Identification of Nonlinear Marine Systems

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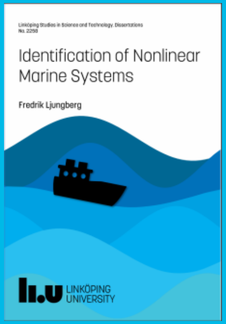

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Why do we want automated ships?

- Safety
  - ~90 % of ship accidents are caused by human error*
- Cutting costs
  - ~80 % of the volume of international trade in goods is carried by sea**
- Sustainability
  - ~3 % of global GHG emissions are from shipping***

Image courtesy of the Earth Science and Remote Sensing Unit, NASA Johnson Space Center

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*Annual Overview of Marine Casualties and Incidents 2021, EMSA
**Review of Martime Transport 2021, UN
***Fourth Greenhouse Gas Study, 2020, IMO

Why are models needed?

- Model-based control
- Simplified development
- Other driver-assistance functionality

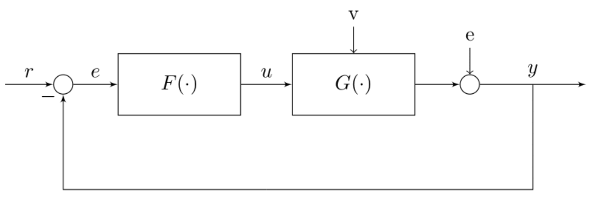


Image courtesy of ABB

Ship commissioning

Experiments for:

- Controller tuning
- Testing of system functionality
- Data collection for system identification

Circle test

Challenges:

- Associated with high costs
  - → Experiment time needs to be kept short
- Planned well in advance
  - → Weather conditions can vary

Zig-zag maneuver

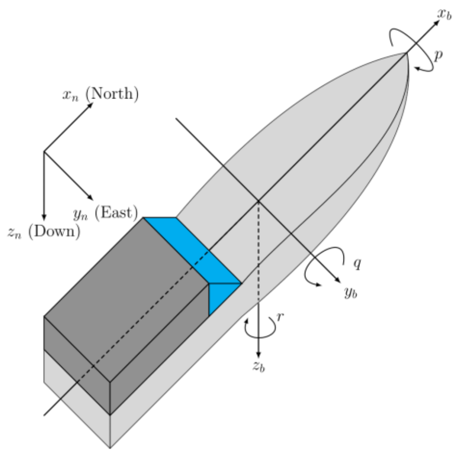
Ship modelling

Disturbed equations of motion

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\boldsymbol{\eta})\boldsymbol{\nu}$$

$$\underbrace{\boldsymbol{M}_{RB}\,\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{Rigid-body effects}} + \underbrace{\boldsymbol{M}_A\,\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r}_{\text{Hydrodynamics}} + \underbrace{\boldsymbol{F}(\boldsymbol{\nu}_q)\boldsymbol{\nu}_q}_{\text{Aerodynamics}} = \boldsymbol{\tau}_{act}$$

Variable	Description
$\boldsymbol{\eta} = [x_n, y_n, z_n, \phi, \theta, \psi]$	Generalized position
$\boldsymbol{\nu} = [u, v, w, p, q, r]$	Generalized velocity
$\boldsymbol{\nu}_c = \boldsymbol{R}^{-1}(\eta)\,\boldsymbol{\nu}_{c,n}$ $\boldsymbol{\nu}_w = \boldsymbol{R}^{-1}(\eta)\,\boldsymbol{\nu}_{w,n}$	Current velocity in *b*-frame Wind velocity in *b*-frame
$\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c$ $\boldsymbol{\nu}_q = \boldsymbol{\nu} - \boldsymbol{\nu}_w$	Relative velocity to current Relative velocity to wind



Problem formulation

Estimate the parameter vector $\boldsymbol{\theta}_0$ in

$$\boldsymbol{x}(k+1) = \boldsymbol{f}_{SOM}\left(\begin{bmatrix}\boldsymbol{x}(k) + \boldsymbol{R}(k)\boldsymbol{v}(k) \\ \boldsymbol{u}(k)\end{bmatrix}, \boldsymbol{\theta}_0\right) + \boldsymbol{w}(k)$$
$$\boldsymbol{y}(k) = \boldsymbol{x}(k) + \boldsymbol{e}(k)$$
$$\boldsymbol{Y}_R(k) = \boldsymbol{R}(k) + \boldsymbol{E}_R(k)$$

- $\boldsymbol{f}_{SOM}(\cdot)$ is a second-order modulus function
- $\boldsymbol{e}(k), \boldsymbol{E}_R(k)$, $\boldsymbol{v}(k)$ and $\boldsymbol{w}(k)$ are disturbance signals
- $\boldsymbol{u}(k)$ is the control signal
- $\boldsymbol{x}(k)$ is the system state
- $\boldsymbol{R}(k)$ is a time-varying matrix
- $\mathbf{y}(k)$ and $\mathbf{Y}_{\mathrm{R}}(k)$ are measured signals

Definition: A second-order modulus function is a function $\boldsymbol{f}_{SOM} : \mathbb{R}^{n_x+n_\theta} \rightarrow \mathbb{R}^{n_f}$ that can be written as

$$\boldsymbol{f}_{SOM}(\boldsymbol{x}, \boldsymbol{\theta}) = \boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{x})\, \boldsymbol{\theta}$$

where each element of the $n_\theta \times n_f$ matrix $\boldsymbol{\Phi}(\mathbf{x})$ is on one of the forms x_i, $|x_i|$, $x_i x_j$ or $x_i|x_j|$ for $i, j \leq n_x$ or zero and $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ is a vector of coefficients.

Instrumental variable (IV) method

$$\hat{\boldsymbol{\theta}} = \text{sol}\left\{\frac{1}{N}\sum_{k=1}^{N} \boldsymbol{Z}(k)(\mathbf{y}(\mathrm{k}) - \boldsymbol{\Phi}^{T}(k)\boldsymbol{\theta}) = 0\right\}$$

- The IV method is a common way of dealing with measurement uncertainty.
- $\text{sol}\{f(x) = 0\}$ is the solution to $f(x) = 0$.
- $\boldsymbol{Z}(k)$ is called the *instrument matrix and* should be correlated with the system state but uncorrelated with the system disturbances.
- The method coincides with LS if $\boldsymbol{Z}(k) = \boldsymbol{\Phi}(k)$.

Dealing with disturbances

- **Scenario 1**: $E\{\boldsymbol{v}(k)\} = 0$.
  - Interpretation: Only bursty wind gusts (or currents).
  - Solution: Excitation offset and zero-mean instruments.
- **Scenario 2**: $E\{\boldsymbol{v}(k)\} = \bar{\boldsymbol{v}}$ and $|\boldsymbol{v}(k)| < |\boldsymbol{x}(k)|$ for $k = 1, \dots, N$.
  - Interpretation: Wind (or current) moves *slower* than the ship and has an unknown first-order moment.
  - Solution: Excitation offset, zero-mean instruments and extended predictor with orientation-dependent regressors.

$$\hat{\boldsymbol{y}}(k|\boldsymbol{\theta}, \boldsymbol{\rho}) = (y(k)|y(k)|, \dots, u(k), \varphi_1(\boldsymbol{Y_R}(k)), \varphi_2(\boldsymbol{Y}_R(k)), \dots)(\theta_1, \dots, \theta_n, \rho_1, \rho_2, \dots)^T$$

- **Scenario 3**: $E\{\boldsymbol{v}(k)\} = \bar{\boldsymbol{v}}$ and $|\boldsymbol{v}(k)| \nless |\boldsymbol{x}(k)|$ for $k = 1, \dots, N$.
  - Interpretation: Wind (or current) occasionally moves *faster* than the ship and has an unknown first-order moment.
  - Solution: Excitation offset, zero-mean instruments and auxiliary disturbance measurement: $\boldsymbol{y_{aux}}(k) = \boldsymbol{R}(k)\mathbf{v}(k) + \boldsymbol{e}_{aux}(k)$.

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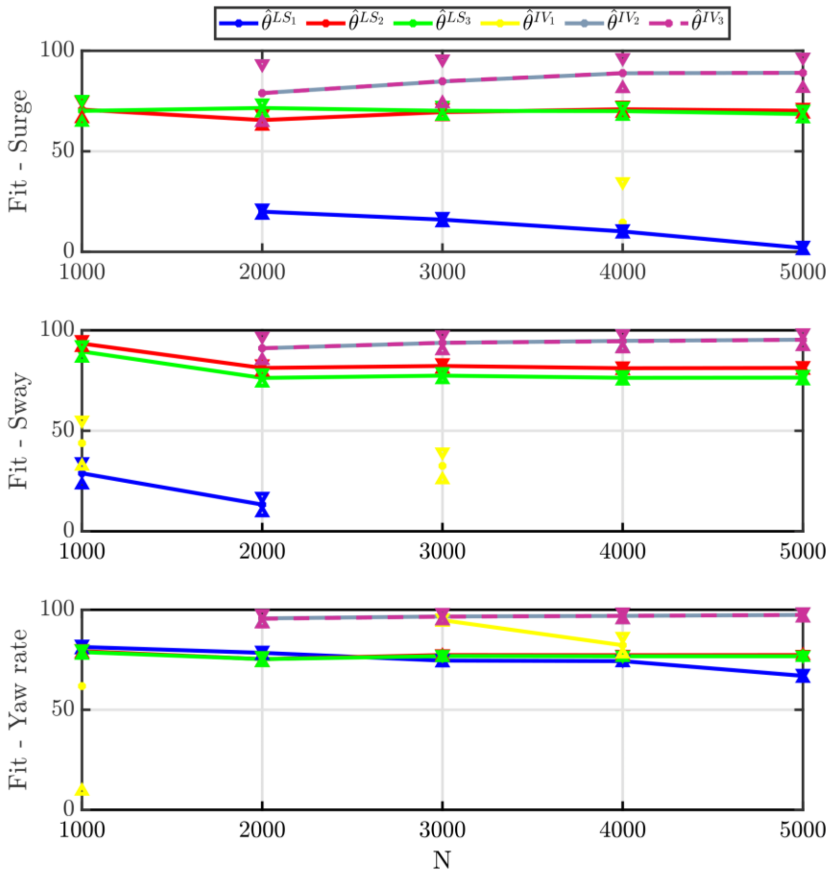
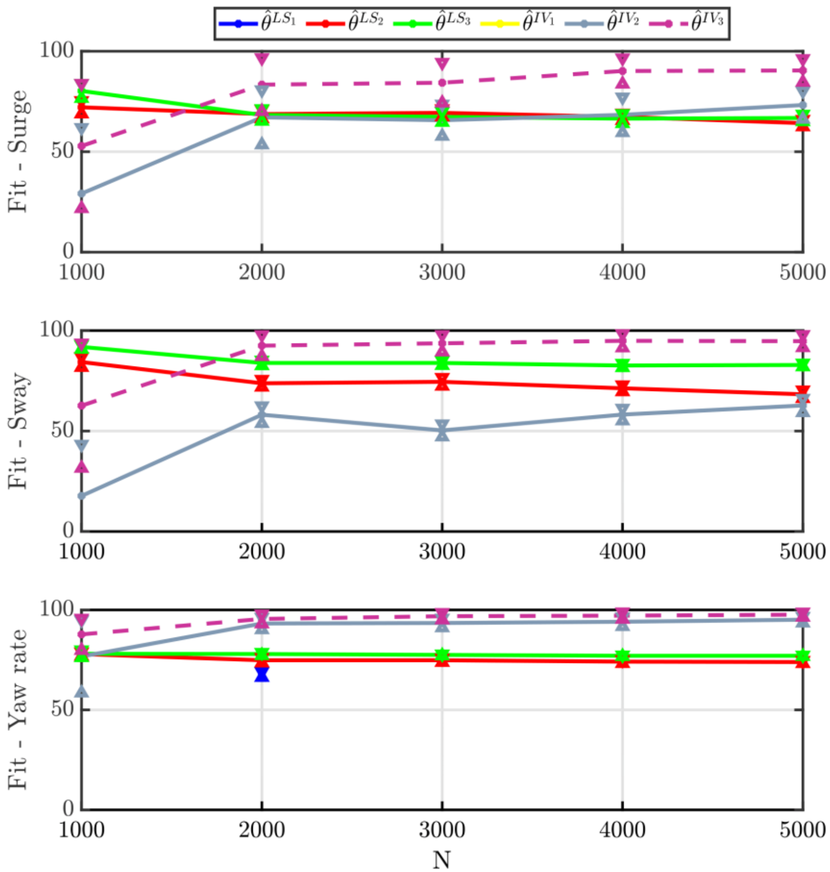
Results I/II (simulated data)



Setup I/II:
Low wind speed
Low current speed

Estimator	ψ-dep. reg.	Wind meas.
$\hat{\theta}_N^{LS_1}$	-	-
$\hat{\theta}_N^{LS_2}$	✓	-
$\hat{\theta}_N^{LS_3}$	✓	✓
$\hat{\theta}_N^{IV_1}$	-	-
$\hat{\theta}_N^{IV_2}$	✓	-
$\hat{\theta}_N^{IV_3}$	✓	✓

Results II/II (simulated data)



Setup II/II:

High wind speed

Low current speed

Estimator	ψ-dep. reg.	Wind meas.
$\hat{\theta}_N^{LS_1}$	-	-
$\hat{\theta}_N^{LS_2}$	✓	-
$\hat{\theta}_N^{LS_3}$	✓	✓
$\hat{\theta}_N^{IV_1}$	-	-
$\hat{\theta}_N^{IV_2}$	✓	-
$\hat{\theta}_N^{IV_3}$	✓	✓

Informative mix of sub-experiments

Goal: Find $\boldsymbol{u}(k)$ that maximizes the determinant of

$$\boldsymbol{G}(N) \triangleq \frac{1}{2}\frac{d^2}{d\theta^2}V_N^{IV}(\theta) = \underbrace{\left[\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\Phi}(k)\boldsymbol{Z}^T(k)\right]}_{\triangleq\, \boldsymbol{\Gamma}(N)}\left[\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\Phi}(k)\boldsymbol{Z}^T(k)\right]^T$$

Solution in steps:

1. Choose candidate signals $\boldsymbol{u}_1(k), \dots, \boldsymbol{u}_Q(k)$ (standard maneuvers).
2. Estimate $\bar{\boldsymbol{\Gamma}}_1, \dots, \bar{\boldsymbol{\Gamma}}_Q$ (information matrices) based on simulation experiments with a nominal model or based on initial experiments with the real platform.
3. Assume that $\boldsymbol{u}_1(k), \dots, \boldsymbol{u}_Q(k)$ are to be applied in sequence and solve

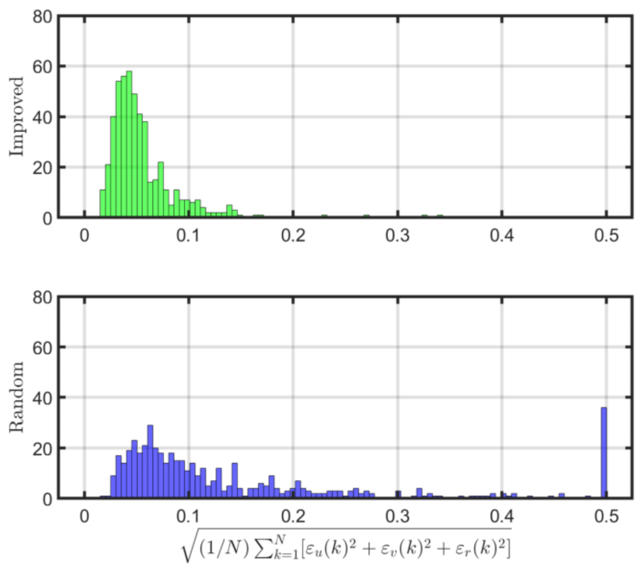
$$(N_1^*, \dots, N_Q^*) = \underset{N_1,\dots,N_Q}{\operatorname{argmax}} \ \log|\det(\boldsymbol{T})|$$

$$\text{s.t.} \quad \begin{aligned} &\boldsymbol{T} = N_1\bar{\boldsymbol{\Gamma}}_1 + \dots + N_Q\bar{\boldsymbol{\Gamma}}_Q \\ &N_1 + \dots + N_Q = N \\ &N_q \geq 0, \qquad q = 1, \dots, Q \end{aligned}$$

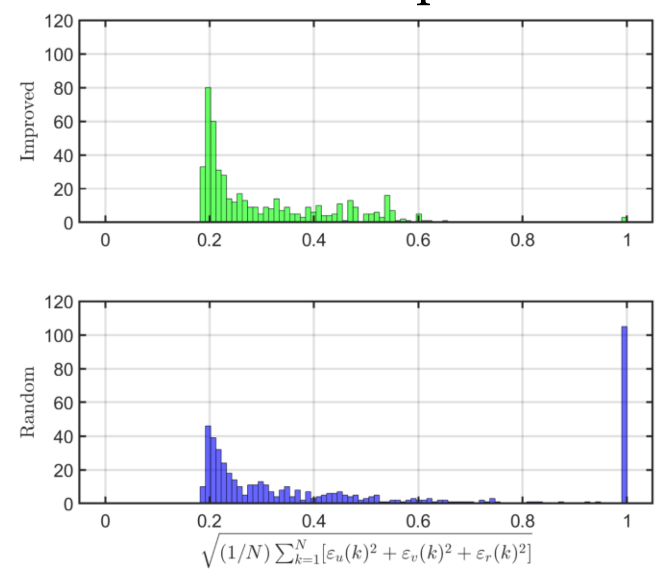
Results

- 11 different candidate input signals, $\boldsymbol{u}_1(k), \ldots, \boldsymbol{u}_{11}(k)$.
- Data collected from 55 sub-experiments (5 of each type).
- Improved design: Pick 6 sub-experiments that work well together.
- Random design: Pick 6 sub-experiments at random.
- Evaluation by comparing simulation accuracy of resulting models.

Simulated data



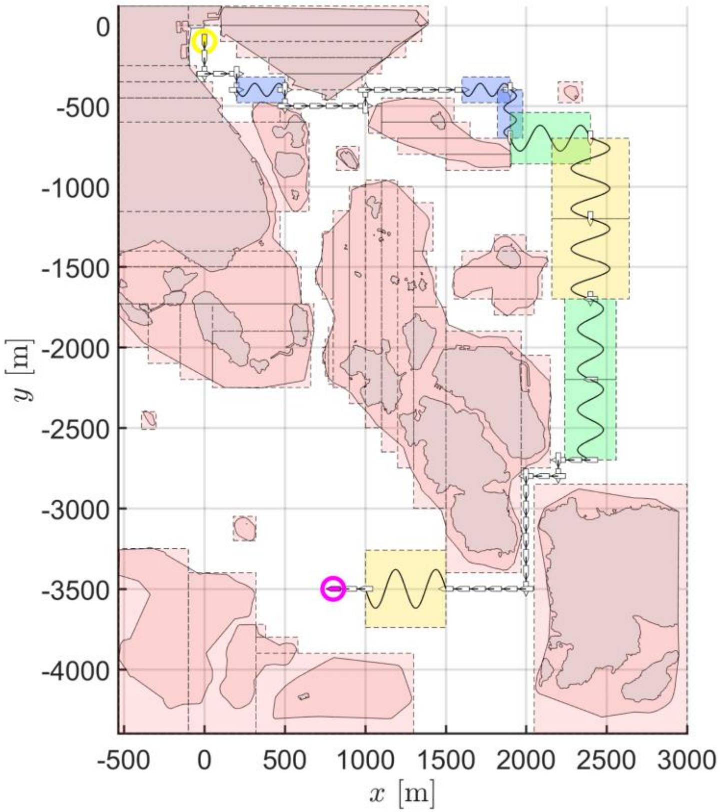
Model-ship data



Obtaining an experiment trajectory

$$
\begin{aligned}
\underset{\{m_k\}_{k=0}^{M-1},M}{\text{minimize}} \quad & \sum_{k=0}^{M-1} J(m_k) \\
\text{s.t.} \quad & \mathbf{x}_0 = \mathbf{x}_s, \quad \mathbf{x}_M = \mathbf{x}_f, \\
& \mathbf{x}_{k+1} = f(\mathbf{x}_k, m_k), \\
& m_k^p \in \Big\{ \underbrace{m^1, \cdots, m^Q}_{\text{informative}}, \underbrace{m^{Q+1}, \cdots, m^{Q+B}}_{\text{basic}} \Big\} \\
& c(m_k, \mathbf{x}_k) \in \mathcal{X}_{\text{free}}.
\end{aligned}
$$

- The signals, $\boldsymbol{u}_1(k), \ldots, \boldsymbol{u}_{\boldsymbol{Q}}(k)$, are used to form motion primitives.
- The ratios found in the previous step are respected by augmenting the state vector with motion-primitive counters.



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Conclusions

Main Results

- A framework for obtaining consistent estimators for the parameters of second-order modulus models that are robust to data having been collected in the presence of
  - Measurement uncertainty.
  - Environmental disturbances.
- A method for improved experiment design for ships that gives informative data as well as a spatially feasible trajectory.

Acknowledgments

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Thank you!

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