LINKÖPINGS UNIVERSITET

Exam in TSDT14 Signal Theory

Exam code:	TEN1
Date:	2019-10-23 **Time:** 14.00–18.00
Place:	KÅRA
Teacher:	Mikael Olofsson, tel: 281343
Visiting exam:	15 and 17
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* A German 10-Mark note of the fourth series (1991-2001). Pocket calculators of all kinds with empty memory.
Number of tasks:	6
Grading:	Task one is not graded in terms of points. At least two of its three sub-tasks have to be treated correctly as partial fulfillment to pass the exam. Tasks 2-6 yield at most 5 points each. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unresonable answers. Totally, you can get 25 points at most. For grade three you need 10 points, for grade four 15 points and for grade five 20 points.
Solutions:	Will be published in the course room no later than three days after the exam, under Previous exams.
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	In the student office of ISY, Building B, between entrances 27-29, ground floor, Corridor D, right next to Café Java, starting about two weeks after the exam.

Important: Solutions and answers must be given in English.

1 These sub-tasks have to be treated correctly as partial fulfillment to pass the exam:

a. Define the concept *time-discrete stochastic process.*

b. A time-continuous filter has frequency response

$$H(f) = \begin{cases} 2, & |f| < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The input $X(t)$ to this filter has PSD $R_X(f) = R_0$. Determine the PSD of the output.

c. The process $Y(t) = X^3(t)$ is created from a time-continuous Gaussian process, $X[n]$, with mean zero and ACF

$$r_X(\tau) = \text{sinc}(\tau)$$

Determine the ACF of the resulting process.

2 Let $X[n]$ be a WSS time-continuous Gaussian process with mean zero and ACF

$$r_X(\tau) = R_0\, \text{sinc}^2(\tau/T).$$

This process is the input to an ideal BP filter with cutoff frequencies

$$\frac{1}{2T} \qquad \text{and} \qquad \frac{3}{2T},$$

with amplification 1 in the passband.

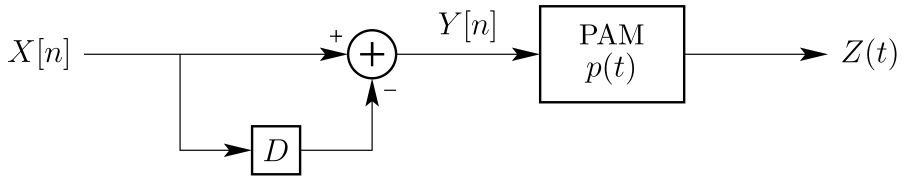
Determine the probability that the output $Y(t)$ is larger than 1. (5 p)

3 Let $X[n]$ be a time-discrete process consisting of independent, identically distributed, binary variables, taking values ± 1. Let p be the probability that $X[n]$ is 1.

a. Determine the mean and ACF of $X[n]$. (2 p)

b. Now consider the process $Y[n] = X[n]X[n-1]$. Is $Y[n]$ WSS? (3 p)

4 Consider the following system.



The time-discrete input $X[n]$ has ACF

$$r_X[k] = \begin{cases} 1, & k = 0 \\ 1/2, & |k| = 1 \\ 0, & \text{elsewhere} \end{cases}$$

The pulse-amplitude modulator gererates pulses of duration T and amplitude $Y[n]$, where T is the sampling interval.

Determine the power spectral density of the time-continuous output $Z(t)$. (5 p)

5 The cross correlation function (CCF) of two jointly WSS complex processes, $X(t)$ and $Y(t)$, is defined as

$$r_{X,Y}(\tau) = \mathrm{E}\{X(t+\tau)Y^*(t)\},$$

and the pseudo-CCF of those processes is defined as

$$\tilde{r}_{X,Y}(\tau) = \mathrm{E}\{X(t+\tau)Y(t)\},$$

Now, let $X(t)$ and $Y(t)$ be circular. What demands do we need to pose on the CCF and/or pseudo-CCF of $X(t)$ and $Y(t)$ to guarantee that

$$Z(t) = X(t) + Y(t)$$

also is circular? (5 p)

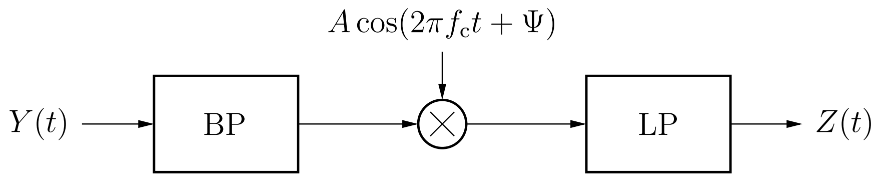
6 Let $M(t)$ be a WSS strictly bandlimited baseband message with bandwidth W. Consider using AM-SC, where the sent signal is

$$X(t) = M(t)\cos(2\pi f_c t + \Psi),$$

where in turn Ψ is uniformly distributed over $[0, 2\pi)$ and where the carrier frequency fulfills $f_c > W$. The channel adds WGN $N(t)$ to the signal. The received signal $Y(t)$ is thus given by

$$Y(t) = X(t) + N(t).$$

$M(t)$, Ψ and $N(t)$ are assumed to be mutually independent. The figure below is a receiver for this situation, where A is non-zero.



The two filters are ideal, with cut-off frequencies chosen so that the useful signal passes the filter unchanged. We assume full synchronization, i.e. Ψ is completely known at the receiver.

Show that the resulting SNR at the output of the receiver does not depend on A.

(5 p)

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $r_X(\tau) = \mathrm{E}\{X(t+\tau)X^*(t)\}$
$= r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$

PSD: $R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R^*_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c.

Passband: $s(t) = \mathrm{Re}\Big\{\tilde{s}(t)e^{j2\pi f_\mathrm{c}t}\Big\} = s_\mathrm{I}(t)\cos(2\pi f_\mathrm{c}t) - s_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c}t)$

Baseband: $\tilde{s}(t) = s_\mathrm{I}(t) + js_\mathrm{Q}(t)$

Spectrum: $S(f) = \frac{1}{2}\big(\tilde{S}(f-f_\mathrm{c}) + \tilde{S}^*(-f-f_\mathrm{c})\big)$, $\qquad \tilde{S}(f) = 2S(f+f_\mathrm{c})u(f+f_\mathrm{c})$

Filtering: $y(t) = (x*h)(t) \quad \Leftrightarrow \quad \tilde{y}(t) = \frac{1}{2}(\tilde{x}*\tilde{h})(t)$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c.

Passband: $S(t) = S_\mathrm{I}(t)\cos(2\pi f_\mathrm{c}t + \Psi) - S_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c}t + \Psi)$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $\tilde{S}(t) = S_\mathrm{I}(t) + jS_\mathrm{Q}(t)$

Mean: $m_S = 0$

ACF: $r_S(\tau) = \frac{r_{S_\mathrm{I}}(\tau) + r_{S_\mathrm{Q}}(\tau)}{2}\cos(2\pi f_\mathrm{c}\tau) - \frac{r_{S_\mathrm{I},S_\mathrm{Q}}(-\tau) - r_{S_\mathrm{I},S_\mathrm{Q}}(\tau)}{2}\sin(2\pi f_\mathrm{c}\tau)$

PSDs: $R_S(f) = \frac{1}{4}\big(R_{\tilde{S}}(f-f_\mathrm{c}) + R_{\tilde{S}}(-f-f_\mathrm{c})\big)$, $\qquad R_{\tilde{S}}(f) = 4R_S(f+f_\mathrm{c})u(f+f_\mathrm{c})$