) \leq h - s(x_p(x_p)) \\ & \vdots \\ & \text{other constraints} \end{aligned}$$

Example

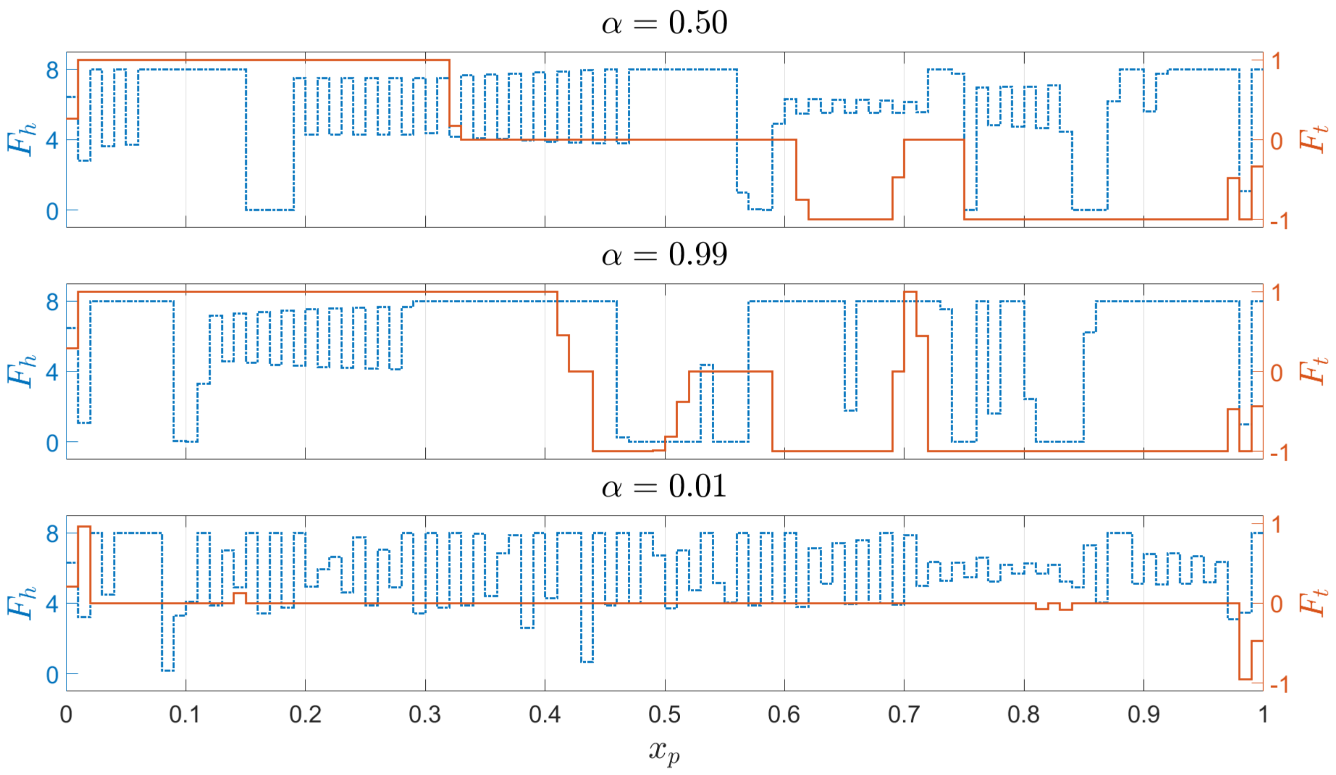


$\alpha = 0.50$
$\alpha = 0.99$
$\alpha = 0.01$
Relative Energy Consumption
Relative Time

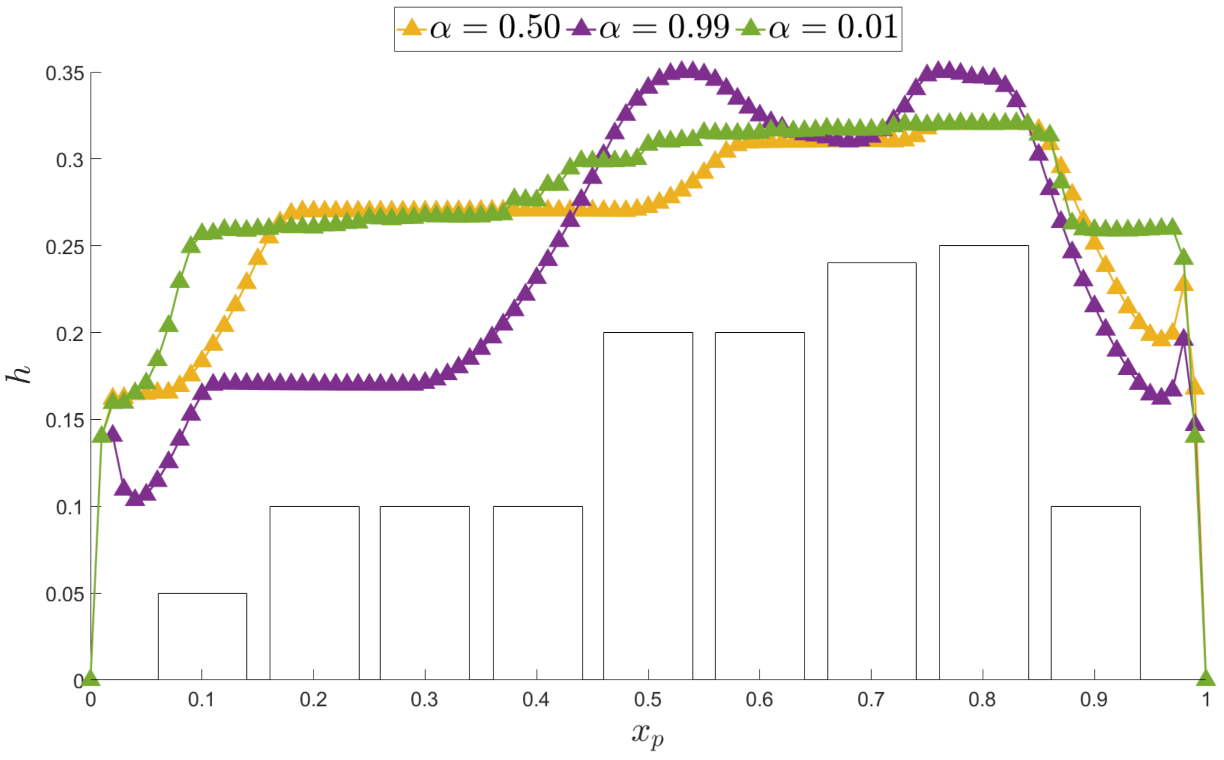


$\alpha = 0.50$ $\alpha = 0.99$ $\alpha = 0.01$
h
x_p

Example

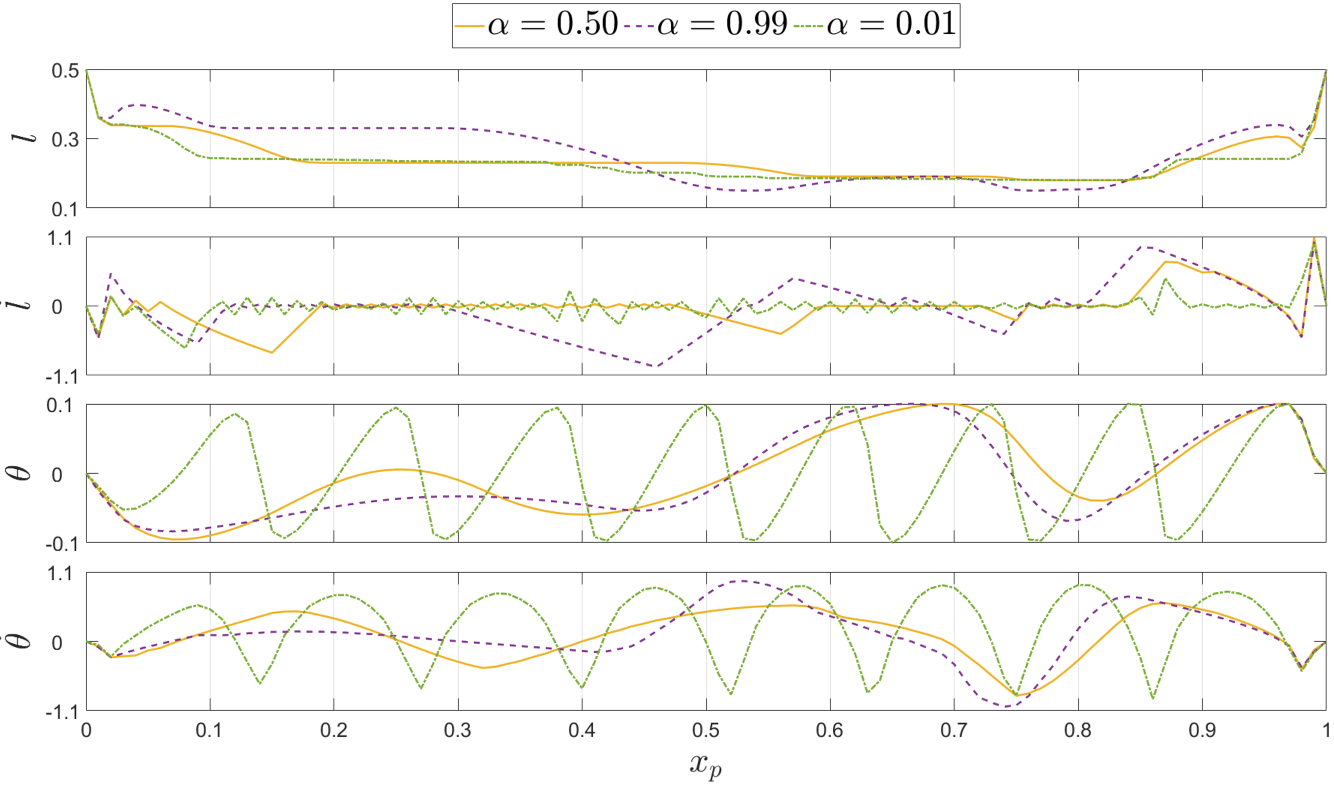


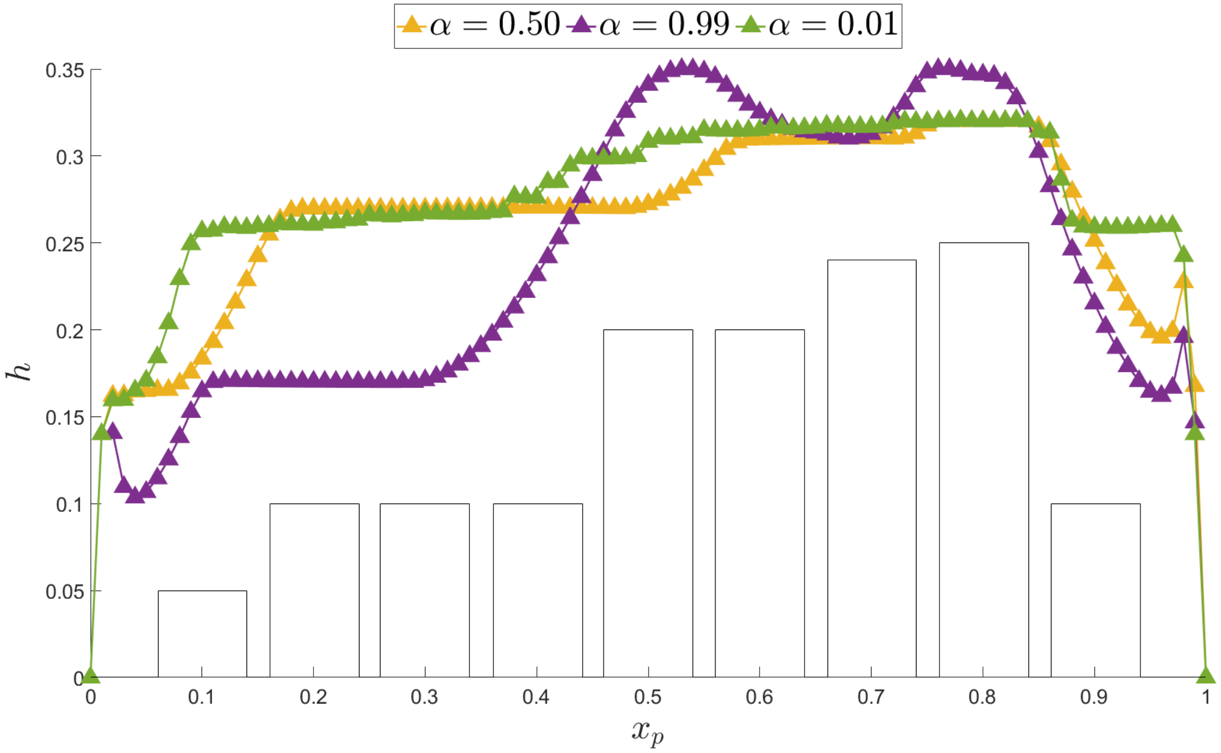
$\alpha = 0.50$
$\alpha = 0.99$
$\alpha = 0.01$
F_h
F_t
x_p



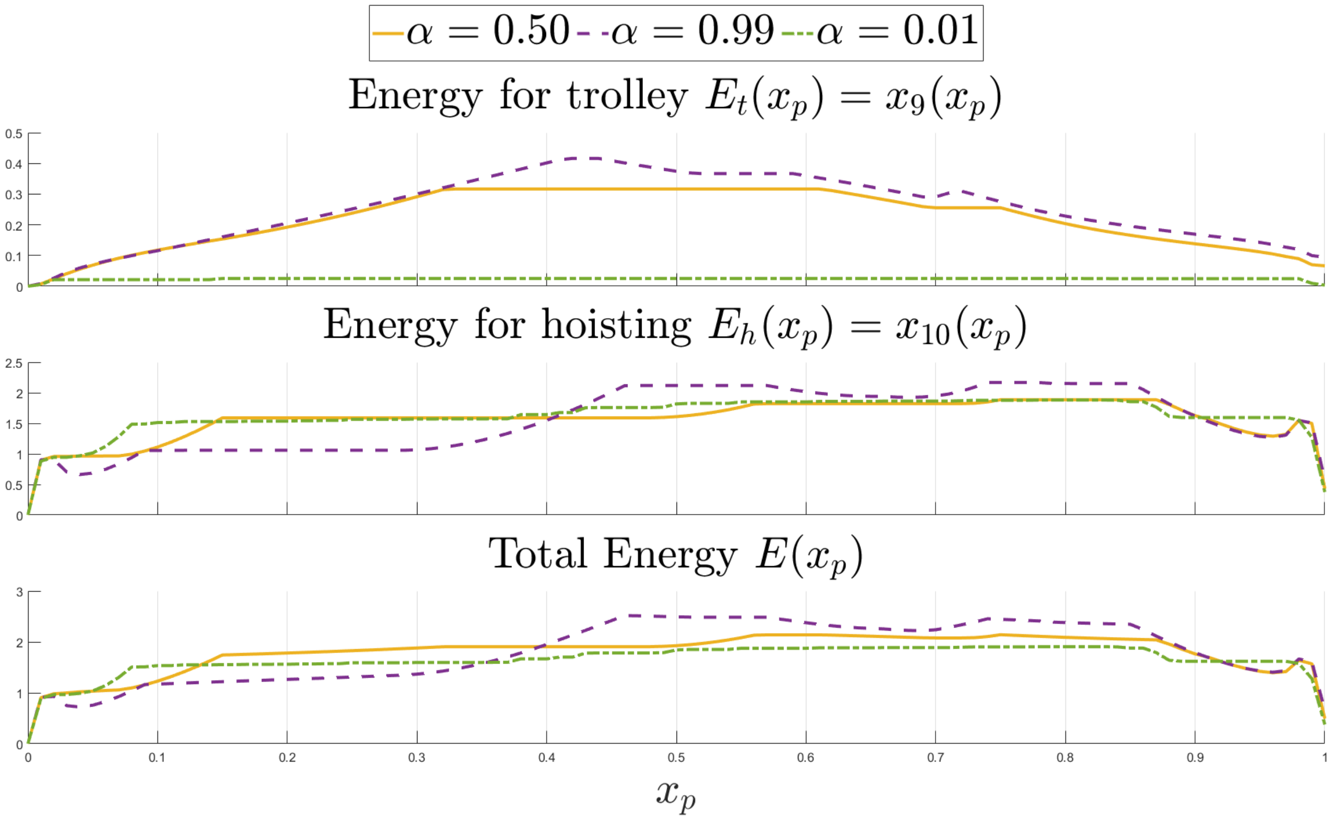
$\alpha = 0.50$ $\alpha = 0.99$ $\alpha = 0.01$
h
x_p

Example

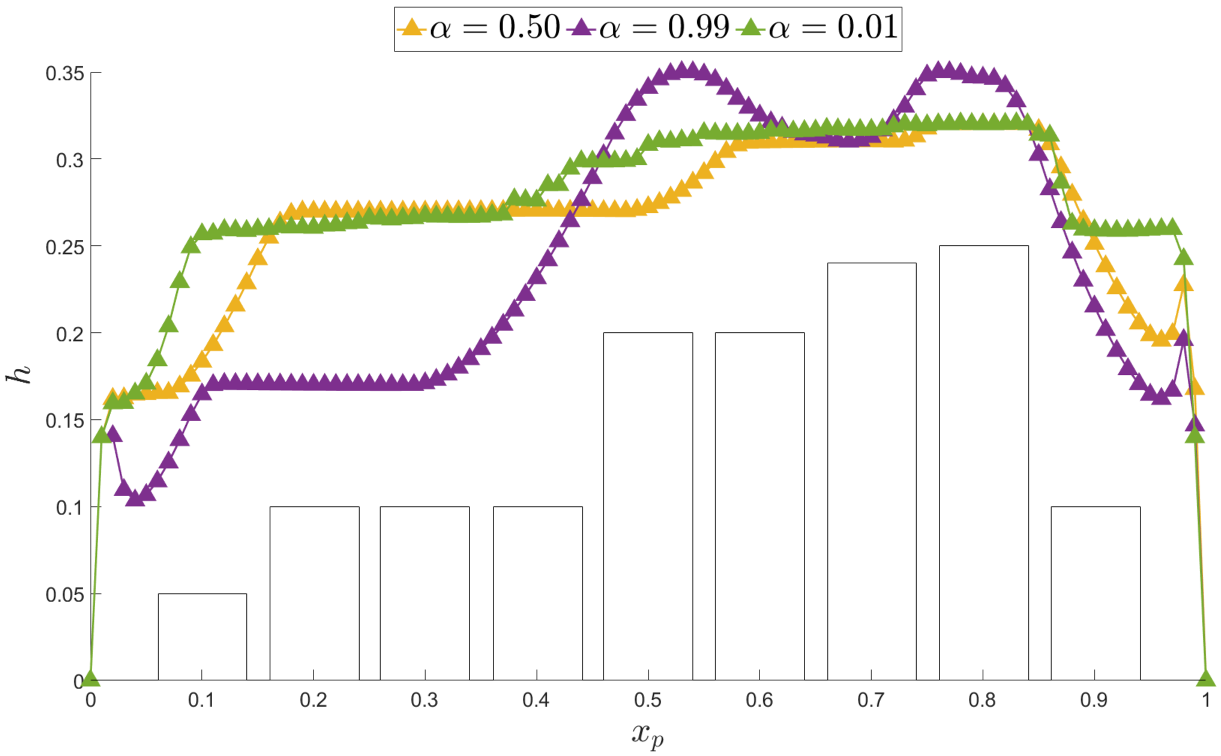




Example



$\alpha = 0.50$ $\alpha = 0.99$ $\alpha = 0.01$
Energy for trolley $E_t(x_p) = x_9(x_p)$
Energy for hoisting $E_h(x_p) = x_{10}(x_p)$
Total Energy $E(x_p)$
x_p



$\alpha = 0.50$ $\alpha = 0.99$ $\alpha = 0.01$
h
x_p

About the reformulations

Advantages

- We no longer define an explicit function $s(x_p)$, but simply function values.
- Description of stack heights becomes trivial and easy to represent numerically.
- By standard epigraph reformulations, the model accounting for energy regeneration can be put in a form with improved numerical properties

About the reformulations

Limitations

- Payload moving monotonically in one direction which enforces no sway condition.
- Unifor discretization may lead to not capturing dynamics of the system in the beginning and the end.
- Dynamics and cost function remain non-convex and may lead to a solution at a local minimum.

About energy consumption

- The model for regeneration is still simple and a more complex one could be incorporated.
- By studying this small example, one note that there’s room for energy reduction with minor increase in the loading time.

Summary

- **Objective:** Trade-off time and energy when loading a container ship.
- **Challenge:** Avoid collision with container stacks.
- **Idea:** Variable change in an optimal control problem and standard epigraph formulations.
- **Outcome:** Non-convex container avoidance constraints become linear bound constraints and energy consumption can be reduced with minor increase in loading time.

Thank you!

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