

CONSTRUCTIVE RELATIONAL BINARY STRUCTURES WITH APARTNESS - UNITY NOT UNIFORMITY

Melanija Mitrović

CAM-FMEN, Faculty of Mechanical Engineering, University of Niš, Serbia
International Chair in Mathematical Physics and Applications,
ICMPA-UNESCO Chair, University of Abomey-Calavi, Benin

Email: melanija.mitrovic@masfak.ni.ac.rs

A. Calder wrote in 1979, [1], that “constructive mathematics is based on the belief that mathematics can have real meaning only if its concepts can be constructed by the human mind, an issue that has divided mathematicians for more than a century”. A. R. D. Mathias in 1992, [2], posted the following question: Is it desirable to press mathematicians all to think in the same way? G. Sommaruga, [4], concluded recently that classical and constructive mathematics “should not be treated as rival domains among which one has to choose one (for life), but they should rather be treated as useful reports about a same grand structure which can help us to construct a whole picture that would be more adequate than each taken alone”.

Inspired by results obtained in interactive theorem proving the approach of formal verifications a new constructive algebraic theory known as the **theory of semigroups with apartness** has been developing by Mitrović and co-authors: Hounkonnou, Baroni, Crvenković, Darpö, Romano. This lecture aims to provide a clear and understandable picture of constructive semigroups with apartness in Bishop’s style of constructive mathematics, **BISH**. In a spirit of Sommaruga’s wording two points of view on constructive semigroups with apartness will be presented: the classical (**CLASS**), which plays a useful role as intuition guides and to at least link with the presentations given in constructive one; the constructive (**BISH**), comprising a binary system with apartness which satisfies a number of extra conditions. An appropriate constructive order theory for sets and semigroups with apartness we develop, its application on semigroups and ordered semigroups will be presented. A comparative analysis between presented classical and constructive results will be given too. Although the lecture will be mostly based on material given in [3], it is, by no means an attempt to give a complete overview of our existing results to the date.

References

- [1] Calder A. (1979). *Constructive Mathematics*. Scientific American, Vol. 241, No. 4, pp. 146-171.
- [2] Mathias A. R. D. (1992). *What is MacLane missing?* Pp. 113-118 in Judah H., Just W., and H. Woodin H. (Eds.), *Set Theory of the Continuum*. Berlin: Springer.
- [3] Mitrović M., Hounkonnou M. N., Catarino P. (2023). *Constructive semigroups with apartness - a state of the art*. In Hounkonnou M. N., Martinovic D., Mitrović M., and Pattison P. (Eds.), *Mathematics for social sciences and arts - algebraic modeling*. Springer (to appear).
- [4] Sommaruga G. (2011). *Introduction*. Pp 1-49 in Sommaruga G. (Ed.), *Foundational Theories of Classical and Constructive Mathematics*, Springer.