One way to get some understanding on the relations between equations and their solutions over complex numbers is to study the projection

$$\text{(an equation, its solutions)} \mapsto \text{(an equation)}$$

over the relevant space of equations. When the latter projection is a covering, we enter the realm of Galois theory. Consider for instance the space of polynomials $f(x) = c_0 + c_1 x + \cdots + c_d x^d$ having d distinct roots in $\mathbb{C}$. The fundamental theorem of algebra ensures that we can permute the roots of $f(x)$ in any possible way by traveling along loops in the space of coefficients $c_0, \cdots, c_d$. In other words, the Monodromy/Galois group of the covering

$$\{(f, x) : \ f(x) = 0\} \mapsto f$$

is the full symmetric group. As a reminder of why these groups matter, recall how Galois himself expressed the solvability by radical of f as some algebraic property of its M/G group, namely the solvability of the group.

One very general context in which we can wonder about M/G groups is for square systems of polynomial equations

$$f_1(x_1, \cdots, x_n) = \cdots = f_n(x_1, \cdots, x_n) = 0$$

where we decide in advance not only the degree of each f_j but its support, that is which monomials show up in f_j. There are some general statements in this context, but already for $n = 2$, the M/G groups of some systems $f_1 = f_2 = 0$ are not known.

In this talk, I would like to report on recent progress on the computation of these groups. In certain classes of examples, I would like to describe the M/G groups, explain why they are not full symmetric groups in general and, eventually, explain how we can compute them using considerations from toric/tropical geometry.