LINKÖPINGS UNIVERSITET

Exam in TSDT14 Signal Theory

Exam code:	TEN1
Date:	2019-08-28 **Time:** 8.00–12.00
Place:	TER4
Teacher:	Mikael Olofsson, tel: 281343
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* A German 10-Mark note of the fourth series (1991-2001). Pocket calculators of all kinds with empty memory.
Number of tasks:	6
Grading:	Task one is not graded in terms of points. At least two of its three sub-tasks have to be treated correctly as partial fulfillment to pass the exam. Tasks 2-6 yield at most 5 points each. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unresonable answers. Totally, you can get 25 points at most. For grade three you need 10 points, for grade four 15 points and for grade five 20 points.
Solutions:	Will be published in the course room no later than three days after the exam, under Previous exams.
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	In the student office of ISY, Building B, between entrances 27-29, ground floor, Corridor D, right next to Café Java, starting about two weeks after the exam.

1 These sub-tasks have to be treated correctly as partial fulfillment to pass the exam:

a. Define the concept *time-continuous stochastic process.*

b. A time-discrete filter has frequency response

$$H[\theta] = 1 - e^{-j2\pi\theta}.$$

The input $X[n]$ of this filter has PSD $R_X[\theta] = R_0$. Determine the PSD of the output.

c. A time-discrete Gaussian process, $X[n]$, with mean zero and ACF

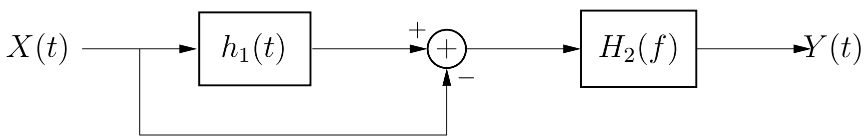
$$r_X[k] = \left(\frac{1}{3}\right)^{|k|}$$

is squared. Determine the ACF of the resulting process.

2 Let $X(t)$ in the figure below be given by

$$X(t) = A + B(t),$$

where A is a Gaussian variable with mean $m_A = 1$ and variance $\sigma_A^2 = 2$, and where $B(t)$ is a Gaussian WSS process with mean zero and ACF $r_B(\tau) = R_0 \operatorname{sinc}(20\tau)$.



The first filter has impulse response

$$h_1(t) = \delta(t) + \delta(t-1),$$

where $\delta(t)$ as usual is the time-continuous unit impulse, and the second filter has frequency response

$$H_2(f) = \operatorname{rect}(f/10).$$

Under the assumption that A and $B(t)$ are independent for all time instances t, determine R_0 such that the power of $Y(t)$ is 2/3 of the power of $X(t)$.

3 Let the time-discrete signal $X[n]$ be a sequence of independent, identically distributed samples with mean zero and variance σ_X^2. This signal is uniformly quantized by a quantizer with N quantization steps of height Δ and with saturation levels $\pm A$. For the input, we have $\Delta \ll \sigma_X$. The probability density function $f_X(x)$ is continuous in the interval $(-A, A)$ and zero outside that interval. Finally, we define the quantization noise

$$Q[n] = Y[n] - X[n],$$

where $Y[n]$ is the output of the quantizer.

a. Determine $\mathrm{E}\left\{Q^2[n]\right\}$. (2 p)

b. Determine $\Pr\{|Q[n]| > \Delta/2\}$. (1 p)

c. Determine the PSD of the quantization noise. (2 p)

4 A time-continuous stochastic process with power-spectral density

$$R_X(f) = \begin{cases} R_0, & |f| < B, \\ 0, & \text{elsewhere} \end{cases}$$

is sampled with sampling frequency $f_\mathrm{s} = 4B$. The sampled signal is filtered by a time-discrete filter with frequency response $H[\theta]$ for which we have

$$|H[\theta]|^2 = \begin{cases} 1 - 2|\theta|, & |\theta| < \frac{1}{2}, \\ |H[\theta + m]|^2, & m \text{ integer.} \end{cases}$$

Determine the quadratic mean of the output.

5 Determine if the following functions are valid ACFs for WSS processes. (1 p each)

a. $r(\tau) = \mathrm{rect}(\tau)$
b. $r(\tau) = \mathrm{triangle}(\tau)$
c. $r(\tau) = \mathrm{rect}(\tau) + \mathrm{triangle}(\tau)$
d. $r(\tau) = \delta(\tau + 1)$
e. $r(\tau) = 3\,\mathrm{triangle}(\tau) - 2\,\mathrm{triangle}(\tau/2)$

6 The stochastic process $X(t)$ has mean zero and ACF $r_X(\tau) = \mathrm{sinc}(10\tau)$. It is modulated using AM-SC as

$$Y(t) = 10X(t)\cos(2000\pi t + \Psi),$$

where Ψ is uniformly distributed over $[0, 2\pi)$ and independent of $X(t)$. The resulting signal $Y(t)$ is demodulated coherently as

$$Z(t) = 2Y(t)\cos(2000\pi t + \Psi),$$

and finally that signal is filtered using an ideal filter with frequency response

$$H(f) = \begin{cases} 1, & |f| < 20, \\ 0, & \text{elsewhere.} \end{cases}$$

Let the output of that filter be denoted $W(t)$.

Determine and draw the power spectral densities of all involved signals.

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $r_X(\tau) = \mathrm{E}\{X(t+\tau)X^*(t)\}$

$$= r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$$

PSD: $R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R^*_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c.

Passband: $s(t) = \mathrm{Re}\Big\{\tilde{s}(t)e^{j2\pi f_\mathrm{c} t}\Big\} = s_\mathrm{I}(t)\cos(2\pi f_\mathrm{c} t) - s_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c} t)$

Baseband: $\tilde{s}(t) = s_\mathrm{I}(t) + js_\mathrm{Q}(t)$

Spectrum: $S(f) = \frac{1}{2}\big(\tilde{S}(f-f_\mathrm{c}) + \tilde{S}^*(-f-f_\mathrm{c})\big)$, $\tilde{S}(f) = 2S(f+f_\mathrm{c})u(f+f_\mathrm{c})$

Filtering: $y(t) = (x*h)(t) \quad \Leftrightarrow \quad \tilde{y}(t) = \frac{1}{2}(\tilde{x}*\tilde{h})(t)$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c.

Passband: $S(t) = S_\mathrm{I}(t)\cos(2\pi f_\mathrm{c} t + \Psi) - S_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c} t + \Psi)$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $\tilde{S}(t) = S_\mathrm{I}(t) + jS_\mathrm{Q}(t)$

Mean: $m_S = 0$

ACF: $r_S(\tau) = \frac{r_{S_\mathrm{I}}(\tau) + r_{S_\mathrm{Q}}(\tau)}{2}\cos(2\pi f_\mathrm{c}\tau) - \frac{r_{S_\mathrm{I},S_\mathrm{Q}}(-\tau) - r_{S_\mathrm{I},S_\mathrm{Q}}(\tau)}{2}\sin(2\pi f_\mathrm{c}\tau)$

PSDs: $R_S(f) = \frac{1}{4}\big(R_{\tilde{S}}(f-f_\mathrm{c}) + R_{\tilde{S}}(-f-f_\mathrm{c})\big)$, $R_{\tilde{S}}(f) = 4R_S(f+f_\mathrm{c})u(f+f_\mathrm{c})$