Decentralized Target Tracking

Under Partially Known Covariances and Communication Constraints

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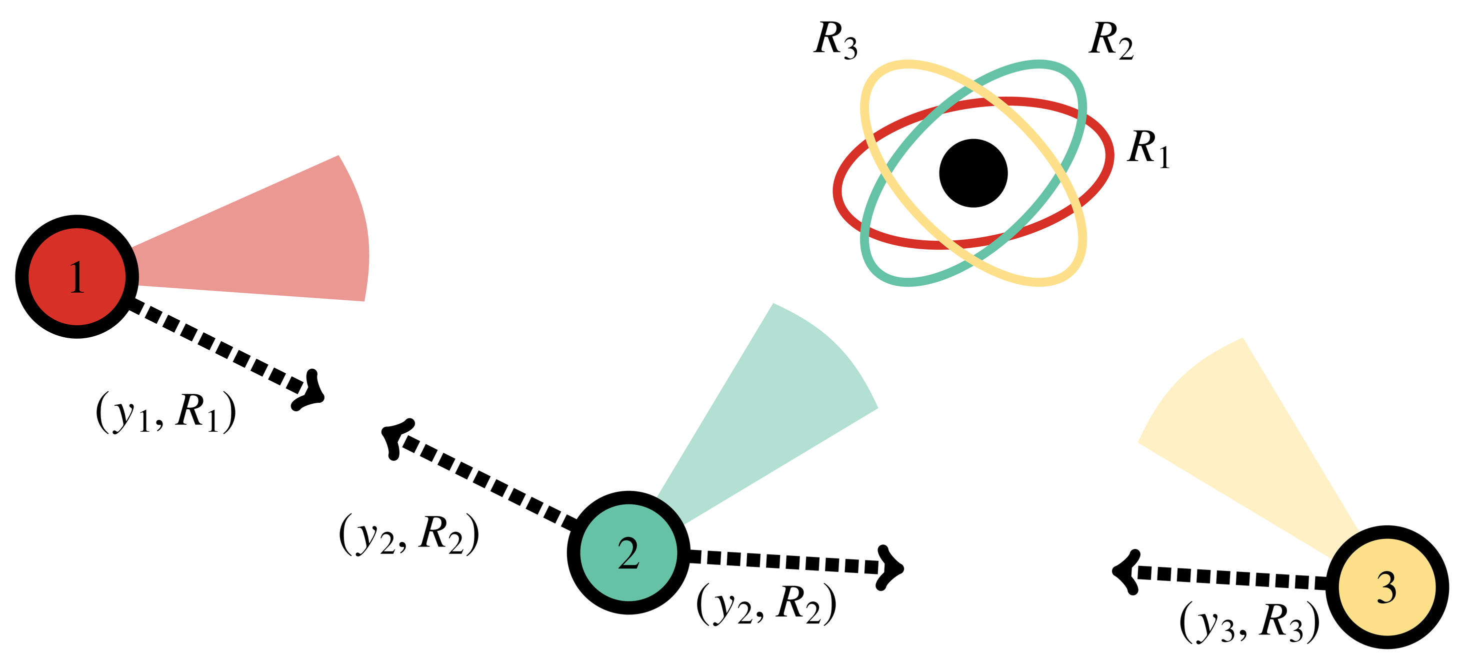
Background

The task is to estimate a *target* state x^0 by utilizing multiple *decentralized agents*, where each agent has *sensor* and *communication* capabilities. By fusion of multiple local estimates improved track estimates are obtained.



Model

The ith local estimate of $x^0 \in \mathbb{R}^n$ is given by

$$y_i = H_i x^0 + v_i, \qquad R_i = \mathrm{cov}(v_i),$$

where $H_i \in \mathbb{R}^{n_i \times n}$, v_i is noise, $y_i \in \mathbb{R}^{n_i}$ is the local state estimate and R_i is its covariance. The cross-covariance of y_i and y_j is $R_{ij} = \mathrm{cov}(v_i, v_j)$. For $N = 3$ estimates, let

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad H = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad R = \begin{bmatrix} R_1 & R_{12} & R_{13} \\ R_{21} & R_2 & R_{23} \\ R_{31} & R_{32} & R_3 \end{bmatrix},$$

and similarly for arbitrary N. Hence $y = Hx^0 + v$.

Two Subproblems

In decentralized target tracking **cross-covariances are often unknown** and the **communication link is limited**. These issues must be handled without *underestimating the uncertainty* and without *degrading the performance* too much. We consider the following **two subproblems**:

- **Conservative estimation under partially known covariances**
- **Optimal fusion of dimension-reduced estimates**

Conservative Linear Unbiased Estimation

Conservative estimation methods are important when R is only partially known. A typical scenario where this occurs is decentralized estimation where the cross-covariances $R_{ij}, i \neq j$ are often unknown. If R is only partially known we say that $R \in \mathcal{A}$, where $\mathcal{A}$ is a set of admissible covariance matrices.

Problem Statement

Given y and $\mathcal{A}$, compute an estimate $\hat{x}$ of x^0 with covariance P, where P **is as small as possible but not smaller** than the true covariance of $\hat{x}$.

- Since we only know $R \in \mathcal{A}$, it is impossible to compute the the true covariance of $\hat{x}$.
- It is assumed $\mathcal{A} \subset \mathbb{S}^m_{++}$ where $\mathbb{S}^m_{++}$ is the set of all $m \times m$ symmetric positive definite matrices.

An estimator $(\hat{x}, P)$ is a *conservative linear unbiased estimator* (CLUE) if it has the following **properties**:

$$\underbrace{\hat{x} = Ky}_{\text{linear}} \qquad \underbrace{KH = I}_{\text{unbiased}} \qquad \underbrace{P \succeq KRK^\mathsf{T}, \forall R \in \mathcal{A}}_{\text{conservative}}$$

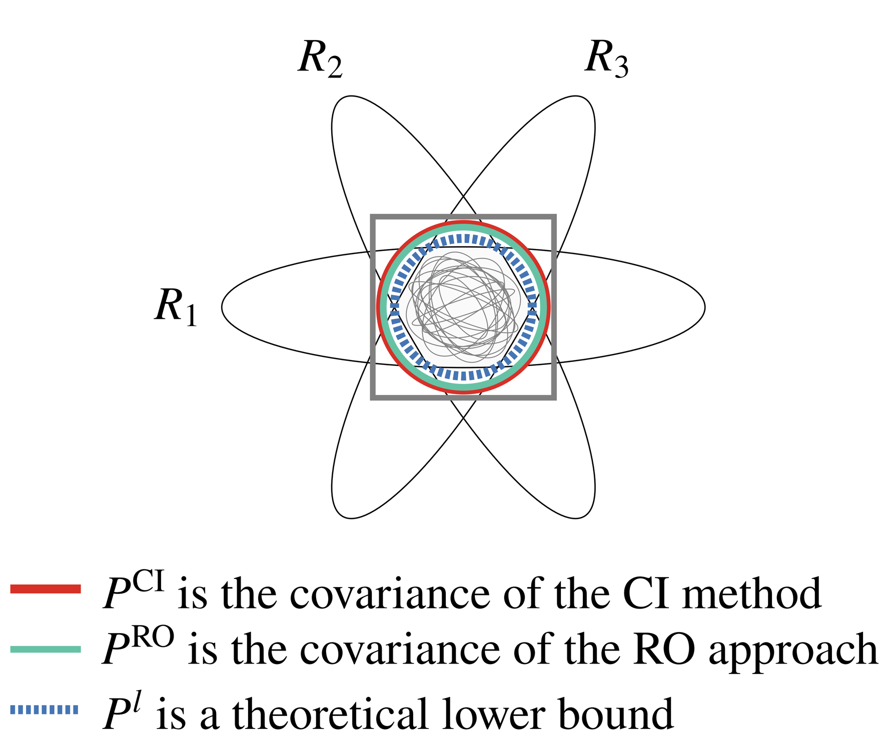
An **optimal CLUE** is defined as follows:

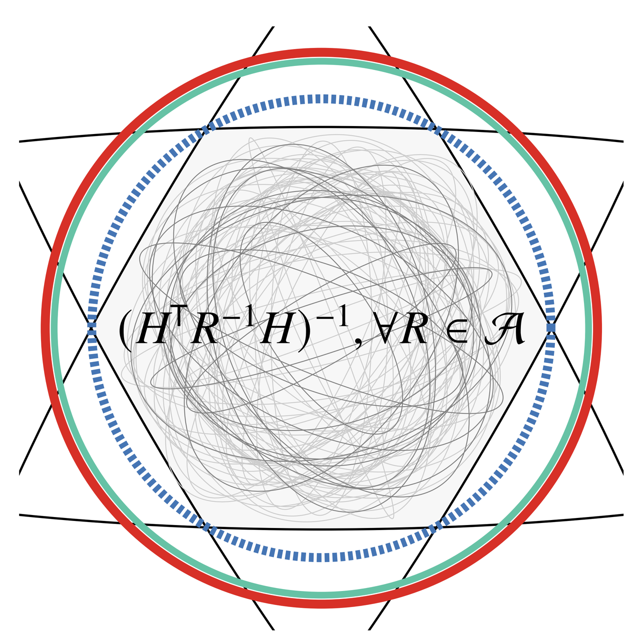
Best CLUE

Let J be a loss function. An estimator reporting $\hat{x}^\star = K^\star y$ and $P^\star$ is called a *best CLUE* if $(K^\star, P^\star)$ is the solution to

$$\begin{aligned} \underset{K,P}{\text{minimize}} \quad & J(P) \\ \text{subject to} \quad & KH = I \\ & P \succeq KRK^\mathsf{T}, \forall R \in \mathcal{A}. \end{aligned}$$

Using *robust optimization* (RO) the CLUE problem can be solved. As an example, consider fusion of (y_i, R_i), where $i = 1, 2, 3$ and $H = \begin{bmatrix} I & I & I \end{bmatrix}^\mathsf{T}$. The RO method is compared with *covariance intersection* (CI):

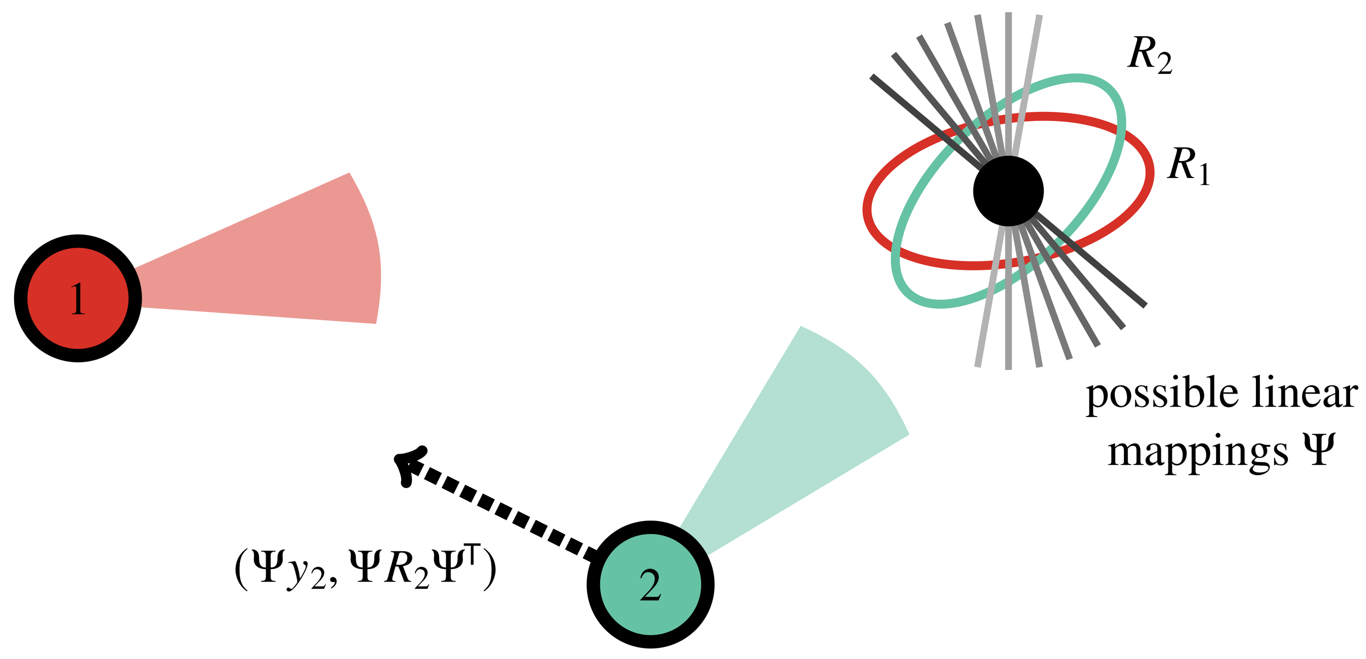




for a certain R the covariance of the optimal linear estimate is $(H^\mathsf{T} R^{-1} H)^{-1}$

Fusion of Dimension-Reduced Estimates

One way of handling communication constraints is by *reducing the dimensionality* of communicated estimates. This can be done e.g., by transmitting $(\Psi y_2, \Psi R_2 \Psi^\mathsf{T})$ instead of (y_2, R_2), where $\Psi \in \mathbb{R}^{m \times n}$ is a wide matrix:



Performance depends on the choice of Ψ.

Problem Statement

Let $(\hat{x}, P)$ be the result of fusing (y_1, R_1) and $(\Psi y_2, \Psi R_2 \Psi^\mathsf{T})$. Then the optimal Ψ, denoted by $\Psi^\star$, is given by

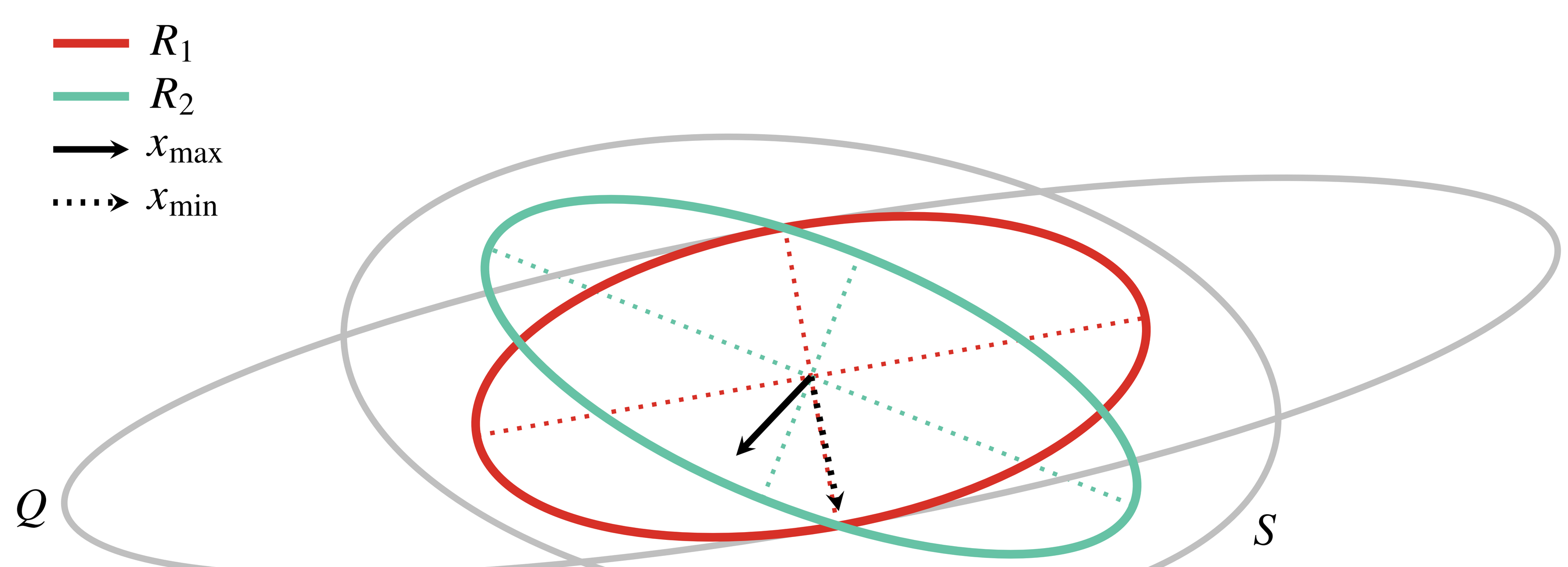
$$\underset{\Psi}{\text{minimize}} \ \mathrm{tr}(P). \tag{1}$$

Since P depends on the particular fusion method and availability of R_{12}, so does the optimal Ψ. Assume $H_1 = H_2 = I$ and that R_{12} is available.

Solution

Let $Q = (R_1 - R_{12})^\mathsf{T}(R_1 - R_{12})$ and $S = R_1 + R_2 - R_{12} - R_{12}^\mathsf{T}$. The solution to the problem in (1) is given by $\Psi^\star = \begin{bmatrix} x_n & \dots & x_{n-m+1} \end{bmatrix}^\mathsf{T}$, where x_i is a generalized eigenvector associated with $\lambda_i(Q, S)$ and $\lambda_1 \leq \cdots \leq \lambda_n$.

As an example, assume $n = 2$ and $R_{12} = 0$:



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