LINKÖPINGS UNIVERSITET

Exam in TSDT14 Signal Theory

Exam code:	TEN1
Date:	2017-10-19 **Time:** 14.00–18.00
Place:	U1, KÅRA
Teacher:	Mikael Olofsson, tel: 281343
Visiting exam:	15 and 17
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* A German 10-Mark note of the fourth series (1991-2001). Pocket calculators of all kinds with empty memory.
Number of tasks:	6
Grading:	Task one is not graded in terms of points. At least two of its three sub-tasks have to be treated correctly as partial fulfillment to pass the exam. Tasks 2-6 yield at most 5 points each. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unresonable answers. Totally, you can get 25 points at most. For grade three you need 10 points, for grade four 15 points and for grade five 20 points.
Solutions:	Will be published no later than three days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/tentor?TSDT14
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	In the student office of ISY, Building B, between entrances 27-29, ground floor, Corridor D, right next to Café Java, starting about two weeks after the exam.

1 At least two of the following three sub-tasks have to be treated correctly as partial fulfillment to pass the exam:

a. Define the concept *wide-sense stationarity* for a time-discrete process.

b. A time-continuous filter has frequency response

$$H(f) = \begin{cases} 3, & 1 < |f|, \\ 0, & \text{elsewhere.} \end{cases}$$

The input $X(t)$ to this filter has PSD $R_X(f) = R_0$. Determine the PSD of the output.

c. Consider a time-continuous Gaussian process, $X(t)$, with mean zero and ACF

$$r_X(\tau) = \operatorname{sinc}(\tau).$$

Determine the ACF of the process $Y(t) = \operatorname{sgn}\big(X(t)\big)$

2 Let $X(t)$ be an ergodic process with ACF

$$r_X(\tau) = 1 + \operatorname{sinc}(\tau).$$

$X(t)$ is the input to an LTI system with impulse response

$$h(t) = e^{-t}u(t),$$

where $u(t)$ as usual is the time-continuous unit step. Determine the mean and PSD of the output.

If there is more than one solution, then determine all solutions.

3 A certain sampler is subject to a quadratic error according to the following expression.

$$Y[n] = X(nT) + \epsilon X^2(nT),$$

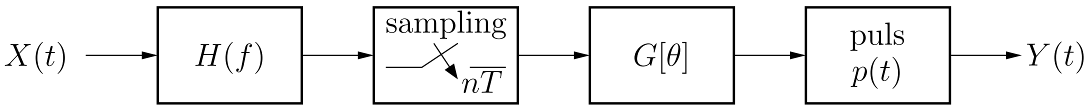
where ϵ is a real constant. The input $X(t)$ is in this case a Gaussian process with mean zero and ACF $r_X(\tau)$.

Express the ACF of the output, $r_Y[k]$, in that of the input, $r_X(\tau)$. (5 p)

4 A time-continuous stochastic signal $X(t)$ with PSD

$$R_X(f) = \begin{cases} 100\left(1 - \frac{|f|}{1000}\right), & |f| < 1000, \\ 0, & |f| \geq 1000, \end{cases}$$

is sampled and pulse-amplitude modulated according to the figure below.



In the figure, there is a time-continuous LP filter with frequency response $H(f)$ and a time-discrete HP filter with frequency response $G[\theta]$, according to

$$H(f) = \begin{cases} 5, & |f| \leq 750, \\ 0, & |f| > 750. \end{cases} \qquad G[\theta] = \begin{cases} 2, & \frac{1}{4} \leq |\theta| \leq \frac{1}{2}, \\ 0, & |\theta| < \frac{1}{4}, \end{cases}$$

with $G[\theta] = G[\theta + m]$, where m is an integer. The pulse-amplitude modulation uses the pulse $p(t) = \mathrm{sinc}(f_s t)$, and the sampling frequency is $f_s = \frac{1}{T} = 1\,\mathrm{kHz}$.

a. Determine and draw the PSD of the output $Y(t)$. (4 p)

b. Determine the power of $Y(t)$. (1 p)

5 The time-continuous stochastic signal $X(t)$ is defined as

$$X(t) = A\cos(2\pi f_0 t) + B\sin(2\pi f_0 t),$$

where A and B are uncorrelated uniformly distributed stochastic variables over the interval $[-1, 1]$. Show that $X(t)$ is WSS! (5 p)

6 Consider the 2-D space-continuous signals

$$x(a_1, a_2) = \begin{cases} 1, & |a_1| < \frac{1}{2}, |a_2| < 1, \\ 0, & \text{elsewhere}, \end{cases} \qquad y(a_1, a_2) = \begin{cases} 1, & |a_1| < 1, |a_2| < \frac{1}{2}, \\ 0, & \text{elsewhere}. \end{cases}$$

a. Calculate the 2-D convolution $(x \circledast y)(a_1, a_2)$. (4 p)

b. Determine and draw $(x \circledast y)(a, a)$ (1 p)

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $$r_X(\tau) = \mathrm{E}\big\{X(t+\tau)X^*(t)\big\} = r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$$

PSD: $$R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c.

Passband: $s(t) = \mathrm{Re}\Big\{\tilde{s}(t)e^{j2\pi f_\mathrm{c}t}\Big\} = s_\mathrm{I}(t)\cos(2\pi f_\mathrm{c}t) - s_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c}t)$

Baseband: $\tilde{s}(t) = s_\mathrm{I}(t) + js_\mathrm{Q}(t)$

Spectrum: $S(f) = \frac{1}{2}\big(\tilde{S}(f-f_\mathrm{c}) + \tilde{S}^*(-f-f_\mathrm{c})\big)$, $\quad \tilde{S}(f) = 2S(f+f_\mathrm{c})u(f+f_\mathrm{c})$

Filtering: $y(t) = (x * h)(t) \quad \Leftrightarrow \quad \tilde{y}(t) = \frac{1}{2}(\tilde{x} * \tilde{h})(t)$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c.

Passband: $S(t) = S_\mathrm{I}(t)\cos(2\pi f_\mathrm{c}t + \Psi) - S_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c}t + \Psi)$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $\tilde{S}(t) = S_\mathrm{I}(t) + jS_\mathrm{Q}(t)$

Mean: $m_S = 0$

ACF: $r_S(\tau) = \frac{r_{S_\mathrm{I}}(\tau) + r_{S_\mathrm{Q}}(\tau)}{2}\cos(2\pi f_\mathrm{c}\tau) - \frac{r_{S_\mathrm{I},S_\mathrm{Q}}(-\tau) - r_{S_\mathrm{I},S_\mathrm{Q}}(\tau)}{2}\sin(2\pi f_\mathrm{c}\tau)$

PSDs: $R_S(f) = \frac{1}{4}\big(R_{\tilde{S}}(f-f_\mathrm{c}) + R_{\tilde{S}}(-f-f_\mathrm{c})\big)$, $\quad R_{\tilde{S}}(f) = 4R_S(f+f_\mathrm{c})u(f+f_\mathrm{c})$