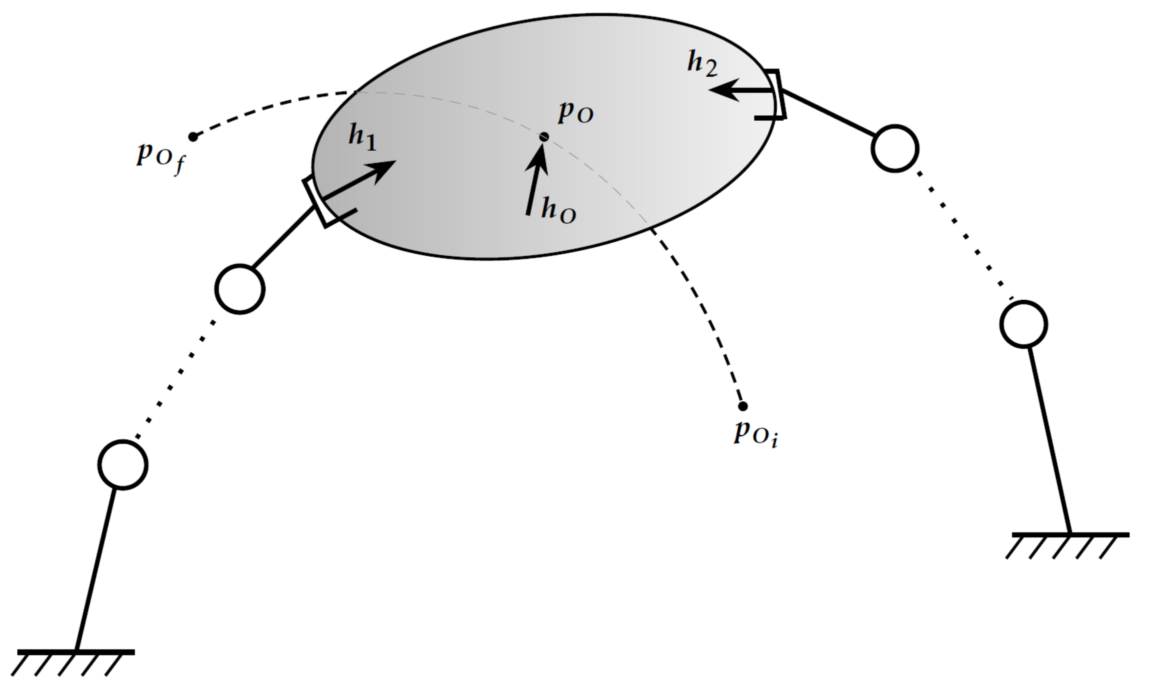
Time-Optimal Cooperative Path Tracking for Multi-Robot Systems

Hamed Haghshenas (hamed.haghshenas@liu.se), Anders Hansson, Mikael Norrlöf



Contributions

- Formulating the time-optimal cooperative path tracking problem as a convex optimization problem and subsequently as an SOCP.
- Extension to different types of contact, namely *point contact with friction* and *soft-finger*.

Time-optimal cooperative path tracking

Problem

- An object is rigidly grasped by multiple manipulators.
- Objective: minimize the traversal time required to move the object with a desired orientation along a prescribed path.
- The path and the orientation are given as functions of a scalar path coordinate.
- Determine the timing law along the path.

Dynamics

Manipulator: $M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i - J_i^T(q_i)h_i$

Object: $M_o(x_o)\dot{v}_o + C_o(x_o,\dot{x}_o)v_o + g_o = h_o$

Relationship between forces h_o and h_i

$$h_o = G(s)h, \qquad h = [h_1^T, \dots, h_N^T]^T$$

Change of variables

$$a(s) = \ddot{s}(t), \quad b(s) = \dot{s}(t)^2, \quad b'(s) = 2a(s),$$

Convex formulation

$$\begin{aligned}
\underset{\substack{a(\cdot),b(\cdot),\tau_i(\cdot),\\ h_i(\cdot),h_O(\cdot)}}{\text{minimize}} \quad & \int_0^1 \frac{1}{\sqrt{b(s)}}ds \\
\text{subject to} \quad & \tau_i(s) = m_i(s)a(s) + c_i(s)b(s) + g_i(s) \\
& \quad + J_i(s)^T h_i(s), \quad i \in \mathcal{N}, \\
& m_O(s)a(s) + c_O(s)b(s) + g_O = h_O(s), \\
& b'(s) = 2a(s), \ \ b(0) = \dot{s}_0^2, \ \ b(1) = \dot{s}_T^2, \\
& b(s) \geq 0, \ \ b(s) \leq \bar{b}(s), \\
& \underline{\tau}_i(s) \leq \tau_i(s) \leq \overline{\tau}_i(s), \quad i \in \mathcal{N}, \\
& \forall s \in [0,1].
\end{aligned}$$

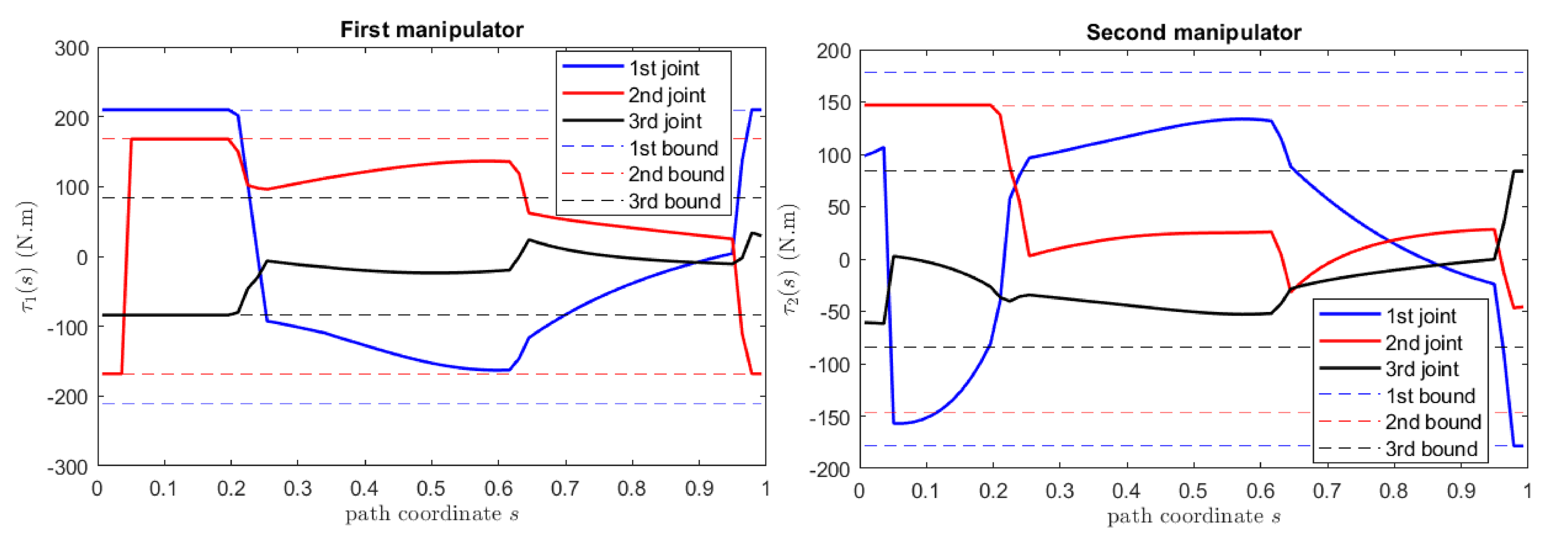


Figure 1: Joint torques $\tau_1(s)$ and $\tau_2(s)$

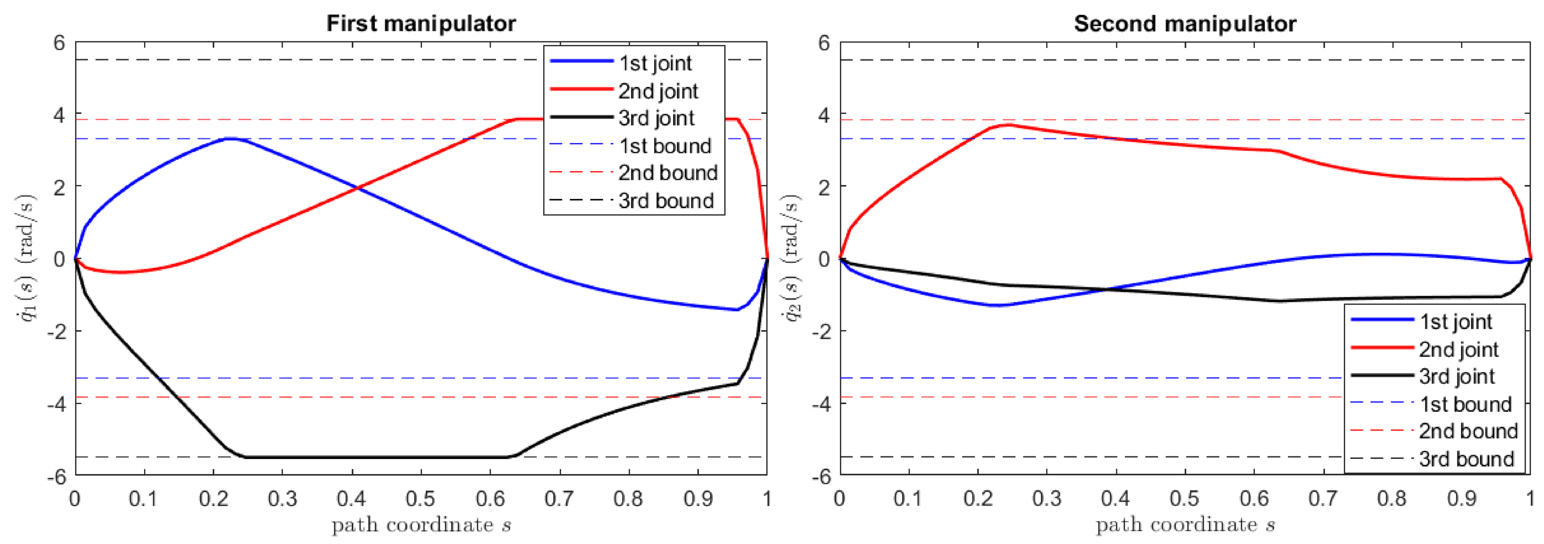
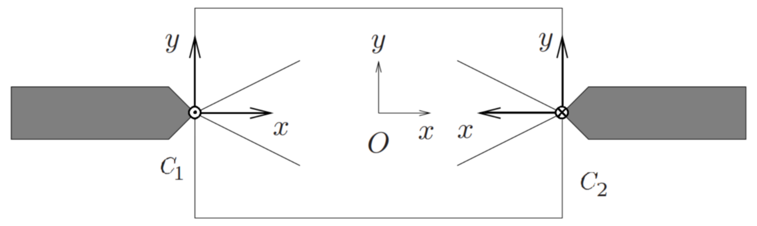


Figure 2: Joint velocities $\dot{q}_1(s)$ and $\dot{q}_2(s)$

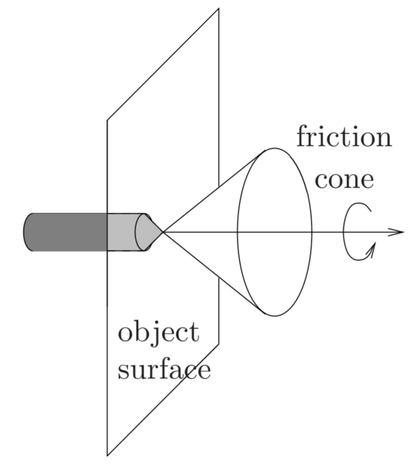
Extension to different types of contact

Point contact with friction: Forces can be exerted in any direction that is within the friction cone for the contact.



$$FC_{c_i} = \left\{ f = [f_x \ f_y]^T \in \mathbb{R}^2 : |f_y| \leq \mu f_x, f_x \geq 0 \right\}.$$

Soft-finger: We allow not only forces to be applied in a cone about the surface normal, but also torques about that normal.



Requirements on grasp

- **Internal force**
  - An internal force is a set of contact forces which result in no net force on the object.
  - Internal forces can be used to insure that contact forces satisfy friction cone constraints.
- **Force-closure grasp**
  - If a grasp can resist any applied wrench on the object, we say that such a grasp is *force-closure*.
  - A key feature of a force-closure grasp is the existence of *internal forces*.

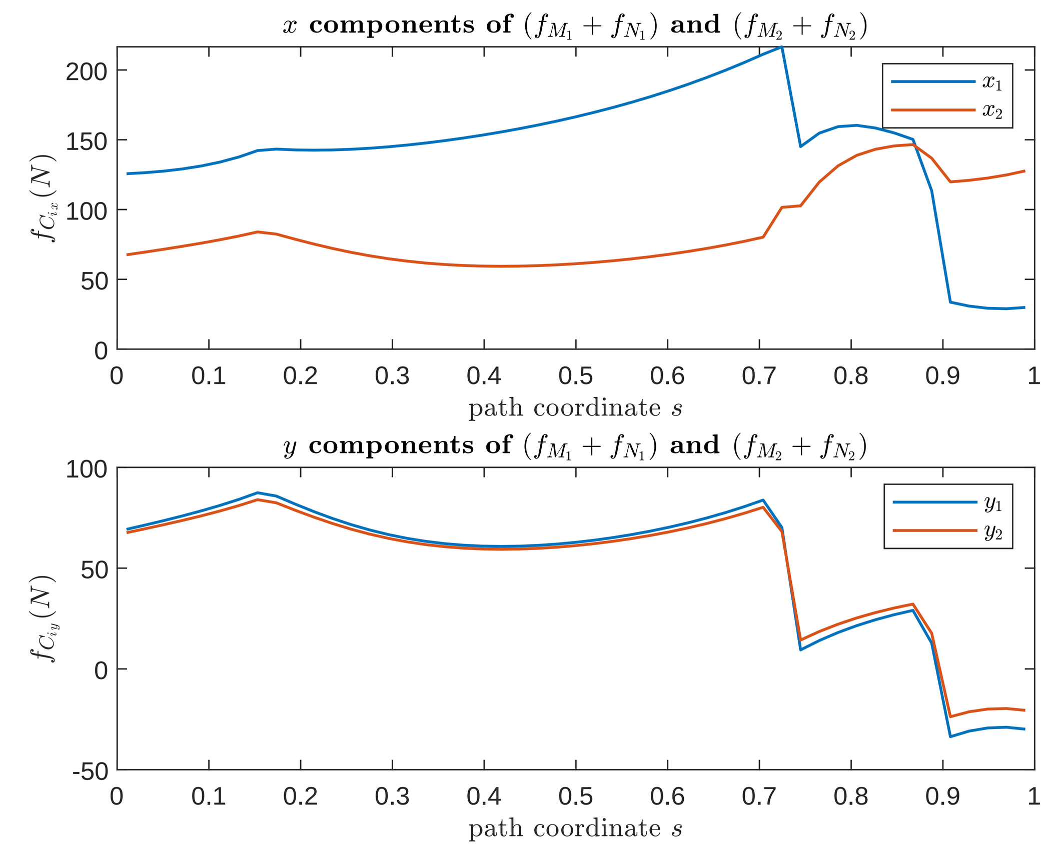


Figure 3: The x and y components of the contact forces.

Future work

- Force-closure test
- Manipulators mounted on a mobile platform

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