LINKÖPINGS UNIVERSITET

Exam in TSDT14 Signal Theory

Exam code:	TEN1
Date:	2018-01-03 **Time:** 8.00–12.00
Place:	
Teacher:	Mikael Olofsson, tel: 281343
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* A German 10-Mark note of the fourth series (1991-2001). Pocket calculators of all kinds with empty memory.
Number of tasks:	6
Grading:	Task one is not graded in terms of points. At least two of its three sub-tasks have to be treated correctly as partial fulfillment to pass the exam. Tasks 2-6 yield at most 5 points each. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unresonable answers. Totally, you can get 25 points at most. For grade three you need 10 points, for grade four 15 points and for grade five 20 points.
Solutions:	Will be published no later than three days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/tentor?TSDT14
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	In the student office of ISY, Building B, between entrances 27-29, ground floor, Corridor D, right next to Café Java, starting about two weeks after the exam.

1 At least two of the following three sub-tasks have to be treated correctly as partial fulfillment to pass the exam:

a. Define the concept *strict-sense stationarity* for a time-continuous process.

b. A time-discrete filter has frequency response

$$H[\theta] = \begin{cases} 2, & |\theta| < 0.1, \\ 0, & 0.1 \le |\theta| \le 0.5, \end{cases} \qquad H[\theta + k] = H[\theta], \quad k \text{ integer.}$$

The input $X[n]$ of this filter has PSD

$$R_X[\theta] = 1 + \cos(2\pi\theta).$$

Determine the PSD of the output.

c. A time-discrete Gaussian process, $X[n]$, with mean zero and ACF

$$r_X[k] = \frac{1}{1 + k^2}$$

is squared. Determine the ACF of the resulting process.

2 Let $X(t)$ be a time-continuous Gaussian process with mean $m_X = 1$ and ACF $r_X(\tau) = 1 + \text{sinc}(\tau)$, and let this $X(t)$ be the input to an LTI system with frequency response $H(f) = 2 \cdot \text{rect}(2f)$. For the output, $Y(t)$, determine (1 p each)

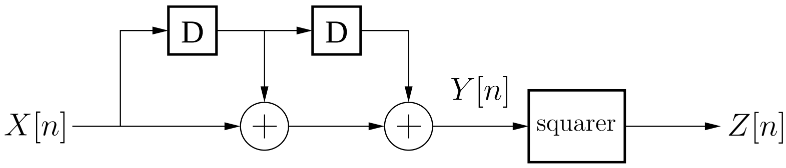
a. its mean m_Y,
b. its ACF $r_Y(\tau)$,
c. its PSD $R_Y(f)$,
d. its signal power P_Y,
e. the probability $\Pr\{Y(t) > 2\}$.

3 Let $X[n]$ be a time-discrete process with PSD

$$R_X[\theta] = 2 + \cos(4\pi\theta) + \cos(8\pi\theta).$$

Assuming that we have observed $X[n]$ for $n \le n_0$, we want to predict $X[n_0+1]$ using a predictor of the form $\hat{X}[n_0 + 1] = aX[n_0] + bX[n_0 - 1]$. Determine the coefficients a and b such that $\epsilon^2 = \mathrm{E}\Big\{\big(\hat{X}[n_0 + 1] - X[n_0 + 1]\big)^2\Big\}$ is minimized. (5 p)

4 Let $X[n]$ in the figure below be a sequence of independent stochastic variables.



Determine the ACF $r_Z[k]$ and the PSD $R_Z[\theta]$ for the case where $X[n]$ is

a. Gaussian with mean zero and variance 1, (2 p)
b. binary, taking values ± 1 with equal probability. (3 p)

5 Let $c(t)$ be a periodic signal given by

$$c(t) = \begin{cases} 1, & 0 \leq t < T/2, \\ -1, & T/2 \leq t < T, \\ c(t + kT), & k \text{ integer}, \end{cases}$$

where T is its period. Also, let $X(t)$ be a process, and consider the process $Y(t) = c(t) \cdot X(t)$. Show that $Y(t)$

a. is WSS if $X(t)$ is white, (3 p)
b. may not be WSS if $X(t)$ is not white. (2 p)

6 Let the space-continuous signal

$$x(a_1, a_2) = \begin{cases} 1, & 0 \leq a_1 < 2 \text{ and } 0 \leq a_2 < 2, \\ 0, & \text{elsewhere}, \end{cases}$$

be the input to an LSI system with impulse response

$$h(a_1, a_2) = \delta(a_1, a_2) + \delta(a_1 + 1, a_2 - 1).$$

Determine the resulting output of the system. (5 p)

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $r_X(\tau) = \mathrm{E}\{X(t+\tau)X^*(t)\}$

$$= r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$$

PSD: $R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c.

Passband: $s(t) = \mathrm{Re}\Big\{\tilde{s}(t)e^{j2\pi f_c t}\Big\} = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$

Baseband: $\tilde{s}(t) = s_I(t) + js_Q(t)$

Spectrum: $S(f) = \frac{1}{2}\big(\tilde{S}(f-f_c) + \tilde{S}^*(-f-f_c)\big)$, $\quad\tilde{S}(f) = 2S(f+f_c)u(f+f_c)$

Filtering: $y(t) = (x*h)(t) \quad\Leftrightarrow\quad \tilde{y}(t) = \frac{1}{2}(\tilde{x}*\tilde{h})(t)$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c.

Passband: $S(t) = S_I(t)\cos(2\pi f_c t + \Psi) - S_Q(t)\sin(2\pi f_c t + \Psi)$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $\tilde{S}(t) = S_I(t) + jS_Q(t)$

Mean: $m_S = 0$

ACF: $r_S(\tau) = \frac{r_{S_I}(\tau)+r_{S_Q}(\tau)}{2}\cos(2\pi f_c\tau) - \frac{r_{S_I,S_Q}(-\tau)-r_{S_I,S_Q}(\tau)}{2}\sin(2\pi f_c\tau)$

PSDs: $R_S(f) = \frac{1}{4}\big(R_{\tilde{S}}(f-f_c) + R_{\tilde{S}}(-f-f_c)\big)$, $\quad R_{\tilde{S}}(f) = 4R_S(f+f_c)u(f+f_c)$