LINKÖPINGS UNIVERSITET

Exam in TSDT14 Signal Theory

Exam code:	TEN1
Date:	2018-08-29 **Time:** 8.00–12.00
Place:	TER2
Teacher:	Mikael Olofsson, tel: 281343
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* A German 10-Mark note of the fourth series (1991-2001). Pocket calculators of all kinds with empty memory.
Number of tasks:	6
Grading:	Task one is not graded in terms of points. At least two of its three sub-tasks have to be treated correctly as partial fulfillment to pass the exam. Tasks 2-6 yield at most 5 points each. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unresonable answers. Totally, you can get 25 points at most. For grade three you need 10 points, for grade four 15 points and for grade five 20 points.
Solutions:	Will be published no later than three days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/tentor?TSDT14
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	In the student office of ISY, Building B, between entrances 27-29, ground floor, Corridor D, right next to Café Java, starting about two weeks after the exam.

1 At least two of the following three sub-tasks have to be treated correctly as partial fulfillment to pass the exam:

a. Define the concept *white noise*.

b. The time-discrete signal $X[n]$ has PSD

$$R_X[\theta] = 1 + \cos(2\pi\theta).$$

This signal is pulse-amplitude modulated using the pulse $p(t)$, with spectrum

$$P(f) = \begin{cases} j \cdot \sin(2\pi f/10), & |f| < 5 \text{ Hz}, \\ 0, & \text{elsewhere.} \end{cases}$$

The sampling frequency is 5 Hz. Sketch the PSD of the output. Do not forget to grade the axes.

c. The process $Y(t) = X^3(t)$ is created from a time-continuous Gaussian process, $X[n]$, with mean zero and ACF

$$r_X(\tau) = e^{-|\tau|}$$

Determine the ACF of the resulting process.

2 $X[n]$ is a stochastic signal with ACF $r_X[k] = (0.5)^{|k|}$. By weighted addition of successive values of $X[n]$, a new sinal is created:

$$Y[n] = \frac{1}{4}(X[n+1] + 2X[n] + X[n-1]).$$

Determine the ACF and PSD of $Y[n]$. Also determine the power of $Y[n]$. (5 p)

3 Consider the deterministic periodic signal

$$x(t) = \begin{cases} 1, & 0 < t \leq T/2, \\ 0, & T/2 < t \leq T, \end{cases}$$

repeated with period T.

Now, form a stochastic signal $Y(t) = x(t + \Psi)$, where Ψ is a stochastic variable that is uniformly distributed on $[0, T)$.

a. Determine if $Y(t)$ is WSS. (2 p)

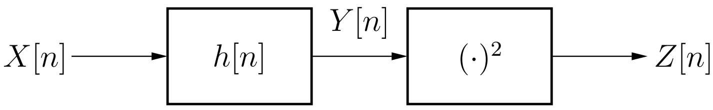
b. Determine if $Y(t)$ is ergodic with respect to its mean. (3 p)

4 The time-discrete stochastic signal $X[n]$ is white noise with mean $m_X = 0$ and PSD $R_X[\theta] = 1$. After filtering $X[n]$ with an LTI filter having impulse response $h[n] = \delta[n] + \delta[n-1]$, we get the signal $Y[n]$. The signal $Z[n]$ is the output from a squarer with input $Y[n]$, i.e. we have $Z[n] = Y^2[n]$. Determine the ACF $r_Z[k]$ if

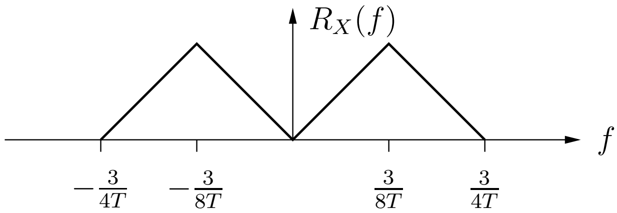
a. $X[n]$ is Gaussian; (2 p)

b. $X[n]$ is binary distributed, taking values ± 1. (3 p)

The system is illustrated below.



5 $X(t)$ is a stochastic process with PSD $R_X(f)$ according to the figure below.



$X(t)$ is sampled with sampling frequency $f_s = 1/T$, and then ideally reconstructed using the pulse $p(t)$ with Fourier transform

$$P(f) = \begin{cases} T, & |f| < \frac{1}{2T} \\ 0, & \text{elsewhere} \end{cases}$$

Let $Z(t)$ be the resulting signal.

a) Determine and draw the PSD of $Z(t)$. (3 p)

b) Determine the smallest power $\varepsilon^2 = E\left\{(X(t) - Z(t))^2\right\}$ of the error if $X(t)$ is bandlimited before the sampling, such that the sampling theorem is fulfilled. (2 p)

6 Show that a two-dimensional signal is separable if and only if its two-dimensional Fourier transform is separable.

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $r_X(\tau) = \mathrm{E}\{X(t+\tau)X^*(t)\}$

$$= r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$$

PSD: $R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_{c}.

Passband: $s(t) = \mathrm{Re}\Big\{\tilde{s}(t)e^{j2\pi f_{\mathrm{c}}t}\Big\} = s_{\mathrm{I}}(t)\cos(2\pi f_{\mathrm{c}}t) - s_{\mathrm{Q}}(t)\sin(2\pi f_{\mathrm{c}}t)$

Baseband: $\tilde{s}(t) = s_{\mathrm{I}}(t) + js_{\mathrm{Q}}(t)$

Spectrum: $S(f) = \frac{1}{2}\big(\tilde{S}(f-f_{\mathrm{c}}) + \tilde{S}^*(-f-f_{\mathrm{c}})\big)$, $\quad \tilde{S}(f) = 2S(f+f_{\mathrm{c}})u(f+f_{\mathrm{c}})$

Filtering: $y(t) = (x * h)(t) \quad \Leftrightarrow \quad \tilde{y}(t) = \frac{1}{2}(\tilde{x} * \tilde{h})(t)$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_{c}.

Passband: $S(t) = S_{\mathrm{I}}(t)\cos(2\pi f_{\mathrm{c}}t + \Psi) - S_{\mathrm{Q}}(t)\sin(2\pi f_{\mathrm{c}}t + \Psi)$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $\tilde{S}(t) = S_{\mathrm{I}}(t) + jS_{\mathrm{Q}}(t)$

Mean: $m_S = 0$

ACF: $r_S(\tau) = \frac{r_{S_{\mathrm{I}}}(\tau) + r_{S_{\mathrm{Q}}}(\tau)}{2}\cos(2\pi f_{\mathrm{c}}\tau) - \frac{r_{S_{\mathrm{I}},S_{\mathrm{Q}}}(-\tau) - r_{S_{\mathrm{I}},S_{\mathrm{Q}}}(\tau)}{2}\sin(2\pi f_{\mathrm{c}}\tau)$

PSDs: $R_S(f) = \frac{1}{4}\big(R_{\tilde{S}}(f-f_{\mathrm{c}}) + R_{\tilde{S}}(-f-f_{\mathrm{c}})\big)$, $\quad R_{\tilde{S}}(f) = 4R_S(f+f_{\mathrm{c}})u(f+f_{\mathrm{c}})$