

# A Learning Approach for Feed-Forward Friction Compensation

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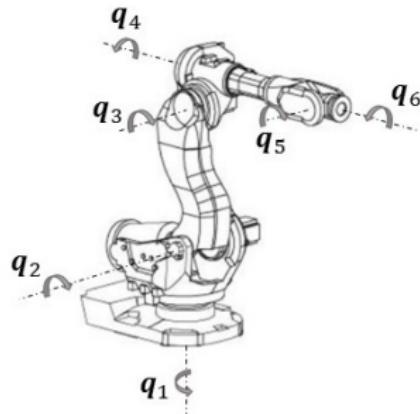
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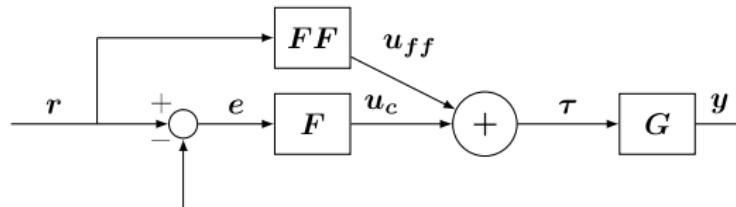
# Main message

- The paper is based on the Master's thesis by Viktor Johansson.
- A comparison of two approaches for feed-forward friction compensation:
  - Grey-box friction model (LuGre) using previously identified parameters.
  - Black-box friction model using B-spline networks (BSN), where the parameters are learned from experiments.
- Experimental evaluation using a six degrees-of-freedom industrial robot.
- The two approaches give comparable performance.
- The BSN approach requires very limited a priori insight about the friction behavior.

# Problem description



$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\dot{\boldsymbol{q}}) = \boldsymbol{\tau}$$



# LuGre friction model

$$\dot{z} = v - \sigma_0 \frac{|v|}{g(v)} z$$

$$g(v) = f_c + (f_s - f_c)e^{-(v/v_s)^2}$$

$$\tau_f(v) = \sigma_0 z + \sigma_1 \dot{z} + f(v)$$

where

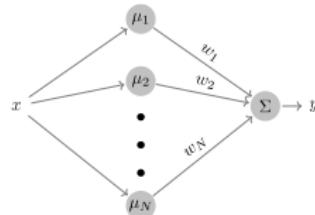
- $z$  is the internal state.
- $v$  is the relative velocity.
- $f(v)$  is the viscous friction. Here  $f(v) = f_v v$ .
- $g(v)$  captures the Coulomb and Stribeck effects.

# B-spline network (BSN) friction model

General:

$$y(x) = \sum_{i=1}^N \mu_i(x) \cdot w_i$$

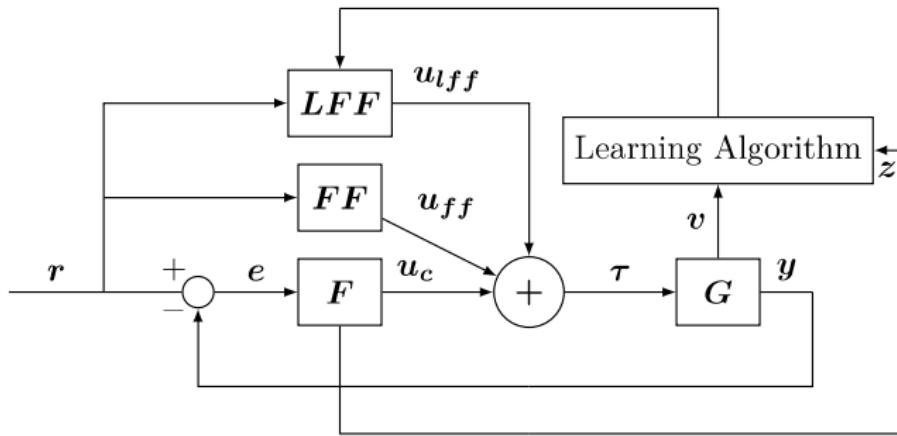
where  $\mu_i(x)$  is the spline function.



Here:

$$\tau_f(v) = \sum_{i=1}^N \mu_i(v) \cdot w_i$$

# Friction compensation using BSN



# Friction compensation using BSN

The BSN is trained using

$$\Delta w_{i,j} = \gamma \frac{\sum_{k=1}^{N_s} u_{i,p}(k) \mu(\dot{q}_i^{ref}(k))}{\sum_{k=1}^{N_s} \mu(\dot{q}_i^{ref}(k))}$$

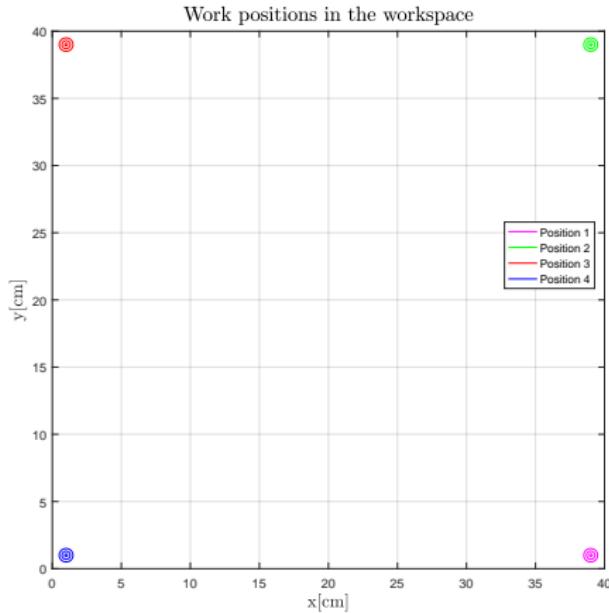
where

- $i$  - joint number,  $j$  - spline number
- $\dot{q}_i^{ref}$  - velocity reference
- $N_s$  - number of samples
- $u_p = u_c - u_I$  where  $u_c$  is the controller output and  $u_I$  is the integral part of the control signal

# Experimental setup

- Six degrees-of-freedom experimental robot.
- Four operating points located in the corners of a square of the size  $40 \times 40$  cm.
- Circular motions with radii 1, 3 and 5 mm respectively.
- TCP velocities 10, 40 and 100 mm/s respectively. Here: Only TCP velocity 40 mm/s.
- The learning process was run for ten iterations using  $\gamma = 0.95$ .
- A suitable number and distribution of the knots in the BSN was selected. A denser distribution was used for low velocities.
- The learning procedure was carried out for each of the three selected TCP velocities.
- Extended version where two different networks were trained and used depending on the sign of the angular acceleration.

# Experimental setup



# Results - Maximum TCP deviation

No feed-forward:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | 0.5    | 0.63   | 0.91   | 0.58   |
| Circ. r3 | 0.41   | 0.35   | 0.38   | 0.46   |
| Circ. r5 | 0.31   | 0.28   | 0.35   | 0.36   |

Feed-forward using static LuGre model:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | 0.48   | 0.52   | 0.93   | 0.59   |
| Circ. r3 | 0.2    | 0.3    | 0.32   | 0.23   |
| Circ. r5 | 0.14   | 0.25   | 0.18   | 0.2    |

Feed-forward using BSN model:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | 0.47   | 0.61   | 0.86   | 0.53   |
| Circ. r3 | 0.17   | 0.3    | 0.4    | 0.2    |
| Circ. r5 | 0.17   | 0.21   | 0.24   | 0.19   |

# Results - RMS of TCP deviation

No feed-forward:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | 0.362  | 0.324  | 0.41   | 0.343  |
| Circ. r3 | 0.187  | 0.188  | 0.206  | 0.209  |
| Circ. r5 | 0.12   | 0.141  | 0.14   | 0.147  |

Feed-forward using static LuGre model:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | 0.307  | 0.263  | 0.378  | 0.28   |
| Circ. r3 | 0.109  | 0.133  | 0.133  | 0.117  |
| Circ. r5 | 0.066  | 0.115  | 0.084  | 0.083  |

Feed-forward using BSN model:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | 0.263  | 0.264  | 0.377  | 0.25   |
| Circ. r3 | 0.099  | 0.131  | 0.145  | 0.112  |
| Circ. r5 | 0.085  | 0.106  | 0.088  | 0.093  |

# Observations

- Larger error for smaller radii.
- Variations in performance between the four positions.
- Comparable performance of the two approaches:
  - Average max error for  $r_5$  over the four positions: 0.1925 for LuGre and 0.2025 for BSN.
  - Average max error for  $r_3$  over the four positions: 0.2625 for LuGre and 0.2675 for BSN.
  - Average max error for  $r_1$  over the four positions: 0.63 for LuGre and 0.6175 for BSN.

## Observations (cont.)

Reduction of the MSE for feed-forward using static LuGre model:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | -15 %  | -19 %  | -8 %   | -18%   |
| Circ. r3 | -42 %  | -29 %  | -35 %  | -44 %  |
| Circ. r5 | -45 %  | -18 %  | -40 %  | -44 %  |

Reduction of the MSE for feed-forward using BSN:

|          | Pos. 1 | Pos. 2 | Pos. 3 | Pos. 4 |
|----------|--------|--------|--------|--------|
| Circ. r1 | -27 %  | -19 %  | -8 %   | -27 %  |
| Circ. r3 | -47 %  | -30 %  | -30 %  | -46 %  |
| Circ. r5 | -29 %  | -25 %  | -37 %  | -37 %  |

# Summary

- A comparison of two approaches for feed-forward friction compensation:
  - Grey-box friction model (LuGre) using previously identified parameters.
  - Black-box friction model using B-spline networks (BSN), where the parameters are learned from experiments.
- Experimental evaluation using a six degrees-of-freedom industrial robot.
- The two approaches give comparable performance.
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Further work:

- Additional experiments.
- ‘Fair’ comparison of the time and efforts needed for the two methods.

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