Time- and energy-optimal control of cranes subject to geometric constraints

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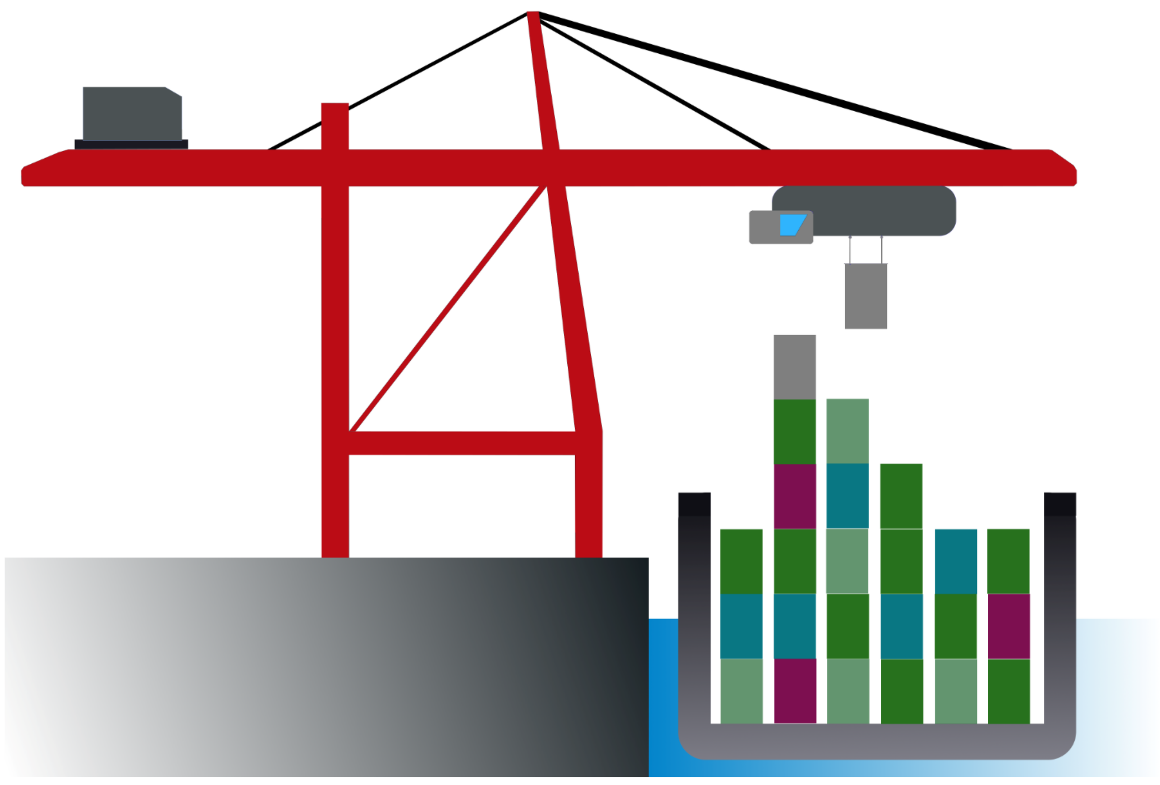
Summary

Objective: Trade-off time and energy when loading a container ship.

Challenge: Avoid collision with container stacks.

Trick: Variable change in an optimal control problem and standard epigraph formulations.

Conclusion: Energy consumption can be reduced with minor increase in the loading time.



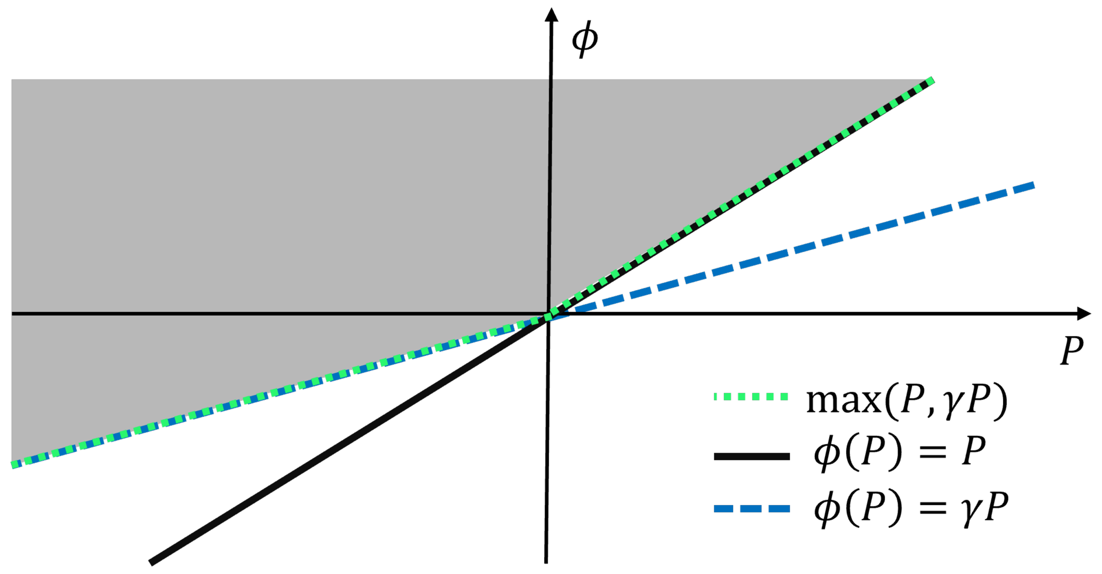
Preliminaries

The nonlinear state-space representation in the original form is

$$\dot{x}(t) = f(t, x(t), u(t)).$$

Taking regeneration into account, the energy consumed is

$$E(t) = \int_0^t \max(P(\tau), \gamma P(\tau)) d\tau.$$



Problem reformulation

1. To deal with a non-smooth integrand in the cost function:

$$\begin{aligned} &\min_u && \int_0^{t_f} z(t)dt \\ &\text{subject to} && z(t) \geq P(t) \\ &&& z(t) \geq \gamma P(t) \end{aligned}$$

with $z(t)$ as an upper bound in an epigraph formulation.

2. Time discretization would lead to

$$0 \leq y_p(t^k) \leq h - s(x_p(t^k)). \quad ✗$$

So, use spatial derivatives deal to with the collision avoidance

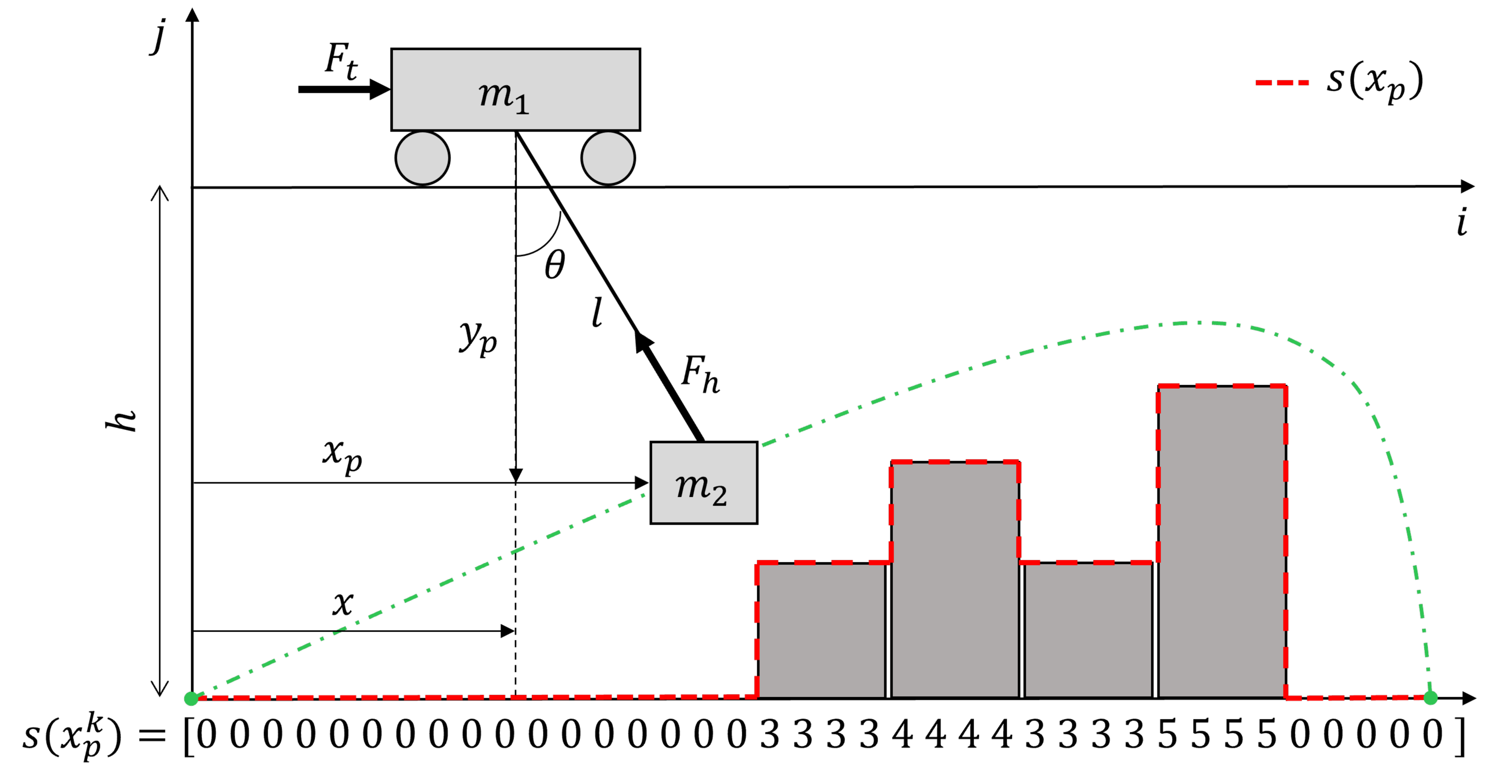
$$\dot{x}_1 = \frac{dx_1}{dt} = x_2 \implies \frac{dt}{dx_1} = \frac{1}{x_2}, \frac{dx_2}{dx_1} = \dots$$

and the collision avoidance becomes

$$0 \leq y_p(x_p^k) \leq h - s(x_p^k). \quad ✓$$

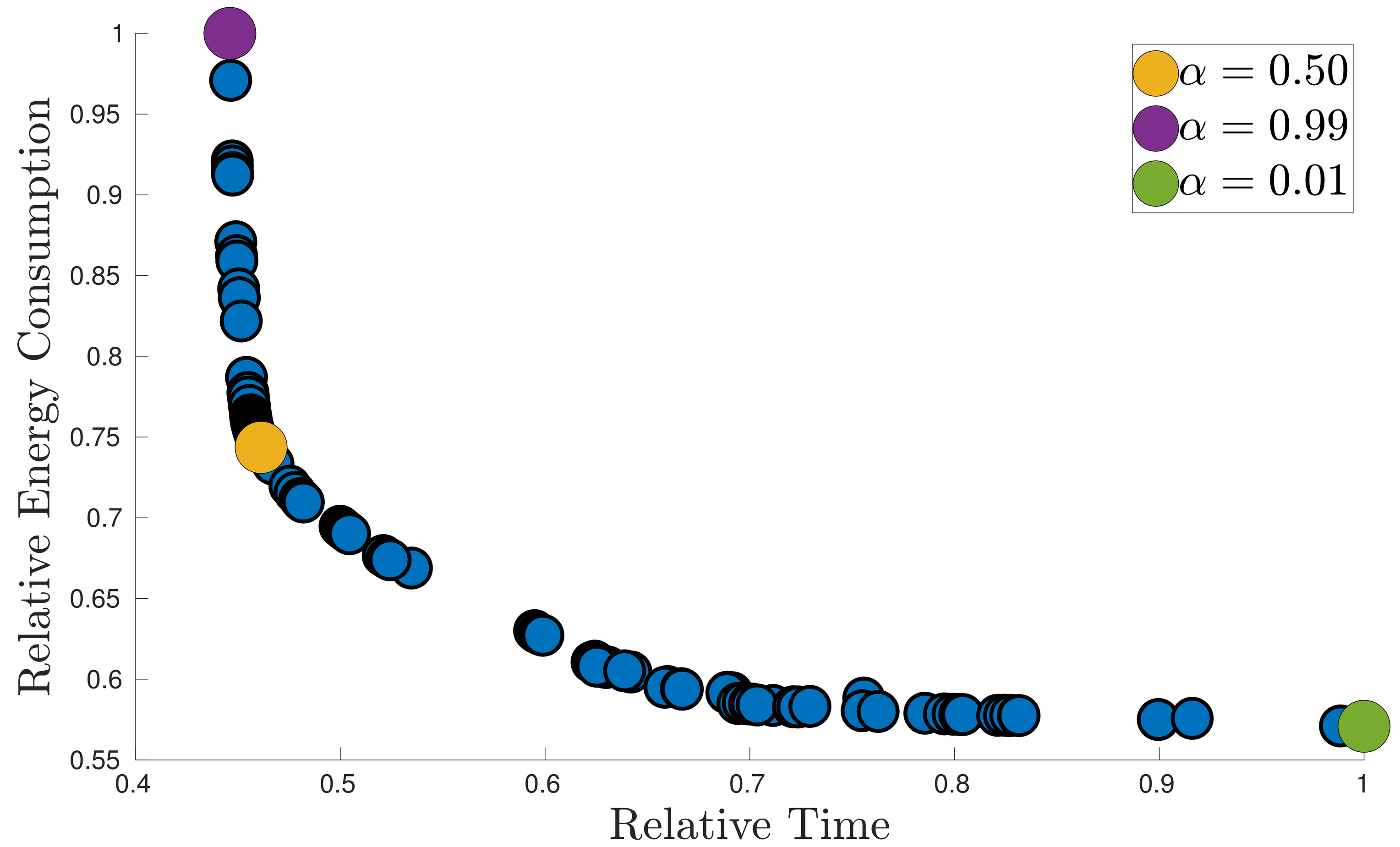
The optimization problem

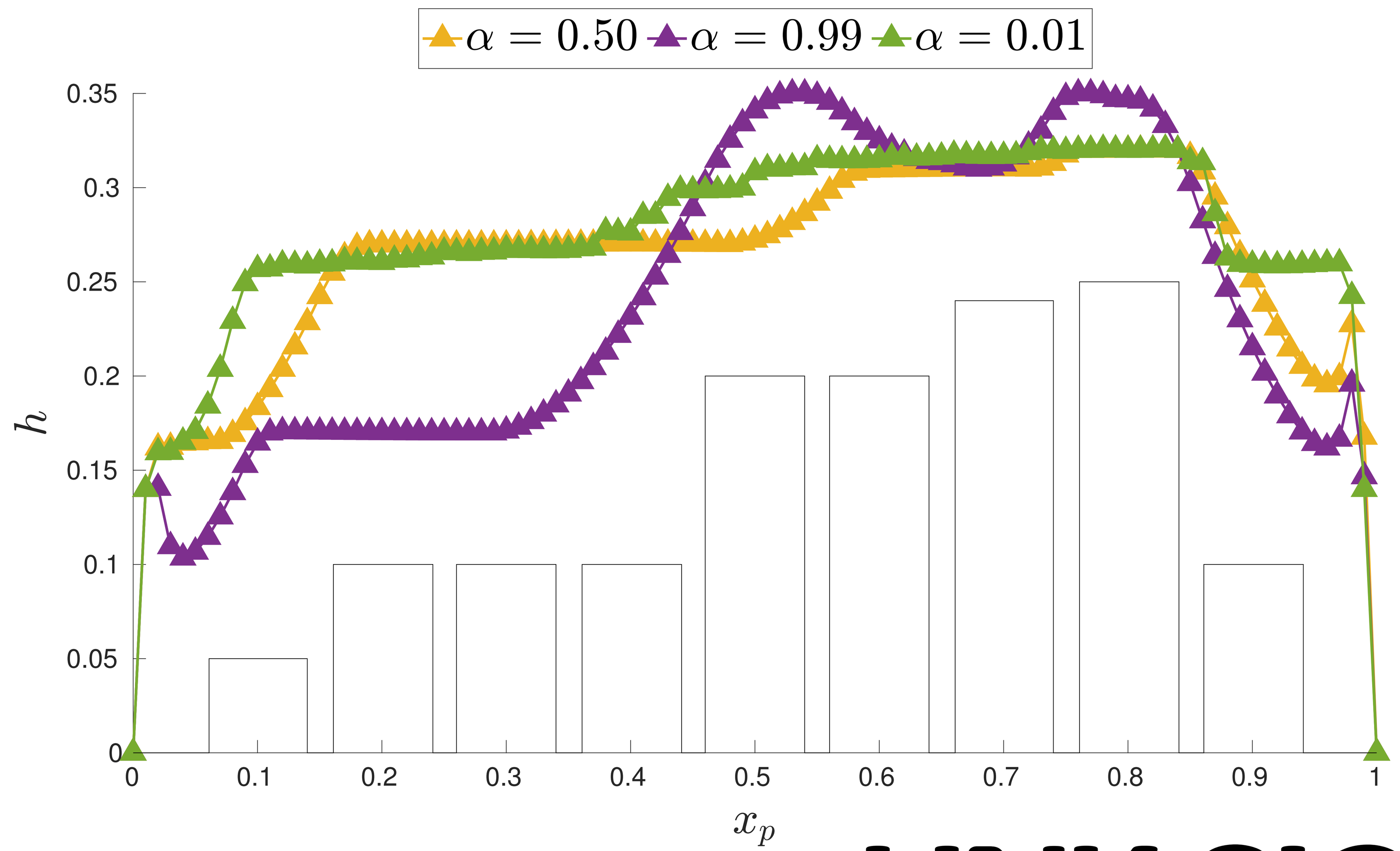
$$\begin{aligned} &\underset{u}{\text{minimize}} && J = \Psi(E, t_f) \\ &\text{subject to} && x_2 \dot{x}(x_p) = f(x_p, x(x_p), u(x_p)) \\ &&& 0 \leq y_p(x_p) \leq h - s(x_p) \leftarrow \text{container constraints} \\ &&& z_t(t) \geq P_t(x_p) \\ &&& z_t(t) \geq \gamma_t P_t(x_p) \\ &&& z_h(t) \geq P_h(x_p) \\ &&& z_h(t) \geq \gamma_h P_h(x_p) \\ &&& \vdots \\ &&& \text{other constraints} \end{aligned}$$



Simulation example

The time and energy trade-off is made through $0 \leq \alpha \leq 1$.





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