Formelblad för TNA007 3 december 2014

Båglängdelement $ds = |\boldsymbol{r}'(u)|du$

Ytelement $dS = |\boldsymbol{r}'_u \times \boldsymbol{r}'_v|dudv$

Integralsatser

Gauss' sats
$$\oint\!\!\!\oint_S \boldsymbol{A}\cdot\hat{\boldsymbol{n}}dS = \iiint_D \nabla\cdot\boldsymbol{A}dV$$

Greens formel
$$\oint_L \big(Pdx + Qdy\big) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy$$

Stokes' sats
$$\oint_L \boldsymbol{A}\cdot d\boldsymbol{r} = \iint_S \nabla\times\boldsymbol{A}\cdot\hat{\boldsymbol{n}}dS$$

Universalintegralsatser

Gauss' universalsats
$$\oint\!\!\!\oint_S \hat{\boldsymbol{n}}dS(\cdots) = \iiint_V dV\nabla(\cdots)$$

Stokes' universalsats
$$\oint_L d\boldsymbol{r}(\cdots) = \iint_S (\hat{\boldsymbol{n}}dS\times\nabla)(\cdots)$$

Kroklinjiga koordinater

Vi har ett kroklinjigt koordinatsystem med koordinater u_1, u_2, u_3 med basvektorer $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ och skalfaktorer h_1, h_2, h_3:

$$d\boldsymbol{r} = h_1\boldsymbol{e}_1du_1 + h_2\boldsymbol{e}_2du_2 + h_3\boldsymbol{e}_3du_3$$
$$\hat{\boldsymbol{n}}dS = \pm h_2h_3\boldsymbol{e}_1du_2du_3,\ \text{ytan } S \text{ ortogonal mot } \boldsymbol{e}_1$$
$$\nabla\phi = \sum_i \frac{1}{h_i}\frac{\partial\phi}{\partial u_i}\boldsymbol{e}_i$$
$$\nabla\cdot\boldsymbol{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\big(h_2h_3A_1\big) + \frac{\partial}{\partial u_2}\big(h_1h_3A_2\big) + \frac{\partial}{\partial u_3}\big(h_1h_2A_3\big)\right]$$
$$\nabla\times\boldsymbol{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\boldsymbol{e}_1 & h_2\boldsymbol{e}_2 & h_3\boldsymbol{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$$
$$\Delta\phi = \nabla\cdot\nabla\phi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(\frac{h_2h_3}{h_1}\frac{\partial\phi}{\partial u_1}) + \frac{\partial}{\partial u_2}(\frac{h_1h_3}{h_2}\frac{\partial\phi}{\partial u_2}) + \frac{\partial}{\partial u_3}(\frac{h_1h_2}{h_3}\frac{\partial\phi}{\partial u_3})\right]$$

Cylinderkoordinater

$$\boldsymbol{r} = (\rho\cos\varphi,\ \rho\sin\varphi,\ z)$$
$$\boldsymbol{e}_\rho = (\cos\varphi,\ \sin\varphi,\ 0),\ \ \boldsymbol{e}_\varphi = (-\sin\varphi,\ \cos\varphi,\ 0),\ \ \boldsymbol{e}_z = (0,0,1)$$

Skalfaktorer: $h_\rho = 1$, $h_\varphi = \rho$, $h_z = 1$.

Sfäriska koordinater

$$\boldsymbol{r} = (r\sin\theta\cos\varphi,\ r\sin\theta\sin\varphi,\ r\cos\theta)$$
$$\boldsymbol{e}_r = (\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta),\ \ \boldsymbol{e}_\theta = (\cos\theta\cos\varphi,\ \cos\theta\sin\varphi,\ -\sin\theta),\ \ \boldsymbol{e}_\varphi = (-\sin\varphi,\ \cos\varphi,\ 0)$$

Skalfaktorer: $h_r = 1$, $h_\theta = r$, $h_\varphi = r\sin\theta$.

Operatorformler

$$\nabla\times(\boldsymbol{A}\times\boldsymbol{B}) = (\boldsymbol{B}\cdot\nabla)\boldsymbol{A} - \boldsymbol{B}(\nabla\cdot\boldsymbol{A}) - (\boldsymbol{A}\cdot\nabla)\boldsymbol{B} + \boldsymbol{A}(\nabla\cdot\boldsymbol{B})$$
$$\nabla(\boldsymbol{A}\cdot\boldsymbol{B}) = (\boldsymbol{B}\cdot\nabla)\boldsymbol{A} + (\boldsymbol{A}\cdot\nabla)\boldsymbol{B} + \boldsymbol{B}\times(\nabla\times\boldsymbol{A}) + \boldsymbol{A}\times(\nabla\times\boldsymbol{B})$$
$$\nabla\times(\nabla\phi) = 0,\quad \nabla\cdot(\nabla\times\boldsymbol{A}) = 0,\quad \nabla\times(\nabla\times\boldsymbol{A}) = \nabla(\nabla\cdot\boldsymbol{A}) - \Delta\boldsymbol{A}$$