Lab-memo for TSDT14 Signal theory

Time-discrete Stochastic Signals

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Linköping August 2018

About this document:
This lab-memo is intended for the engineering course TSDT14 Signal Theory for Master students at Linköping University.

Differences compared to the 2017 version:
Only minor changes. Misprints have been corrected and some formulations have been changed for clarity. Some minor layout issues have been addressed. Details about how to submit the report.

Acknowledgements:
The Swedish original of this lab-memo used in 2007 was a complete rewrite of even older lab memos in TSDT06 Signal Theory, and it has since been translated to English and gradually rewritten somewhat. This memo therefore builds on the work of teachers that used to give that course, i.e. Ulf Henriksson, Anders Lindman and Danyo Danev.

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This document is written in LaTeX2$_\varepsilon$ under CentOS 6.

Contents

1 Introduction

The goal of these laborations in signal theory is to provide a feeling of how you can create, manage and analyze outcomes of stochastic processes using a computer. The laborations are also meant to increase the understanding of a model as opposed to the reality.

The laborations is divided into four studies, namely

1. modelling signals,
2. improving estimates,
3. non-LTI-systems, and
4. special operations.

You will be using Matlab, which offers an intuitive environment for managing matrices and vectors. Signals can be represented as vectors. The laborations assume that you have some knowledge of Matlabs syntax for managing matrices.

There is not much theory presented in this memo. You are expected to use the course book and other literature if you need to find such information.

The idea is that each study should provide a deeper understanding of the connection between the model and the reality, or the lack thereof. It is therefore important to consider the task thoroughly before you sit down in front of a computer. It is also a good idea to read through the appendix to get a clear picture of useful functions that are available.

2 Examination

2.1 Formalia

The laborations are carried out in groups of two students and consist of four studies that are described in this memo. The examination of the laborations is based on one written report that is to be submitted in PDF format to the examiner via Submissions in the course room in Lisam. Note that you may have to provide additional details in written or oral form before your report can be approved.

For approval, your report has to be submitted no later than *the last day of the exam period in october (after HT1).* Any additional details can be provided during HT2. If this is not met, you will have to do the laborations again next year.

2.2 The Report

The lab report is supposed to be a *report*, i.e. not just short notes about what you have done, but a *complete description* of the tasks and the solution of them. *This means that relevant theory should be accounted for, as well as methodology, results, interpretations of results, and conclusions.* The report can be written in English or in Swedish. You need to include figures and graphs to help the description of theory and results. Do not forget to mention what the figures and graphs are meant to illustrate. Finally, remember to define identifiers and provide grading of the axes of graphs. The report is approved based on its outline, as well as on how well it describes the problems and their solutions, apart from pure facts.

The following information is supposed to be on the front page:

- Title
- Name
- Swedish social security number (personnummer)
- Student-ID (email address)

We do *not* like to see reports with MatLab code. We want to know what you have done, not exactly how. *Reports with MatLab code will be returned ungraded.*

3 Initial Instructions

The estimations of ACFs and PSDs that you are supposed to do *should not be done using Matlab's built-in functions* for that purpose. Instead you are supposed to develop functions for that yourselves.

DFT – The Discrete Fourier Transform

MatLab implements DFT according to the following:

$$Y[k] = \mathrm{DFT}\{y[n]\} = \sum_{n=0}^{N-1} y[n] e^{-j2\pi kn/N}$$

DFT is an important tool in signal analysis. A DFT is most often calculated using a fast algorithm called FFT - Fast Fourier Transform. Your functions will most likely use DFT, and the Matlab command for that – `fft` – should be used for that purpose.

4 The Studies

4.1 Study 1 – Modelling Signals

Modelling Signals

A sequence of independent identically distributed stochastic variables is not always a good model of a real-life signal. In signal theory, we introduce a number of tools that can be used to describe signals that are random. There are also useful formulas that for instance describe the power spectral density after LTI filtering. By filtering white noise, you can get a signal with a power spectral density that better models a real signal.

The problem that you face is then to adapt the parameters of the model to a given signal. If you are to measure the quality of the adaptation of the model, you end up in an optimization problem. There are also problems that have to do with the connection between time averages and the corresponding ensemble averages, and there are also issues with the DFT. In the course book, e.g., the *Bartlett estimate* is mentioned as a common way to estimate the auto correlation function and the *periodogram* is introduced as a way to estimate the power spectral density.

When you have a model, it can be used as a basis for prognoses. Good models are also useful for noise reduction, packing of data, automatic control, and so on.

Preparations

The following should be done before starting the actual laboration. This is especially important if you attend the teacher led lab sessions, so that you can make good use of your teacher. You are not supposed to do those preparations at the lab session. That is a waste of time and resources.

- Take a deep breath. This is more than you think it is.
- Read through the additional material *Short Matlab Manual* that you can download from the course room.
- Read up on White processes.
- Read up on Gaussian processes.
- Read up on LTI filtering of stochastic processes.
- Do the theoretical analysis of the situation (see Tasks below).
- Read up on estimation of ACFs and PSDs.
- Outline program functions that perform estimation of ACFs and PSDs.

Tasks

By filtering white noise, you can obtain a signal with known power spectral density. Therefore, construct two filters:

- A *simple* (low degree) low-pass filter, so that it is easily analyzed by hand.
- A low-pass filter of high degree, that can be approximated as an ideal filter.

Calculate the power spectral densities and auto correlation functions of the outputs of those filters *theoretically* if the input is white Gaussian noise, and plot these curves. In the first case, do the calculations exact. In the second case, approximate the filter as an ideal filter.

Write functions that estimate ACFs and PSDs. Based on that, estimate the auto correlation function and the power spectral density of the output of your two filters. The length (number of samples) of your input signal should be 2^{16}.

Plot your estimations, and compare them to the theoretical results.

The Report

Your report should contain *at least* the following:

- An overview of the theoretical background of this study.
- A theoretical analysis of the situation, resulting in graphs of the ACF and PSD of the outputs of the filters.
- Graphs of raw estimated ACFs and PSDs of the outputs of the filters.
- A comparison of the theoretical and estimated graphs with comments about the suitability of the different estimation methods that you have used.

All ACFs should be plotted as line-plots using `plot` for all indices k that you have available and with `stem` for $|k| \leq 20$. All PSDs should be plotted as line-plots in linear scale using `plot` for normalized frequencies θ in the interval $0 \leq \theta < 1$. Make sure that the frequency axis is graded for normalized frequency.

4.2 Study 2 – Improving Estimates

Modelling Signals

The raw estimated spectra that you got using periodograms in Study 1 are very noisy. In this study, you are supposed to improve on those estimates using averaged spectra and smoothing.

Preparations

As for Study 1, do this in advance.

- Make sure that you have not lost the functions you wrote during Study 1.
- Read up on improved estimates of PSDs. See Section 10.3 in the course book.
- Outline program functions that perform those improved estimates. Those should reasonably use the functions you wrote in Study 1.

Tasks

Here, you use your filters from study 1, i.e. the following:

- Your low degree low-pass filter.
- Your high degree low-pass filter.

Write functions that provide improved estimates of ACFs and PSDs. Based on that, estimate the auto correlation function and the power spectral density of the output of your two filters.

Plot your improved estimations, and compare them to the theoretical results from Study 1. Vary parameters in those improvement methods. Can you identify any trade-offs?

The Report

Your report should contain *at least* the following:

- Graphs of improved estimations of ACFs and PSDs of the outputs of the filters using both averaging of periodograms and smoothing, together with specifications of parameters used. Try out a few different sets of parameters.
- A comparison of the theoretical and estimated graphs with comments about the suitability of the different estimation methods that you have used.

All ACFs should be plotted as line-plots using `plot` for all indices k that you have available and with `stem` for $|k| \leq 20$. All PSDs should be plotted as line-plots in linear scale using `plot` for normalized frequencies θ in the interval $0 \leq \theta < 1$. Make sure that the frequency axis is graded for normalized frequency.

4.3 Study 3 – Non-LTI-systems

Non-LTI-systems are in general hard to analyze, and we only deal with fairly simple such systems in this course. Refer to the course book for details.

Preparations

As before, do this in advance.

- Make sure that you have not lost the functions you wrote during Study 1.
- Read up on nonlinearities.
- Read up on amplitude modulation of stochastic processes.
- Do the theoretical analysis of the three situations below, assuming that the input is SSS.

Tasks

Create low-pass-filtered noise in the same way as you did in Studies 1 and 2, using a low-pass filter of high degree. As before, you can approximate the filter as an ideal filter, which makes a theoretical comparison fairly simple. Use this filtered noise as input to the following systems:

- Squarer, $Y[n] = X^2[n]$.
- Half-wave rectifier, $Y[n] = \begin{cases} X[n], & n: \ X[n] > 0, \\ 0, & n: \ X[n] \leq 0. \end{cases}$
- AM-SC modulator, $Y[n] = X[n]\cos(\Omega_0 n)$. AM-SC is short for Amplitude Modulation - Suppressed Carrier.

It is important that the power spectral density of the input is such that it can be used to demonstrate non-LTI-properties of the systems. Therefore, use a well chosen normalized cut-off frequency. Also, in the AM case, use a well chosen normalized carrier frequency, taking the normalized cut-off frequency into account. Specifically, do *not* use 1/2 as the normalized carrier frequency. Find phenomena, if possible, *both* in the periodogram and the amplitude distribution of the output that demonstrates that the systems are non-LTI. The power spectral densities should be estimated using your functions from Study 1 (sic). The amplitude distribution can be studied using the histogram of the output.

Account for expected (theoretical) results and compare them with your measurements and estimations. Do not forget that a process has to be stationary – at least in the wide sense – for the power spectral density to be well defined. Check if your outputs are stationary in the wide sense, and if they are not: Adjust them so that they are stationary in the wide sense, at least in the theoretical analysis.

The Report

Your report should contain *at least* the following:

- An overview of the theoretical background of this study.
- A theoretical analysis of the three situations, resulting in graphs of the PSD of the outputs of the three systems.
- Graphs of raw estimated PSDs of the outputs of the systems.
- Histograms of the outputs of the systems.
- A comparison of the theoretical and estimated graphs.

All PSDs should be plotted as line-plots in linear scale using `plot` for normalized frequencies θ in the interval $0 \leq \theta < 1$. Make sure that the frequency axis is graded for normalized frequency. You should also identify interesting parts of the spectra and make separate plots of those parts, reasonably scaled.

4.4 Study 4 – Special Operations

Signal processing is an area where many different methods to manipulate signals are used. Here you are supposed to study a few simple manipulations.

Preparations

And again, do this in advance.

- Make sure that you have not lost the functions you wrote during Study 1.
- Do the theoretical analysis of the two situations below. Are the outputs WSS? If not, make them WSS by adjusting the situation. You can find inspiration in the analysis of AM in the course book.

Tasks

Investigate what happens with the power spectral density when

- a sequence is multiplied by alternating $+1$ and -1. In other words, you construct a new sequence $Y[n] = X[n] \cdot (-1)^n$, where $X[n]$ is the input.
- a sequence is decimated, by which we mean that every second sample is replaced by zero. This can be implemented using a multiplication by the sequence $\ldots, 0, 1, 0, 1, 0, 1, \ldots$

Create low-pass-filtered noise in the same way as you did in Studies 1 and 2, using a low-pass filter of high degree, again using a well chosen normalized cut-off frequency. As before, you can approximate the filter as an ideal filter, which makes a theoretical comparison fairly simple. Use this filtered noise as input to the two systems. As before, compare theory and practical results. Make sure that the outputs are WSS at least, so that the power spectral densities exist. Among other things, express the power spectral density of the output in that of the input.

The Report

Your report should contain *at least* the following:

- A theoretical analysis of the two situations, resulting in graphs of the PSDs of the outputs of the two systems.
- Graphs of raw estimated PSDs of the outputs of the systems.
- A comparison of the theoretical and estimated graphs.

All PSDs should be plotted as line-plots in linear scale using `plot` for normalized frequencies θ in the interval $0 \leq \theta < 1$. Make sure that the frequency axis is graded for normalized frequency.